

# Pro-Consumer Price Ceilings under Regulatory Uncertainty\*

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## Abstract

We examine optimal price ceilings when the regulator is uncertain about demand and maximizes expected consumer surplus. With perfect competition, large enough regulatory uncertainty calls for softer intervention, with the price ceiling set at a relatively high level compared to a full information scenario. In an imperfectly competitive setting where symmetric firms compete in supply functions, with large enough uncertainty, the optimal ceiling increases in the degree of competition, so greater competitive pressure justifies less restrictive regulation. Under perfect competition, we also pin down a cut-off level of rationing efficiency below which a price ceiling should not be used.

**Keywords:** price regulation, consumer surplus, uncertainty

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# 1 Introduction

Although controversial, price ceilings have been used in a wide range of markets, including those for rental accommodation, gas, electricity, telecommunications, pay-day lending, and basic consumption goods. One rationale for price ceiling regulation is to correct inefficiency stemming from insufficient competition, while another is the protection of consumers. The latter may explain the use of price controls in nearly competitive markets, including those for rental accommodation in developed economies (see Glaeser and Luttmer, 1997 and 2003, on US rent controls) and those for basic food products in developing economies (see, Mbaye et al., 2015, on price caps in Senegal).<sup>1</sup>

Price ceilings have been used extensively in utility markets, where competition is typically restricted, to secure low prices for consumers. Sappington and Weisman (2010) discuss their use in telecommunications, while Davis and Kilian (2011) examine empirically price caps in the US residential market for natural gas. A survey of regulators in transition and developing economies by Kirkpatrick et al. (2005) found that price controls had been used in 40% of the countries, most commonly in telecommunications (76%).

To correct market inefficiency, price regulation would aim to maximize total welfare. But, recent years have witnessed an increase in the focus of policy makers and regulators on consumer welfare and greater involvement of consumer groups in policy debate.<sup>2</sup> Following the 2002 Enterprise Act, approved UK bodies were designated ‘super-complainants’ at the Office of Fair Trading to ‘strengthen the voice of consumers’ and protect their interests.<sup>3</sup>

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<sup>1</sup>Other developing economies with price caps include Kenya and Argentina. (See ‘Kenya Enacts Price Control Law’ on 16/09/2011 and ‘Argentine Peso Hits New Lows as Food Price Controls Take Effect’ on 7/04/2014, both on Reuters).

<sup>2</sup>Empirical evidence suggests that lobby groups influence regulation, with small supplier groups being more effective than consumers (a large but fragmented group). See Viscusi et al. (2005, Ch.10).

<sup>3</sup>See Office of Fair Trading 511 (2003). Consumer organization *Which?* - one of the first ‘super-complainants’ - played a part in shaping tariff regulation in the UK energy sector in 2014. Consumers’ changed role may relate to utility market deregulation, increased market complexity (e.g., banking and financial services), and growing evidence on demand-side biases.

Moreover, in markets where the suppliers are foreign-owned, consumer surplus may be an appropriate measure of domestic welfare. So the realities of regulation are more complex and call for a better understanding of a consumer surplus standard in economic analysis.

This paper explores how market conditions affect the level of a price ceiling that maximizes expected consumer surplus in a setting where the regulator is uncertain about demand. Our framework allows for varying degrees of competition, so it fits a gamut of market structures. We first investigate the impact of regulatory uncertainty on the optimal ceiling for perfect competition and then the relationship between competitive pressure and the optimal level of intervention. We also examine the effect of arbitrary levels of rationing inefficiency on the optimal ceiling.

In our model, private agents have all relevant information. So, uncertainty is entirely on the part of the regulator, who is aware of this disadvantage.<sup>4</sup> This information structure also covers the possibility that the price ceiling is a long term regulatory decision, while short run market conditions may change. We focus on a frictionless homogeneous product market where trade occurs at a uniform price. If the price ceiling binds, it lowers the cost of purchase to consumers, but results in a shortage. For tractability, we assume quadratic cost and benefit functions, but we argue that the qualitative results are more general.

A preliminary analysis of perfect competition with efficient rationing shows that with no uncertainty the ceiling that maximizes consumer surplus always binds (i.e., it is strictly lower than the equilibrium price). Consider a price ceiling that is marginally below the market clearing price: this has a negative effect on consumer surplus as some consumers are excluded from the market and a positive one as those who still purchase pay a lower price. As the marginal consumers excluded from the market have the lowest valuations, the negative effect is small and completely offset by the surplus gains of all those who still

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<sup>4</sup>For example, a regulator may be uncertain of the demand for new products.

purchase. The optimal ceiling balances the gains and losses. We further show that this result is robust for relatively low levels of demand uncertainty.

We next introduce arbitrary levels of uncertainty, and characterize the optimal price ceiling.<sup>5</sup> We show that sufficient regulatory uncertainty calls for a price ceiling set at a relatively high level and thus for softer intervention, compared to the no uncertainty case. At this level the ceiling may not bind ex post, and so may not benefit consumers. However, it offers protection against potentially high free market prices. To see the intuition, suppose the optimal price ceiling under no uncertainty were set. If the realized demand is low enough, the market clearing price will be lower than this ceiling and the intervention will not bind. Then a marginal increase in the price ceiling would not affect consumer surplus. In contrast, if the realized demand is large enough, this ceiling binds and a marginal increase brings it closer to the (higher) level that is optimal for that particular high demand, and so increases consumer surplus.

Our perfect competition model is relevant for interventions in fragmented markets, like rent controls or price caps on basic consumption goods. To analyze optimal ceilings in concentrated markets (e.g., telecommunications or utility markets), we employ an imperfectly competitive framework, which can be interpreted as the reduced form of a model where identical firms compete in linear supply functions. We draw on the fact that in this setting aggregate supply in equilibrium is a fraction of the perfectly competitive one, which allows us to explore varying degrees of market power. The reduced form model can also represent a monopoly, so our results apply to this limiting case.

If there is no regulatory uncertainty the optimal ceiling is the same for any degree of

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<sup>5</sup>Although we focus on regulatory uncertainty regarding demand in the main text, in an online appendix we show that qualitatively similar results obtain in a model with supply uncertainty only (e.g., the regulator may be uncertain of what innovations are in the pipeline). We also outline in this appendix the corresponding model with two-sided (demand and supply) uncertainty and explore its solution numerically.

competition. As this ceiling lies below the perfectly competitive equilibrium price, it is also below the (higher) imperfect competition equilibrium prices. But, with sufficient regulatory uncertainty, we show that the appropriate level of the ceiling depends on the degree of competition, as this determines the unregulated market price. The more concentrated the market, the greater is the expected consumer surplus loss if there is no regulation. This creates a stronger incentive to lower the price ceiling, triggering a positive relationship between the optimal ceiling and the degree of competition. Thus, with enough uncertainty, a higher price ceiling - that is, less restrictive intervention - is justified in an environment with greater competitive pressure.

We also explore the impact of rationing inefficiency on the optimal price ceiling under perfect competition.<sup>6</sup> We determine a cut-off level of rationing efficiency below which a price ceiling should not be used, and show that our previous findings are qualitatively robust when rationing efficiency exceeds this level. Rationing is fully efficient when the goods supplied are allocated to the consumers with the highest valuations amongst those willing to purchase. At a binding ceiling all consumers who buy the product enjoy a higher surplus than in the absence of regulation. But with inefficient rationing some of these consumers displace others who value the product more. If rationing is sufficiently inefficient the loss in consumer surplus from such misallocations, together with lower supply, fully offsets the benefits for the consumers (who buy) at the lower price.

A vast economics literature analyzes the rationales, design, and impact of price regulation (see Armstrong and Sappington, 2007, for a comprehensive review). In this context, our information structure was first formalized by Weitzman (1974) to analyze the choice of regulatory instrument and subsequently used in studies of optimal regulation (e.g., Baron and Myerson, 1982, and Lewis and Sappington, 1988) that consider more sophisticated

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<sup>6</sup>In our setting, the regulator cannot affect the extent of rationing efficiency in the market.

mechanisms. Lewis and Sappington (1988) show that in a monopoly market, a regulator facing demand uncertainty could implement the first best solution by using a menu of price-transfer pairs, when marginal cost is increasing. In contrast to these studies, we focus on a price ceiling, a simple but commonly used instrument, which is particularly relevant in markets where the intervention is fixed over a period in which demand may fluctuate.<sup>7</sup>

Recent work has examined price cap regulation in oligopoly markets with homogeneous products, where both the firms and the regulator face demand uncertainty. Earle et al. (2007) point out that an increase in a price ceiling near marginal cost may raise welfare. Grimm and Zöttl (2010) show that there is always a range of ceilings that increase production and welfare from the unregulated market levels and, for non-degenerate uncertainty, this range is strictly above marginal cost.<sup>8</sup> In contrast to their analyses, ours focuses on a consumer surplus standard with only the regulator facing uncertainty.

Consumer welfare has sometimes been used in regulation in conjunction with a welfare standard (e.g., for merger policy in Europe and the US). The consumer surplus standard has been regarded as a way to redress the regulatory and policy making balance in favour of consumers, a less represented group. Besanko and Spulber (1993) show that this standard may offset some of the firms' informational advantage over the regulator.<sup>9</sup> Nevertheless, to be sure, a consumer surplus standard has various caveats, including adverse dynamic consequences on firms' innovation and investment incentives and their long run survival.<sup>10</sup>

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<sup>7</sup>Furthermore, some of the instruments required for optimal regulation (e.g., transfers) may not be available in practice. See Armstrong and Sappington (2007) for discussion of practical regulatory policies and Crew and Kleindorfer (2002) for more on the limitations of optimal regulation models.

<sup>8</sup>In related models, Reynolds and Rietzke (2016) consider endogenous entry and identify conditions under which a price cap is welfare improving, while Lemus and Moreno (2017) explore the optimal ceiling when a monopolist pre-commits to a capacity level before the demand is realized.

<sup>9</sup>Neven and Röller (2005) analyze how institutional settings, e.g. transparency and accountability, interact with the choice of an appropriate standard.

<sup>10</sup>Moreover, to the extent to which consumers own the firms (e.g., through pension funds), this standard may also be detrimental to consumers. See Motta (2004).

Our work is also related to an emerging literature that explores inefficient rationing under price regulation. Bulow and Klemperer (2012) examine the impact of price regulation on consumers in a model with general demand and supply and random rationing, but no uncertainty. They show that if supply is locally more elastic than demand, a price ceiling always hurts consumers when demand is convex. By considering arbitrary levels of rationing inefficiency and regulatory uncertainty, our findings complement theirs. Glaeser and Luttmer (1997, 2003) and Davis and Kilian (2011) analyze empirically price regulation and the welfare loss associated with inefficient rationing in competitive markets.<sup>11</sup> We contribute to this literature by exploring the interplay between rationing inefficiency and regulatory uncertainty. By parameterizing rationing efficiency, we propose a flexible theoretical framework that allows for a range of outcomes and extends previous work.

We explore how market characteristics affect a pro-consumer price ceiling in a static framework. However, the introduction of a price ceiling is likely to have long term consequences, most notably by affecting firms' investment incentives. Dobbs (2004) finds that under uncertainty regarding technological change, a price cap reduces a monopolist's incentive to invest in capacity expansion. Also, Biglaiser and Riordan (2000) show that dynamic price ceiling adjustments can distort incentives for capital replacement. Nonetheless, as argued by Joskow (2014), a price ceiling gives a strong incentive for cost reduction, although consumers could only benefit later, when the ceiling is revised.<sup>12</sup> A fuller analysis of pro-consumer price ceilings would need to investigate the dynamic effects, of both the instrument and the consumer-surplus standard, and to assess the relative performance of different regulatory interventions in this context.

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<sup>11</sup>Focusing on misallocations due to rent control, Glaeser and Luttmer show that under conservative assumptions, 20% of rented apartments in New York City were in the wrong hands. Davis and Kilian (2011) find substantial allocative costs in the US residential market for natural gas over the 1950-2000 period.

<sup>12</sup>Price ceilings also have dynamic effects in the presence of search frictions. For related work, see Fershtman and Fishman (1994), Armstrong et al. (2009) and Rauh (2004).

After formulating the model, we present a preliminary analysis of perfect competition with little or no regulatory uncertainty. Section 4 introduces arbitrary levels of demand uncertainty and section 5 analyzes the imperfect competition case. Section 6 explores the implications of rationing inefficiency. Section 7 presents conclusions. All proofs missing from the text are relegated to appendices.

## 2 The Model

Consider a market in which the regulator may be uncertain about the demand for a homogeneous product. There is no uncertainty on the part of private agents. On this basis, the regulator chooses a price ceiling that maximizes expected consumer surplus. The regulatory intervention is announced to all private agents (producers and consumers).

First we consider a perfectly competitive market. Let consumers' gross benefit (or utility) from the consumption of  $q$  units of the product be given by  $\mathfrak{B}(q, \eta) = (B + \eta)q - bq^2/2$  where  $b > 0$ ,  $B + \eta > 0$ . From the regulator's point of view  $\eta$  is a random variable with zero mean ( $E(\eta) = 0$ ) distributed according to a twice continuous and differentiable c.d.f.  $\mathcal{F}(\eta)$  defined on a closed interval  $[n_{\min}, n_{\max}]$ . We assume that the hazard rate  $\mathcal{F}'(\eta)/(1 - \mathcal{F}(\eta))$  is strictly increasing. The suppliers' cost of producing  $q$  units is given by  $\mathfrak{C}(q) = Cq + cq^2/2$  where  $c > 0$ ,  $C > 0$ . Then, inverse demand and supply are, respectively,  $P_d(q, \eta) = \partial\mathfrak{B}(q, \eta)/\partial q = B + \eta - bq$  and  $P_s(q) = \partial\mathfrak{C}(q)/\partial q = C + cq$ .

Writing  $p$  for unit price, it follows that direct demand and supply are given, respectively, by

$$q^d(p) = \frac{B + \eta - p}{b} \text{ and } q^s(p) = \frac{p - C}{c} . \quad (1)$$

We assume that consumers and producers can observe  $\eta$ , while the regulator cannot. This captures the informational advantage that producers may have over the regulator



regarding demand.<sup>13</sup>

Denote by  $p^*$  the ex post market clearing price (where  $q^d(p^*) = q^s(p^*)$ ) and by  $q^*$  the corresponding output level. Then,

$$p^* = \frac{c(B + \eta) + bC}{b + c} \text{ and } q^* = \frac{B + \eta - C}{b + c}. \quad (2)$$

We assume that  $B + \eta > C$  for any  $\eta$ . This guarantees a well defined equilibrium output ex post. Note that ex ante (before the demand shock  $\eta$  is realized), the regulator views  $p^*(\eta)$  as a random variable with expected value

$$p_e^* = \frac{cB + bC}{b + c}. \quad (3)$$

We explore regulatory intervention that takes the form of a price ceiling  $\bar{p}$ , assuming that resale of rationed goods is not possible. A price ceiling stipulates a maximal trade price and only binds if the unregulated market price lies above the regulated level. If it lies at or below the ceiling, the outcome coincides with the unregulated market equilibrium; that is, for a given price ceiling  $\bar{p}$ , if  $p^* \leq \bar{p}$ , then  $q(p) = q^*$  - as given by (2) - and if  $p^* > \bar{p}$ , then  $q(p) = q^s(\bar{p})$  - as given by (1). The c.d.f. of  $p^*(\eta)$  is determined by the c.d.f. of  $\eta$  ( $\mathcal{F}(\eta)$ ). Since  $\eta$  is defined on a closed interval, so is the c.d.f. of  $p^*(\eta)$ .

We then examine price ceilings in a model of imperfect competition, maintaining our assumptions regarding the demand side of the market and the quadratic cost presented above. We assume that there are  $N$  identical suppliers and each firm  $i$ 's cost of producing  $q_i$  units is given by  $\mathfrak{C}(q) = Cq_i + Ncq_i^2/2$ .<sup>14</sup> We interpret our model as one where the firms compete by choosing linear supply functions. Building on Klemperer and Meyer (1989),

<sup>13</sup> $\eta$  may be unknown when the regulator sets a price ceiling, but revealed before the producers make supply decisions; or producers may be able to adjust their behavior as information about  $\eta$  is revealed by the market.

<sup>14</sup>This individual cost function guarantees that when the total cost of producing  $q$  using  $N$ -plants in the industry is minimised, total cost is the same as under perfect competition ( $\mathfrak{C}(q) = Cq + cq^2/2$ ). Then, the marginal cost curve coincides with  $P^s(q)$ .

each firm chooses a supply function  $S_i(p-C) = d_i(p-C)$ , where the slope  $d_i > 0$ .<sup>15</sup> In the linear supply function equilibrium, a firm's supply is only a fraction of its supply in the perfect competition model (i.e., where the firms are price takers).<sup>16</sup> Thus, with imperfect competition, the firms restrict their output compared to that in the perfectly competitive market. It then follows that the aggregate quantity supplied in the symmetric supply function equilibrium at price  $p$  can be written as

$$q^s(p, \delta) = \delta \frac{p - C}{c},$$

where  $\delta < 1$  captures the restriction in output below the competitive level. Note that  $\delta$  increases in the degree of competition, captured by the number of firms,  $N$ . As the market becomes nearly competitive ( $N \rightarrow \infty$ ),  $\delta$  converges to 1. Alternatively,  $q^s(p, \delta)$  can be regarded as an ad hoc way of capturing the restriction in aggregate supply in a market where the firms have market power.

Using  $q^s(p, \delta)$ , we can derive the equilibrium outcome in the unregulated imperfectly competitive market. Denote by  $p^\delta$  the ex post unregulated market price (where  $q^d(p^\delta) = q^s(p^\delta, \delta)$ ) and by  $q^\delta$  the corresponding output level. Then,

$$p^\delta = \frac{c(B + \eta) + \delta b C}{\delta b + c} \text{ and } q^\delta = \frac{\delta (B + \eta - C)}{\delta b + c}.$$

As in the competitive framework, the regulator views the unregulated market price  $p^\delta(\eta)$  with imperfect competition as a random variable.

In general, price ceiling regulation may result in excess demand, in which case the scarce output will be rationed. However, if there is some degree of rationing inefficiency,

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<sup>15</sup>Klemperer and Meyer (1989) show that with unbounded demand uncertainty and a symmetric industry there is a unique equilibrium where firms choose linear supply functions of the form we consider. This result holds for any distribution of uncertainty (even if it degenerates into a mass point), so long as the support is unbounded.

<sup>16</sup>Akgün (2004) employs a related framework. We present a supply function competition microfoundation for our model in the online appendix.

this will limit the scope for pro-consumer price regulation. We therefore adapt our perfect competition model by introducing a parameter that captures rationing inefficiency. Specifically, we write the regulator’s objective function as a linear combination of the expected consumer surplus for efficient and for extremely inefficient rationing. With efficient rationing, the goods supplied are allocated to the consumers with the highest valuations among those willing to buy. With extremely inefficient rationing, the available supply is allocated to the consumers whose valuations are the lowest. Our objective function puts weight  $\alpha$  on expected consumer surplus with efficient rationing and  $1 - \alpha$  on expected consumer surplus with inefficient rationing, where  $\alpha \in [0, 1]$ . This covers fully efficient rationing ( $\alpha = 1$ ), extremely inefficient rationing ( $\alpha = 0$ ) and all intermediate cases, including random rationing which corresponds to  $\alpha = 1/2$ .

In our model, the extent of rationing inefficiency is determined by market conditions and is independent of regulatory uncertainty.<sup>17</sup> For expositional simplicity, our initial analysis assumes efficient rationing. We then allow for arbitrary levels of rationing efficiency and show that the optimal ceiling that obtains under efficient rationing carries over for  $\alpha$  above a critical level which depends on the slopes of demand and supply. This critical level may lie above or below  $\alpha = 1/2$ .<sup>18</sup>

### 3 Preliminary Analysis

This section introduces some of our results in a ‘textbook’ framework of perfect competition and efficient rationing (i.e., we assume that  $\delta = 1$  and  $\alpha = 1$ ). We start with a situation where demand (as well as supply) is deterministic, and show that consumer surplus can be increased from the free market benchmark by setting an appropriate binding

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<sup>17</sup> $\alpha$  may instead also be interpreted as the probability, for the regulator, that rationing will be efficient, if this is independent of demand uncertainty.

<sup>18</sup>Random rationing could be used as an alternative benchmark, but then we would have to cover two cases (with the critical level of efficiency above or below  $1/2$ ), which would complicate the exposition.

price ceiling. We then discuss why this result still holds when there is a limited amount of demand uncertainty. The next section provides a formal analysis, allowing for arbitrary degrees of uncertainty.<sup>19</sup>

Using the model introduced in section 2, let us initially assume that  $\eta = 0$ . Demand becomes  $q^d(p) = (B - p)/b$ , and the equilibrium price in the unregulated market is  $p^* = (cB + bC)/(b + c)$ . For any price ceiling  $\bar{p} \geq p^*$ , the market price is  $p^*$  and consumer surplus is given by  $CS(q^d(p^*)) = b(B - C)^2/2(b + c)^2$ .

However, any price ceiling  $\bar{p} < p^*$  is binding so that output is  $\min[q^d(\bar{p}), q^s(\bar{p})] = q^s(\bar{p}) = (\bar{p} - C)/c$ . With efficient rationing, consumer surplus is

$$CS(q^s(\bar{p})) = Bq^s(\bar{p}) - \frac{1}{2}b(q^s(\bar{p}))^2 - \bar{p}q^s(\bar{p}) = \frac{(\bar{p} - C)[2c(B - \bar{p}) - b(\bar{p} - C)]}{2c^2} \equiv CS_d^L. \quad (4)$$

Using  $CS(q^d(p^*))$  and  $CS_d^L$  we obtain the following result.

**Lemma 1** *With no uncertainty, the price ceiling  $\hat{p}$  that maximizes consumer surplus under perfect competition with efficient rationing is given by*

$$\hat{p} = \frac{cB + (b + c)C}{b + 2c} < p^*. \quad (5)$$

If a price ceiling were set marginally below the market clearing price  $p^*$ , there would be a first-order gain in consumer surplus because the lower price would be paid by all consumers who still buy the good, but a second-order loss of consumer surplus from the marginal reduction in quantity supplied. With further marginal reductions in the price ceiling the gain is made over steadily smaller quantities supplied, while the loss is steadily greater because consumers with higher marginal willingness to pay are excluded. Thus, the optimal pro-consumer price ceiling  $\hat{p}$  is strictly lower than the free market equilibrium price  $p^*$  and balances the trade-off. Panel A of Figure 1 illustrates this trade-off at the

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<sup>19</sup>Under perfect competition, a price ceiling cannot improve (expected) total surplus. However, our analysis focuses on intervention that aims to maximize consumer surplus.

optimal price ceiling  $\hat{p}$ . The consumer surplus gain is captured by the dotted rectangle, and the loss by the dotted triangle.<sup>20</sup>

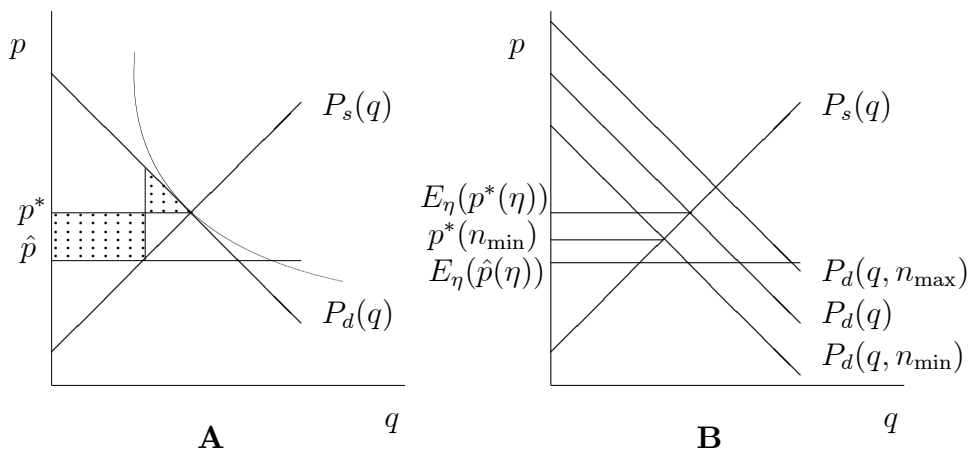


Figure 1: Optimal Pro-Consumer Price Ceiling with Efficient Rationing

Before introducing uncertainty, we briefly discuss how the analysis above would be affected by nonlinearity of demand and supply. It can be seen that, for a given demand-supply intersection and given slopes of demand and supply at this intersection, the same qualitative conclusion holds if there is strict convexity or strict concavity of either curve. However, strict convexity results in a higher optimal ceiling, while strict concavity results in a lower one than in the linear case. Consider the strictly convex demand tangent to the straight line  $P_d(q)$  in Figure 1A at the intersection with  $P_s(q)$ . The consumer surplus loss from the ceiling  $\hat{p}$  is then larger than in the figure, but the gain is the same. Still, there are binding ceilings for which the net gain is positive (e.g., a ceiling marginally below  $p^*$ ). A similar argument holds if supply is strictly convex. Then the loss is the same and the gain is smaller than in the linear case. These conclusions are reversed for strict concavity.

We now introduce a small amount of demand uncertainty. Panel B in Figure 1 illustrates the highest and lowest demand functions,  $P_d(q, n_{\max})$  and  $P_d(q, n_{\min})$ , respectively. In this case,  $P_d(q)$  is the expected demand and captures a situation where the realized

<sup>20</sup>(1) can be written as  $\hat{p} [1 - (b - c) / c\varepsilon_s(\hat{p})] = MR(\hat{p})$ , where  $\varepsilon_s(\hat{p})$  is supply elasticity and  $MR(\hat{p})$  market marginal revenue at price  $p = \hat{p}$ . This gives a parallel with the Lerner index, but with an additional relative slope term.

value of  $\eta$  is equal to the expectation of  $\eta$ ,  $E(\eta) = 0$ . For any realization of  $\eta$ , consumer surplus is maximized at a price ceiling

$$\hat{p}(\eta) = \frac{c(B + \eta) + (b + c)C}{b + 2c} < p^*(\eta) . \quad (6)$$

This follows immediately from replacing  $B$  with  $B + \eta$  in Lemma 1. The expected optimal ceiling is the same as the one in the lemma as  $E(\hat{p}(\eta)) = \hat{p}$ . This is because the objective function only depends on the expectation of  $\eta$ ,  $E(\eta) = 0$ . This simple reasoning is correct so long as  $\hat{p} < p^*(n_{\min})$  - which is the case if there is only relatively little uncertainty. Therefore, our result from the no uncertainty case carries over to a model with relatively little uncertainty, and the price ceiling that maximizes expected consumer surplus binds regardless of the realization of demand.<sup>21</sup> In contrast, if  $\hat{p} > p^*(n_{\min})$ , the analysis will be different: such a ceiling may bind for some realizations of demand but not for others. That is, ex post the market clearing price may be above or below the price ceiling.

## 4 Perfect Competition

This section fully investigates the impact of arbitrary levels of demand uncertainty, focusing on price regulation where the ceiling satisfies  $\bar{p} > C$ , so that supply is positive.

For a given price ceiling  $\bar{p}$ , we define  $n^*(\bar{p})$ , the specific value of  $\eta$  for which the market clears, i.e.  $\bar{p} = p^*(\eta)$ . Using (2), we obtain

$$n^*(\bar{p}) = \frac{(b + c)(\bar{p} - p_e^*)}{c} , \quad (7)$$

where  $p_e^*$ , the expectation of  $p^*(\eta)$ , is defined in (3). Using (7), we identify three price regions where the intervention has different implications. We then examine each of these

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<sup>21</sup>This analysis relates to the case in which the regulator faces a small amount of uncertainty over the demand intercept, though the slope is known. If instead the regulator knows the intercept or the market equilibrium point, for instance, but faces a small amount of uncertainty over the slope, the price ceiling that maximizes expected consumer surplus is below the equilibrium price, as in our framework.

regions as a potential location for the optimal value of  $\bar{p}$ .

**(a) The price ceiling always binds.**  $\bar{p} < p^*(n_{\min})$  (i.e.,  $\bar{p}$  is such that  $n^*(\bar{p}) < n_{\min}$ ) so a price ceiling binds regardless of  $\eta$  and this results in excess demand.

**(b) The price ceiling never binds.**  $\bar{p} > p^*(n_{\max})$  (i.e.,  $\bar{p}$  is such that  $n^*(\bar{p}) > n_{\max}$ ) so a price ceiling does not bind regardless of  $\eta$  and the free market outcome prevails.

**(c) The price ceiling may or may not bind.**  $\bar{p} \in [p^*(n_{\min}), p^*(n_{\max})]$  (i.e.,  $\bar{p}$  is such that  $n^*(\bar{p}) \in [n_{\min}, n_{\max}]$ ) and so the effect of a price ceiling depends on the value of  $\eta$ : in particular, for low demand (when  $\eta \in [n_{\min}, n^*(\bar{p})]$ ) the price ceiling does not bind, whereas for high demand (when  $\eta \in [n^*(\bar{p}), n_{\max}]$ ) it results in excess demand.

**(a)** In this case, as supply is a binding constraint regardless of  $\eta$ , output is  $\min[q^d(\bar{p}), q^s(\bar{p})] = q^s(\bar{p}) = (\bar{p} - C)/c$ . Consumer surplus is  $CS(q^s(\bar{p}), \eta) = (\bar{p} - C)[2c(B + \eta - \bar{p}) - b(\bar{p} - C)]/2c^2$ , and expected consumer surplus is  $E(q^s(\bar{p}), \eta) = CS_d^L$ , as given in (4). The price ceiling that solves the f.o.c. of the optimization problem in this region is  $\hat{p}$  in (5).

$\hat{p}$  is also the optimal ceiling with little or no demand uncertainty, as we show in Section 3. The limit on uncertainty for which this result holds can be specified in terms of  $n_{\min}$  ( $< 0$ ). If  $n_{\min}$  is close enough to zero (i.e., uncertainty is small) the price ceiling  $\hat{p}$  will bind for all realizations of  $\eta$ , but for  $n_{\min}$  further from zero it may or may not bind, depending on  $\eta$ . Using (2) and (5), let  $n_0$  be the value of  $\eta$  at which  $\hat{p}$  is equal to the market clearing price.

**Definition 1** Let  $n_0 = -\frac{c(B - C)}{b + 2c}$ .

If there is little uncertainty (i.e. if  $n_{\min} > n_0$ ), as  $d^2CS_d^L/d\bar{p}^2 < 0$ , (5) is a well defined local maximum within the region  $(C, p^*(n_{\min}))$ . But, with greater uncertainty, (i.e., if  $n_{\min} \leq n_0$ ),  $CS_d^L$  is increasing for all  $\bar{p} < p^*(n_{\min})$  and the critical value  $\hat{p}$  is weakly larger than  $p^*(n_{\min})$ , which is inconsistent with the region  $(C, p^*(n_{\min}))$ . This proves the

following result.

**Lemma 2** *With small enough demand uncertainty (i.e.,  $n_{\min} > n_0$ ), in the region where  $\bar{p} < p^*(n_{\min})$ , the price ceiling that maximizes expected consumer surplus under perfect competition is given by (5). But, if  $n_{\min} \leq n_0$ , an optimal price ceiling cannot be strictly lower than  $p^*(n_{\min})$ .*

After analyzing the other possible regions for the price ceiling, we will explore under what conditions the price ceiling in (5) is globally optimal.

**(b) The price ceiling never binds.** With a price ceiling in the region  $\bar{p} > p^*(n_{\max})$ , regardless of the realization of  $\eta$ , the outcome is the same as with no intervention. For a given  $\eta$ , consumer surplus is  $b(B + \eta - C)^2/2(b + c)^2$ . So expected consumer surplus becomes

$$\frac{b[(B - C)^2 + E(\eta^2)]}{2(b + c)^2} \equiv CS_d^H, \quad (8)$$

and is the same as in the free market equilibrium.

**(c) The price ceiling may or may not bind.** In this case  $\bar{p} \in [p^*(n_{\min}), p^*(n_{\max})]$ , so  $n^*(\bar{p}) \in [n_{\min}, n_{\max}]$ . We summarize below the main steps of the analysis, and relegate the derivation of the expected consumer surplus to the appendix.

For any price ceiling  $\bar{p} \geq p^*(\eta)$  (or, equivalently,  $\eta \leq n^*(\bar{p})$ ), the intervention does not bind, and so the market clearing price  $p^*(\eta)$  prevails and the quantity traded is  $q^d(p^*, \eta) = q^s(p^*)$ . Since  $q^d(p^*, \eta) = (B + \eta - p^*)/b = (B + \eta - C)/(b + c) \equiv q_d^*$ , consumer surplus in this case is given by  $CS(q_d^*) = b(B + \eta - C)^2/2(b + c)^2$ . We use  $CS(q_d^*)$  to derive expected consumer surplus conditional on  $\bar{p} \geq p^*(\eta)$ . For  $\bar{p} \leq p^*(\eta)$  (or, equivalently,  $\eta \geq n^*(\bar{p})$ ), the intervention leads to excess demand. The realized consumer surplus from the  $q^s(\bar{p}) = (\bar{p} - C)/c$  units produced is the same as in region (a), where the price ceiling always binds.

In the appendix, we derive the expressions for expected consumer surplus in these two cases. Then, we obtain total expected consumer surplus in region (c), which we denote



by  $CS_d$ . An interior optimum solves  $dCS_d/d\bar{p} = 0$ , where

$$\frac{dCS_d}{d\bar{p}} = -(1 - \mathcal{F}(n^*(\bar{p}))) \frac{(b + 2c)(\bar{p} - C) - c(B - C)}{c^2} - \mathcal{F}(n^*(\bar{p})) \frac{1}{c} \frac{\epsilon_d^L(n^*(\bar{p}))}{\mathcal{F}(n^*(\bar{p}))}, \quad (9)$$

with  $\epsilon_d^L(n^*(\bar{p})) = \int_{n_{\min}}^{n^*(\bar{p})} \eta d\mathcal{F}(\eta)$ .

The two terms in (9) are related to the fact that a price ceiling  $\bar{p}$  may either bind or not and they are weighted by their respective probabilities. The fraction  $-[(b + 2c)(\bar{p} - C) - c(B - C)]/c^2$  is the value of  $dCS_d/d\bar{p}$  when there is no uncertainty and supply is the binding constraint, as in our preliminary analysis. The second term in (9) shows the impact of demand uncertainty.  $\epsilon_d^L(n^*(\bar{p}))/\mathcal{F}(n^*(\bar{p}))$  ( $< 0$ ) is the expected value of  $\eta$ , conditional on the ceiling not binding, and a more negative value represents greater demand uncertainty.

The next result combines the analyses of the three regions and uses (4), (8) and (14).

**Lemma 3** *With demand uncertainty, expected consumer surplus under perfect competition is (i) continuous and differentiable for all values of  $\bar{p} > C$ ; (ii) independent of  $\bar{p}$  at any price ceiling  $\bar{p} \geq p^*(n_{\max})$  because the intervention is not binding in this range.*

In the appendix, using Lemma 3 and as the hazard rate of  $\eta$  is strictly increasing, we show that the objective function is single-peaked over the three regions and the location of the optimal ceiling depends on the degree of uncertainty. This leads to the following result.

**Proposition 1** *Under perfect competition, if demand uncertainty is small (i.e.,  $n_{\min} > n_0$ ), the unique price ceiling that globally maximizes expected consumer surplus always binds and is given by (5); but if demand uncertainty is large (i.e., if  $n_{\min} \leq n_0$ ), it lies in the interval  $[p^*(n_{\min}), p^*(n_{\max})]$  and may bind or not ex post.*

This proposition highlights the effects of the degree of uncertainty on optimal pro-consumer regulation. The solution for  $n_{\min} > n_0$  is related to the effects discussed in section 3. It generalizes the result that, under no uncertainty, a price ceiling  $\bar{p} = \hat{p}$  below

the market equilibrium maximizes consumer surplus. When demand is stochastic, it is not known ex ante whether supply will be a binding constraint on consumption. But, with small uncertainty,  $CS_d$  is maximized with the same price ceiling,  $\bar{p} = \hat{p}$ . Moreover, the analysis preceding Proposition 1 shows that an optimal ceiling cannot lie in  $[p^*(n_{\min}), p^*(n_{\max})]$ . Instead it is strictly lower than  $p^*(n_{\min})$  and, implicitly, than the expected market clearing price  $p_e^*$ . Nevertheless, if the regulator faces enough uncertainty ( $n_{\min} \leq n_0$ ), this favours setting a price ceiling that is relatively high and, therefore, less likely to bind.

Intuitively, take the no uncertainty case: at the optimal ceiling  $\hat{p}$ , supply is a binding constraint and so  $q = q^s(\hat{p})$ . If we allow for small uncertainty (i.e.,  $n_{\min} > n_0$ ), then at price ceiling  $\hat{p}$ , supply remains a binding constraint and  $q = q^s(\hat{p})$  for any  $\eta$ , and  $\hat{p}$  remains the optimal ceiling. If instead uncertainty is large (i.e.,  $n_{\min} \leq n_0$ ) and the regulator still sets  $\bar{p} = \hat{p}$ , this ceiling would not bind for very low demand realizations (as the equilibrium price is below  $\hat{p}$ ), but it would still bind for high demand realizations. Suppose the regulator marginally increases the ceiling to  $\bar{p} = \hat{p} + \varepsilon$  for small  $\varepsilon > 0$ . For very low demand realizations,  $\bar{p} = \hat{p} + \varepsilon$  would still not bind and ex post output (and consumer surplus) would be the same as at  $\bar{p} = \hat{p}$ . For high demand realizations, both  $\bar{p} = \hat{p}$  and  $\bar{p} = \hat{p} + \varepsilon$  would bind. But  $\bar{p} = \hat{p} + \varepsilon$  would result in higher consumer surplus because  $\bar{p} = \hat{p}$  is the optimal ceiling when  $\eta = 0$  and it is lower than the optimal ceiling that would apply for any high demand realization (where  $\eta > 0$ ). Hence, with great enough uncertainty, expected consumer surplus is larger at  $\bar{p} = \hat{p} + \varepsilon$  than at  $\bar{p} = \hat{p}$ , and so the optimal price ceiling is higher than when there is no or little uncertainty.<sup>22</sup>

We now compare the optimal ceiling first to the expected market clearing price and

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<sup>22</sup>This intuition also applies when the regulator knows the demand intercept but faces uncertainty over the slope, which suggests that our results carry over unchanged to such an environment.

then to the optimal ceiling under no uncertainty.

**Corollary 1** *With demand uncertainty and perfect competition, if  $n_{\min} > n_0$ , the optimal ceiling lies below the expected market clearing price  $p_e^*$ . But if  $n_{\min} \leq n_0$ , the optimal ceiling may be greater or smaller than  $p_e^*$ , as  $CS_d$  is maximized at  $\bar{p} \begin{cases} \geq \\ \leq \end{cases} p_e^*$  when  $-(1 - F(0))c(B - C) - (b + c)\epsilon_d^L(0) \begin{cases} \geq \\ \leq \end{cases} 0$ .*

Thus, with large enough uncertainty the optimal pro-consumer price ceiling may exceed  $p_e^*$ . The following example illustrates this.

**Example 1** *Let  $\eta \sim U[-n, n]$ . Then,  $\epsilon_d^L(0) = -n/4$ ,  $\mathcal{F}(0) = 1/2$ , and  $n_{\min} \leq n_0 \Leftrightarrow n \geq c(B - C)/(b + 2c)$ .  $CS_d$  is maximized at  $\bar{p} \begin{cases} \geq \\ \leq \end{cases} p_e^*$  as  $n \begin{cases} \geq \\ \leq \end{cases} 2c(B - C)/(b + c)$ . So, for  $n \in [c(B - C)/(b + 2c), 2c(B - C)/(b + c))$  (for  $n > 2c(B - C)/(b + c)$ ), the optimal ceiling is lower (higher) than  $p_e^*$ .*

Although this section focuses on the role of regulatory uncertainty, the setting with no uncertainty in section 3 is a special case where  $\eta$  has a degenerate distribution so that  $n_{\min} = n_{\max} = 0$ . With greater uncertainty (for  $n_{\min} \leq n_0$ ) a price ceiling in the middle region,  $\bar{p} \in [p^*(n_{\min}), p^*(n_{\max})]$ , is optimal. In this case, the no uncertainty optimal ceiling  $\hat{p}$  also belongs to the middle region. Evaluating (9) at  $\hat{p}$ , we obtain

$$\frac{dCS_d(\bar{p})}{d\bar{p}} \Big|_{\bar{p}=\hat{p}} = -\frac{1}{c}\epsilon_d^L(n^*(\hat{p})) \geq 0 ,$$

where the weak inequality follows from  $\epsilon_d^L(n^*(\hat{p})) \leq 0$ , and holds with equality only for  $n_{\min} = n_0$  (or,  $p^*(n_{\min}) = \hat{p}$ ).

**Corollary 2** *With large enough uncertainty ( $n_{\min} \leq n_0$ ), the optimal price ceiling is strictly higher than  $\hat{p}$ , the optimal ceiling with smaller or no uncertainty.*

Thus, larger regulatory uncertainty calls for softer intervention, with the price ceiling set at a relatively high level compared to a case where there is no uncertainty.

## 5 Imperfect Competition

We now analyze the price ceiling that maximizes expected consumer surplus in the imperfectly competitive model presented in section 2. This framework, where suppliers have

(some) market power, allows us to study the impact of the degree of competition on the optimal pro-consumer ceiling, and can be interpreted as the reduced form of a model where firms compete in supply functions. However, we show that our analysis is also applicable in the limiting case of a monopoly market.  $\delta < 1$  measures competitive pressure, and a larger value corresponds to more intense competition.

With no uncertainty, the optimal pro-consumer price ceiling from section 3 still obtains with imperfect competition as consumer surplus is maximized at the same price level. Recall that aggregate marginal cost is the same under perfect and imperfect competition (see the discussion in section 2 and footnote 14). A binding price ceiling becomes marginal revenue for all firms, and total quantity is given by equating this ceiling with aggregate marginal cost, as in the perfectly competitive case. However, as we show below, this is no longer true with large enough uncertainty.

Consider a realization of demand  $\eta$ . Then the equilibrium price in the imperfectly competitive market is given by

$$p^\delta(\eta, \delta) = \frac{c(B + \eta) + \delta bC}{\delta b + c} > p^*(\eta) ,$$

where  $p^*(\eta)$  is the perfectly competitive price (see Figure 2).

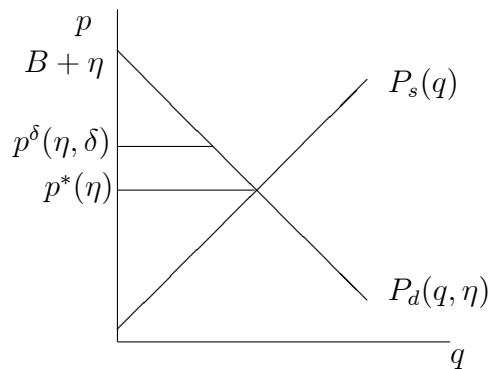


Figure 2: Imperfectly Competitive Price

A price ceiling  $\bar{p}$  binds or not depending on its position relative to  $p^\delta(\eta, \delta)$ . If  $\bar{p} \geq$

$p^\delta(\eta, \delta)$  the intervention does not bind and the imperfectly competitive market outcome prevails. If  $\bar{p} < p^\delta(\eta, \delta)$  the intervention binds and either demand or supply may be a constraint depending on the position of  $\bar{p}$  relative to the perfectly competitive price  $p^*(\eta)$ . More specifically, if  $\bar{p} < p^*(\eta)$  the quantity traded will be  $q^s(\bar{p})$  and there is excess demand. If  $p^*(\eta) < \bar{p} < p^\delta(\eta)$  the quantity traded will be  $q^d(\bar{p})$  and there is excess supply.

Consider our supply function competition interpretation. If the market is unregulated, the firms compete by choosing price-quantity schedules. However, in the presence of a binding price ceiling, the firms can no longer adjust the price, so they can only choose optimally the output levels.<sup>23</sup> When the intervention binds,  $\bar{p}$  becomes the (constant) marginal revenue. Therefore the firms will supply the quantity at which  $\bar{p}$  equals marginal cost  $P^s(q)$  unless demand is a binding constraint, in which case they supply the quantity demanded at  $\bar{p}$ . So, the quantity traded in the market at the regulated price is  $\min\{q^d(\bar{p}), q^s(\bar{p})\}$ , where  $q^s(\bar{p})$  is the quantity at which marginal cost equals  $\bar{p}$ .<sup>24</sup> This is different from the perfectly competitive model where a binding ceiling could only result in excess demand.

As before, we examine different price regions as potential locations for the optimal price ceiling. The same three regions from section 4 apply, depending on the realization of  $\eta$ . For clarity, in this section we will refer to these regions as (a'), (b') and (c'). With imperfect competition the potential outcomes for region (c') are more complicated than for region (c) in section 4 as either demand or supply may bind. We outline the analysis for each region here but leave the details to the appendix. There is a minor difference in the argument depending on whether  $p^\delta(n_{\min}, \delta)$ , the ex post equilibrium price under imperfect

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<sup>23</sup>In this sense, in a market where without intervention, firms compete in supply functions, the introduction of a binding price ceiling leads to a change in competition as the firms become price-takers.

<sup>24</sup> $q^s(p)$  is the same as the perfectly competitive supply. In contrast,  $q^s(p, \delta)$  is the aggregate supply in the unregulated market where the firms compete in supply functions and  $q^s(p, \delta) = \delta q^s(p)$ .

competition for the lowest demand realization, is greater or smaller than  $p^*(n_{\max})$ , the ex post equilibrium price under perfect competition for the highest demand realization.<sup>25</sup>

We proceed on the assumption that  $p^\delta(n_{\min}, \delta) < p^*(n_{\max})$  and cover the reverse case in footnote 27.<sup>26</sup>

**(a´) The price ceiling always binds ( $\bar{p} < p^*(n_{\min})$ ).** The corresponding analysis in section 4 carries over unchanged. So, if  $p^\delta(n_{\min}, \delta) < p^*(n_{\max})$  and  $n_{\min} > n_0$ , the optimal pro-consumer price ceiling in this region is given by (5). But, if  $n_{\min} \leq n_0$ , an optimal ceiling cannot be strictly lower than  $p^*(n_{\min})$  as expected consumer surplus is increasing in  $\bar{p}$ .

**(b´) The price ceiling never binds ( $\bar{p} > p^*(n_{\max})$ ).** For values of  $\eta$  at which the ceiling binds, demand is a constraint, that is  $\min\{q^d(\bar{p}), q^s(\bar{p})\} = q^d(\bar{p})$ . Hence, a small decrease in  $\bar{p}$  leads to an increase in consumer surplus. At values of  $\eta$  where the ceiling does not bind, a decrease in the ceiling either has no effect on consumer surplus or, if it has, it boosts consumer surplus as the unregulated market price is above the competitive level for  $\delta < 1$ . In effect, an optimal price ceiling cannot belong to this region.

**(c´) The price ceiling may or may not bind ( $\bar{p} \in [p^*(n_{\min}), p^*(n_{\max})]$ ).** We identify two sub-regions as potential locations for the optimal price ceiling,  $[p^*(n_{\min}), p^\delta(n_{\min}, \delta)]$  and  $(p^\delta(n_{\min}, \delta), p^*(n_{\max})]$ . We sketch the arguments below and relegate the details to the appendix.

(c´1) Consider first  $\bar{p} \in [p^*(n_{\min}), p^\delta(n_{\min}, \delta)]$ . Since the upper bound  $p^\delta(n_{\min}, \delta)$  is the lowest possible unregulated market price, the ceiling binds regardless of  $\eta$ . However,

<sup>25</sup>The sign of  $p^\delta(n_{\min}, \delta) - p^*(n_{\max})$  depends on all the parameters in the model. But, at least for a symmetric distribution where  $\eta \in [-n, n]$  there is a cut-off level of uncertainty at  $n = b(1 - \delta)(B - C) / b(1 + \delta) + 2c \equiv \hat{n}$ . For  $n > (<) \hat{n}$ ,  $p^\delta(n_{\min}, \delta) - p^*(n_{\max}) < (>) 0$ .

<sup>26</sup>Even if for some values of  $\delta$ ,  $p^\delta(n_{\min}, \delta) \geq p^*(n_{\max})$ , there exists a cut-off value  $\delta_0$ , so that for  $\delta \in [\delta_0, 1)$ ,  $p^\delta(n_{\min}, \delta) < p^*(n_{\max})$ . This is because  $\lim_{\delta \rightarrow 1} p^\delta(n_{\min}, \delta) = p^*(n_{\min}) < p^*(n_{\max})$ . When  $\delta \rightarrow 1$ , the market becomes almost perfectly competitive. The cut-off value  $\delta_0$  is implicitly defined by  $p^*(n_{\max}) = p^\delta(n_{\min}, \delta_0)$ .

depending on  $\eta$ , the intervention may lead to excess demand or excess supply. The derivation is the same as in the perfectly competitive case (with the critical value  $\eta = n^*(\bar{p})$ ) except that the upper bound to the region is at a higher price ceiling.

(c'2) For  $\bar{p} \in (p^\delta(n_{\min}, \delta), p^*(n_{\max}))$  the price ceiling may bind or not depending on the value of  $\eta$ . The critical value  $\eta = n^*(\bar{p})$  again applies, and for high enough demand realizations ( $\eta > n^*(\bar{p})$ ) there is excess demand, so that  $q = q^s(\bar{p})$ . However, for lower demand realizations ( $\eta \leq n^*(\bar{p})$ ) excess supply only obtains if demand is not too low. If, for example,  $\eta = n_{\min}$ , the equilibrium unregulated price is  $p^\delta(n_{\min}, \delta)$ , which is below this sub-region, and so the price ceiling does not bind. More generally, for values of  $\eta$  below a cut-off value  $n^{**}(\bar{p}, \delta)$ , the price ceiling does not bind and the quantity traded is  $q^d(p^\delta(\eta, \delta))$ .<sup>27</sup>

In the online appendix we show that for  $n_{\min} > n_0$ , the globally optimal price ceiling is strictly lower than  $p^*(n_{\min})$  and given by (5), whereas for  $n_{\min} \leq n_0$  it lies in the interval  $[p^*(n_{\min}), p^*(n_{\max})]$ .<sup>28</sup> We are now ready to state the following result.

**Proposition 2** *For sufficiently large demand uncertainty ( $n_{\min} \leq n_0$ ), the optimal pro-consumer price ceiling with imperfect competition is increasing in the degree of competition. For smaller uncertainty ( $n_{\min} > n_0$ ), it is independent of the degree of competition.*

With no uncertainty, the optimal ceiling is independent of  $\delta$ . But, with sufficient uncertainty, it is increasing in the degree of competition and, therefore, lower than the one in the perfectly competitive market. A decrease in the optimal ceiling generates significant gains in consumer surplus under high demand realizations. If the optimal ceiling is lower than  $p^\delta(n_{\min}, \delta)$ , so that it binds regardless of the realized demand, then it is independent of the degree of competition and is weakly lower than in the competitive

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<sup>27</sup>If  $p^\delta(n_{\min}, \delta) \geq p^*(n_{\max})$  the analysis is qualitatively the same as we have outlined in the text, except in one respect. By definition, sub-region (c'2) does not exist. Intuitively, the analysis in (c'1) above applies now to the entire range.

<sup>28</sup>We show in the appendix that expected consumer surplus is single peaked.

case. If the optimal ceiling is higher than  $p^\delta(n_{\min}, \delta)$ , it may bind or not depending on the realization of demand and it is smoothly increasing in the degree of competition.

With great enough uncertainty, the optimal ceiling is lower if the market is less competitive. Denote by  $\bar{p}_c$  the price ceiling that is optimal under perfect competition and suppose it were set in an imperfectly competitive market. For those high demand realizations for which  $\bar{p}_c$  binds under perfect competition, it will also bind under imperfect competition and so the same outcome obtains regardless of the degree of competition. In contrast, for those demand realizations for which  $\bar{p}_c$  does not bind under perfect competition, the price that obtains under imperfect competition (either  $p^\delta(\eta)$  or  $\bar{p}_c$ , depending on the value of  $\eta$ ) is higher than the perfectly competitive price (i.e., the market clearing price  $p^*(\eta)$ ). However, for any given demand realization  $\eta$ , consumer surplus is maximized at a price  $\hat{p}(\eta) < p^*(\eta)$ . Thus, for each possible lower demand realization, price under perfect competition exceeds the corresponding optimal price ceiling, and under imperfect competition is greater still. To limit such effects the optimal ceiling under imperfect competition is lower than that under perfect competition, and it decreases in the degree of market power. In this sense, our analysis shows that, with sufficient regulatory uncertainty, competitive pressure weakens the scope for pro-consumer regulatory intervention.<sup>29</sup>

In our imperfect competition model, aggregate output is a fraction  $\delta$  of the competitive supply. This result builds on the supply function competition microfoundation and, in the online appendix, we show that  $\delta \geq 2c/(c + \sqrt{c(2b + c)})$  for  $N \geq 2$ . However, the reduced form model with an output restriction can also capture a monopoly market when  $\delta = c/(b + c)$ . Therefore, the analysis in this section directly applies to a market with a

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<sup>29</sup>We focus here on the optimal ceiling that maximizes expected consumer surplus in the presence of regulatory uncertainty under imperfect competition. Our intuition is that, in this setting, the expected welfare maximizing ceiling would be at or above the expected market clearing price and - as the optimal pro-consumer ceiling - is increasing in the degree of competition.



monopoly supplier.

## 6 Inefficient Rationing

We now focus on the impact of rationing inefficiency on the optimal pro-consumer price ceiling under perfect competition. We show that, if rationing efficiency  $\alpha$  is great enough, the qualitative results from section 4 still hold, although the level of the optimal ceiling may depend on the value of  $\alpha$ . We identify a cut-off rationing efficiency below which a price ceiling should not be set.

We use the three regions for the optimal ceiling specified in section 4. In region (b), as a price ceiling never binds ( $\bar{p} > p^*(n_{\max})$ ), expected consumer surplus is unaffected by rationing inefficiency and so is given by (8). However, in region (c) supply may be a binding constraint and in region (a) it surely is, so for these regions inefficiency of rationing plays an important role. For (a) and (c), we write the regulator's objective function as the weighted sum of expected surplus under fully efficient rationing (with weight  $\alpha \in [0, 1]$ ) and under extremely inefficient rationing (with weight  $1 - \alpha$ ). Below we outline the basic reasoning underlying the analysis of inefficient rationing, but the full derivation is relegated to the online appendix.

When supply binds for a given price ceiling  $\bar{p}$ , output  $q^s(\bar{p}) = (\bar{p} - C)/c$  is bought by the consumers with the lowest valuations amongst those willing to pay at least  $\bar{p}$ . Then the  $q^s(\bar{p}) = (\bar{p} - C)/c$  units available are purchased by consumers along the lowest part of the demand curve at and above  $\bar{p}$ , i.e., from  $q^d(\bar{p}) - q^s(\bar{p})$  to  $q^d(\bar{p})$ . For a given realization of demand  $\eta$ , at  $q^d(\bar{p}) - q^s(\bar{p})$  consumer surplus per unit is  $B + \eta - b(q^d(\bar{p}) - q^s(\bar{p})) - \bar{p}$ , while at  $q^d(\bar{p})$  it is zero. So, using (1), total consumer surplus is then  $b(\bar{p} - C)^2/2c^2 \equiv CS_{d0}^L$  (and independent of  $\eta$ ). We use  $CS_{d0}^L$  to calculate expected consumer surplus under extremely inefficient rationing, taking into account that in (a) supply always binds, whereas in (c)

it binds only for  $\eta \geq n^*(\bar{p})$ .

Below we extend Proposition 1 for arbitrary values of  $\alpha$ . To do so, we define a critical level of the price ceiling and a threshold degree of uncertainty, which generalize the value in (5) and Definition 1, respectively, for arbitrary rationing efficiency  $\alpha$ .

$$\hat{p}(\alpha) = \frac{\alpha c(B + C) - (1 - 2\alpha)bC}{2\alpha(b + c) - b} \quad \text{and} \quad n_0(\alpha) = -\frac{[\alpha(b + c) - b](B - C)}{2\alpha(b + c) - b}. \quad (10)$$

Both  $\hat{p}(\alpha)$  and  $n_0(\alpha)$  are well defined for  $\alpha \neq b/2(b + c)$ . For  $\alpha = 1$ ,  $\hat{p}(\alpha)$  reduces to the value specified in (5) and  $n_0(\alpha) = n_0$ .

**Proposition 3** *Assume perfect competition. For rationing efficiency  $\alpha > b/(b + c)$ , there is a unique price ceiling that globally maximizes expected consumer surplus: if  $n_{\min} > n_0(\alpha)$ , this optimal ceiling is given by (10) and always binds; while if  $n_{\min} \leq n_0(\alpha)$ , it lies in the interval  $[p^*(n_{\min}), p^*(n_{\max})]$ . For rationing efficiency  $\alpha \leq b/(b + c)$ , any price ceiling decreases expected consumer surplus.*

This proposition highlights the combined effects of rationing efficiency and the degree of uncertainty on the optimal ceiling. Provided that  $\alpha > b/(b + c)$ , the qualitative conclusions in section 4 carry over. If uncertainty is relatively small ( $n_{\min} > n_0(\alpha)$ ) a price ceiling in the low region (a), that is sure to bind, is optimal. This situation is illustrated in Figure 1B. If uncertainty is greater ( $n_{\min} \leq n_0(\alpha)$ ), the optimal ceiling is in region (c) and may or may not bind. The critical uncertainty level, as represented by  $n_0(\alpha)$ , is decreasing in  $\alpha$ . Thus, the greater is rationing efficiency, the greater is the maximum amount of uncertainty for which the optimal ceiling lies in the low region, and binds for sure. The condition  $\alpha > b/(b + c)$  is consistent with a wide range of inefficiency, e.g. if demand is less steep than supply ( $b < c$ ), it is consistent with random rationing ( $\alpha = 1/2$ ).

For large uncertainty ( $n_{\min} \leq n_0(\alpha)$ ) and sufficiently efficient rationing ( $\alpha > b/(b + c)$ ), the optimal ceiling may lie above or below the expected market clearing price. In the online appendix, we present a condition which generalizes Corollary 1 in section 4 and

also shows that, when rationing efficiency  $\alpha$  is smaller (but  $\alpha > b/(b+c)$ ), the level of demand uncertainty required for  $\bar{p}$  to exceed  $p_e^*$  is greater.

When instead  $\alpha \leq b/(b+c)$  a price ceiling should not be set, regardless of the amount of demand uncertainty. If rationing efficiency is this low, price regulation has a large allocative cost which eliminates the scope for pro-consumer intervention. If  $b > c$ , this non-intervention result holds even for random rationing; but, if  $b < c$ , intervention can increase expected consumer surplus even with worse than random rationing.

If uncertainty is great enough ( $n_{\min} \leq n_0$ ), then, generalizing our result for efficient rationing, the optimal price ceiling lies in region (c) and is strictly higher than  $\hat{p}(\alpha)$ , the optimal ceiling with smaller or no uncertainty. Moreover, Proposition 3 allows us to characterize fully the relationship between the efficiency of rationing  $\alpha$  and the optimal price ceiling.

**Corollary 3** *With perfect competition, for  $\alpha \geq b/(b+c)$ , the optimal pro-consumer price ceiling is strictly decreasing in the rationing efficiency  $\alpha$ .*

Thus, whenever it is optimal to set a price ceiling that may bind, there is a negative relationship between the optimal level and the efficiency of rationing.

## 7 Conclusions

We explore the impact of market conditions on optimal price ceiling regulation in a setting where the regulator maximizes expected consumer surplus and is imperfectly informed about demand. Under perfect competition, regulatory uncertainty does not eliminate the rationale for price ceiling intervention, but sufficient uncertainty calls for softer intervention, with the ceiling set at a relatively high level. Under imperfect competition, if there is sufficient uncertainty, the optimal ceiling is increasing in the degree of competition and so competitive pressure justifies less restrictive regulatory intervention. This result is

broadly consistent, for example, with Oftel's decision in 2002 to increase the price ceiling imposed on British Telecom once new suppliers entered the telecommunications market. We extend the perfect competition model to allow for rationing inefficiency and identify a cut-off level of rationing inefficiency above which a ceiling should not be used. Above this level, our qualitative results from the model with efficient rationing carry over unchanged.

We focus on demand uncertainty, but our results are qualitatively robust in settings with supply uncertainty only, or where the regulator faces both demand and supply uncertainty, as we formally show in the online appendix. For tractability, we focus on linear demand and supply, but the intuition suggests that our results are qualitatively robust to more general assumptions.<sup>30</sup> Likewise, the main insights would be relevant in settings where the regulatory standard is a weighted sum of consumer surplus and profit.

## 8 Appendix

### Preliminary Analysis

**Proof of Lemma 1.** (i) At  $\bar{p} \geq p^*$ ,  $dCS(q^d(p^*))/d\bar{p} = 0$ . (ii) When  $\bar{p} < p^*$ , the f.o.c.  $\partial CS_d^L / \partial \bar{p} = 0$  gives (5). The s.o.c. is always satisfied. The optimal ceiling lies in  $(C, p^*)$ , so is well defined. ■

### Perfect Competition

(c) **The price ceiling may or may not bind.**

**Remark 1** ( $\gamma$ ) implies that  $\text{prob}(p^* \leq \bar{p}) = \text{prob}(\eta \leq n^*(\bar{p})) = \mathcal{F}(n^*(\bar{p}))$ .

Using Remark 1, expected consumer surplus conditional on  $\bar{p} \geq p^*(\eta)$  is given by

$$E(CS(q_d^*) \mid \eta \leq n^*(\bar{p})) = \left( \int_{n_{\min}}^{n^*(\bar{p})} CS(q_d^*) d\mathcal{F}(\eta) \right) / \mathcal{F}(n^*(\bar{p})) \equiv CS_d^* . \quad (11)$$

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<sup>30</sup>More precisely, so long as demand and supply (marginal cost) are well-behaved, demand is finite for all positive prices, and there is scope for trade ex-post, the intuitive arguments carry over.

Substituting for  $CS(q_d^*) = b(B + \eta - C)^2/2(b + c)^2$  in (11), and using  $\epsilon_d^L(n^*(\bar{p})) = \int_{n_{\min}}^{n^*(\bar{p})} \eta d\mathcal{F}(\eta)$  and  $\Upsilon_d^L(n^*(\bar{p})) = \int_{n_{\min}}^{n^*(\bar{p})} \eta^2 d\mathcal{F}(\eta)$ ,

$$CS_d^* = b [(B - C)^2 \mathcal{F}(n^*(\bar{p})) + 2(B - C)\epsilon_d^L(n^*(\bar{p})) + \Upsilon_d^L(n^*(\bar{p}))] / 2(b + c)^2 \mathcal{F}(n^*(\bar{p})).$$

The expected consumer surplus conditional on  $\bar{p} \leq p^*(\eta)$  is

$$E(CS(q^s(\bar{p}), \eta) \mid \eta \geq n^*(\bar{p})) = \left( \int_{n^*(\bar{p})}^{n_{\max}} CS(q^s(\bar{p}), \eta) d\mathcal{F}(\eta) \right) / [1 - \mathcal{F}(n^*(\bar{p}))] \equiv CS_d^S. \quad (12)$$

Substituting for  $CS(q^s(\bar{p}))$  in (12), and letting  $\epsilon_d^H(n^*(\bar{p})) = \int_{n^*(\bar{p})}^{n_{\max}} \eta d\mathcal{F}(\eta)$ , we obtain

$$\frac{(\bar{p} - C) [2c(B - \bar{p}) - b(\bar{p} - C)]}{2c^2} + \frac{(\bar{p} - C) \epsilon_d^H(n^*(\bar{p}))}{c(1 - \mathcal{F}(n^*(\bar{p})))} \equiv CS_d^S. \quad (13)$$

Total expected consumer surplus for any given price ceiling  $\bar{p} \in [p^*(n_{\min}), p^*(n_{\max})]$  is

$$E(CS(\bar{p})) = \mathcal{F}(n^*(\bar{p}))CS_d^* + (1 - \mathcal{F}(n^*(\bar{p})))CS_d^S \equiv CS_d. \quad (14)$$

**Proof of Lemma 3.** It is easy to see from (4), (8) and (14) that expected consumer surplus is piece-wise continuous and differentiable. So we focus on the continuity and differentiability of the expected consumer surplus at  $p^*(n_{\min})$  and  $p^*(n_{\max})$ .

Continuity at  $p^*(n_{\min}) = [c(B + n_{\min}) + bC]/(b + c)$ :  $\lim_{\bar{p} \nearrow p^*(n_{\min})} CS_d^L(\bar{p}) = CS_d^S(p^*(n_{\min}))$ , where  $CS_d^S$  is given in (13). The equality follows as  $\mathcal{F}(n_{\min}) = \epsilon_d^H(n_{\min}) = 0$ . Since  $\Upsilon_d^L(n_{\min}) = \epsilon_d^L(n_{\min}) = 0$ , then  $\lim_{\bar{p} \nearrow p^*(n_{\min})} CS_d^L(\bar{p}) = CS_d(n_{\min})$  so the result follows.

Continuity at  $p^*(n_{\max})$ : As  $\mathcal{F}(n_{\max}) = 1$ ,  $\epsilon_d^L(n_{\max}) = 0$  and  $\Upsilon_d^L(n_{\max}) = E(\eta^2)$ , then  $CS_d(n_{\max}) = CS_d^*(n_{\max}) = \lim_{\bar{p} \searrow p^*(n_{\max})} CS_d^H(\bar{p}) = CS_d^H$  and the result follows.

Differentiability at  $p^*(n_{\min})$ : In region (a),  $\lim_{\bar{p} \nearrow p^*(n_{\min})} \partial CS_d^L / \partial \bar{p} =$

$[c(B - C) - (p^*(n_{\min}) - C)(b + 2c)] / c^2$ . In region (c), as  $\epsilon_d^L(n_{\min}) = \mathcal{F}(n_{\min}) = 0$ ,

$\partial CS_d(p^*(n_{\min})) / \partial \bar{p} = \lim_{\bar{p} \nearrow p^*(n_{\min})} \partial CS_d^L / \partial \bar{p}$ . The result follows.

Differentiability at  $p^*(n_{\max})$ : In region (c), using  $\partial CS_d / \partial \bar{p}$  given in (9), along with

$\epsilon_d^L(n_{\max}) = 0$  and  $\mathcal{F}(n_{\max}) = 1$ , it follows that  $\partial CS_d(p^*(n_{\max}))/\partial \bar{p} = 0$ . As (8) is independent of  $\bar{p}$ ,  $\lim_{\bar{p} \searrow p^*(n_{\max})} \partial CS_d^H / \partial \bar{p} = 0$ . The result follows.

The second part of the Lemma follows as expected consumer surplus is continuous at  $p^*(n_{\max})$  and (8) is independent of  $\bar{p}$ . ■

**Proof of Proposition 1.** ► Suppose that  $n_{\min} \leq n_0$ . From (9),  $d^2 CS_d / d\bar{p}^2 = [-(b+2c)(1-\mathcal{F}(n^*(\bar{p}))) + (b+c)(\bar{p}-C)\mathcal{F}'(n^*(\bar{p}))]/c^2$ . As  $\epsilon_d^L(n_{\max}) = E(\eta) = 0$  and  $\mathcal{F}(n_{\max}) = 1$ ,  $p^*(n_{\max})$  satisfies the f.o.c. in region (c).  $CS_d$  is convex at this point, and so  $p^*(n_{\max})$  is a local minimum. Hence,  $CS_d$  is decreasing in  $\bar{p}$  as it approaches  $p^*(n_{\max})$  from below. Also, by Lemma 3, all  $\bar{p} > p^*(n_{\max})$  give the same level of expected consumer surplus as  $\bar{p} = p^*(n_{\max})$ . Using (7) and (9),  $CS_d$  is increasing at  $p^*(n_{\min})$  iff  $n_{\min} \leq n_0$ . Moreover, by Lemma 2, if  $n_{\min} \leq n_0$ , there is no candidate optimal ceiling strictly below  $p^*(n_{\min})$ . Hence, if  $n_{\min} \leq n_0$ , the global optimal ceiling must belong to  $[p^*(n_{\min}), p^*(n_{\max})]$  and is unique, and  $\text{sign} \frac{d^2 CS_d}{d\bar{p}^2} = \text{sign} \left( \frac{\mathcal{F}'(n^*(\bar{p}))}{1-\mathcal{F}(n^*(\bar{p}))} - \frac{b+2c}{c^2(b+c)} \frac{1}{(\bar{p}-C)} \right)$ , where  $\mathcal{F}'(n^*(\bar{p})) / (1-\mathcal{F}(n^*(\bar{p})))$  is, by assumption, strictly increasing on  $[p^*(n_{\min}), p^*(n_{\max})]$ . As  $1/(\bar{p}-C)$  is strictly decreasing in  $\bar{p}$ ,  $CS_d$  has a unique inflexion point,  $p_I^d$ . Since at  $p^*(n_{\max})$ ,  $d^2 CS_d / d\bar{p}^2 > 0$ ,  $CS_d$  is convex for all  $\bar{p} \in (p_I^d, p^*(n_{\max}))$ . But, if  $\bar{p} < p_I^d$ ,  $1/(\bar{p}-C)$  is greater and the hazard rate smaller, so that  $d^2 CS_d / d\bar{p}^2 < 0$  for all  $\bar{p} \in (p^*(n_{\min}), p_I^d)$ . Thus, when  $n_{\min} \leq n_0$  there is a unique globally-optimal price ceiling in  $[p^*(n_{\min}), p^*(n_{\max})]$ .

► Now suppose  $n_{\min} > n_0$ . For  $\bar{p} > p^*(n_{\max})$ , the ceiling does not bind and  $CS_d$  is the same as for  $\bar{p} = p^*(n_{\max})$ . For  $\bar{p} < p^*(n_{\min})$ , by Lemma 2  $CS_d$  is maximized at  $\bar{p} = \hat{p}$ . Lastly, consider the region  $\bar{p} \in [p^*(n_{\min}), p^*(n_{\max})]$ . From (7) and (9)  $d CS_d / d\bar{p} < 0$  at  $p^*(n_{\min})$  and so, from our argument above,  $d CS_d / d\bar{p} < 0$  for the whole region. Hence, across the three regions  $CS_d$  is maximized at  $\bar{p} = \hat{p}$ . ■

**Proof of Corollary 1.** For  $n_{\min} > n_0$ , the result follows from (3) and (5). For

$n_{\min} \leq n_0$ , we evaluate  $dCS_d/d\bar{p}$  at  $\bar{p} = p_e^*$  as given in (3), and obtain  $dCS_d(p_e^*)/d\bar{p} = -(1 - \mathcal{F}(0))(B - C)/(b + c) - \epsilon_d^L(0)/c$ . The second term is non-positive, while the first is non-negative as  $\epsilon_d^L(0) < 0$ . Depending on the sign of  $dCS_d(p_e^*)/d\bar{p}$ ,  $CS_d$  is maximized at  $\bar{p} \gtrless p_e^*$ . ■

## Imperfect Competition

### Analysis for region (c')

As  $p^\delta(n_{\min}, \delta) \in [p^*(n_{\min}), p^*(n_{\max})]$ , we explore two sub-regions  $[p^*(n_{\min}), p^\delta(n_{\min}, \delta)]$  and  $(p^\delta(n_{\min}, \delta), p^*(n_{\max})]$ .

(c'1) If  $\bar{p} \in [p^*(n_{\min}), p^\delta(n_{\min}, \delta)]$ , the ceiling binds regardless of  $\eta$ . For a given  $\bar{p}$ ,  $\exists \eta = n^*(\bar{p})$  s.t.  $p^*(\eta) = \bar{p}$ . This is the same as  $n^*(\bar{p})$  in (7). If  $\bar{p} \geq p^*(\eta)$  (i.e., if  $\eta \leq n^*(\bar{p})$ ), there is excess supply, so consumer surplus is determined by  $q^d(\bar{p})$ . If  $\bar{p} < p^*(\eta)$  (i.e., if  $\eta > n^*(\bar{p})$ ), then there is excess demand, so consumer surplus is determined by  $q^s(\bar{p})$ , the quantity at which marginal cost equals  $\bar{p}$ . Let  $CS_\delta^L$  be the expected consumer surplus in this case. Then, using  $\epsilon_d^L(n^*(\bar{p}))$  as defined in section 4,  $dCS_\delta^L/d\bar{p}$  is given by

$$-\frac{1}{b}(B - \bar{p}) - \frac{b + c}{bc} \epsilon_d^L(n^*(\bar{p})) + \frac{(b + c) [c(B - C) - (b + c)(\bar{p} - C)]}{bc^2} (1 - \mathcal{F}(n^*(\bar{p}))). \quad (15)$$

(c'2) For  $\bar{p} \in (p^\delta(n_{\min}, \delta), p^*(n_{\max})]$  the ceiling may bind or not depending on  $\eta$ . There is a cut-off  $\eta$ , let it be  $n^{**}(\bar{p}, \delta)$ , implicitly defined by  $p^\delta(n^{**}, \delta) = \bar{p}$ , so that for  $\eta < n^{**}(\bar{p}, \delta)$  the ceiling does not bind and the quantity traded is  $q^d(p^\delta(\eta, \delta))$ . For  $\eta \geq n^{**}(\bar{p}, \delta)$ , the ceiling may lead to excess demand or excess supply depending on the value of  $\eta$ . Specifically,  $\exists \eta = n^*(\bar{p})$  for which  $p^*(\eta) = \bar{p}$ . This is the same as the one in (7), and  $n^{**}(\bar{p}, \delta) < n^*(\bar{p})$ . If  $n^{**}(\bar{p}, \delta) \leq \eta \leq n^*(\bar{p})$ , there is excess supply, so that consumer surplus is determined by  $q^d(\bar{p})$ . If  $\eta > n^*(\bar{p})$ , there is excess demand and consumer surplus is determined by  $q^s(\bar{p})$ , the quantity at which marginal cost equals  $\bar{p}$ . Let  $CS_\delta^H$

be the expected consumer surplus in this case. Then,  $dCS_{\delta}^H/d\bar{p}$  is given by

$$-\frac{1}{b}(B - \bar{p})(\mathcal{F}(n^*(\bar{p})) - \mathcal{F}(n^{**}(\bar{p}, \delta))) - \frac{b+c}{bc}\epsilon_d^L(n^*(\bar{p})) + \frac{1}{b}\epsilon_d^L(n^{**}(\bar{p}, \delta)) \quad (16)$$

$$+ \frac{c(B - \bar{p}) - (b+c)(\bar{p} - C)}{c^2}(1 - \mathcal{F}(n^*(\bar{p}))) ,$$

where  $n^{**}(\bar{p}, \delta) = [(\delta b + c)p - cB - \delta bC]/c$  and  $\epsilon_d^L(n^{**}(\bar{p})) = \int_{n_{\min}}^{n^{**}(\bar{p})} \eta d\mathcal{F}(\eta)$ .

**Proof of Proposition 2.** Suppose  $n_{\min} > -c(B - C)/(b + 2c)$ . Then the globally optimal price ceiling is given by (5) and independent of  $\delta$ .

Suppose  $n_{\min} < -c(B - C)/(b + 2c)$ . Let  $p^{\delta}(n_{\min}, \delta) < p^*(n_{\max})$ . If the globally optimal ceiling lies in  $[p^*(n_{\min}), p^{\delta}(n_{\min}, \delta))$ , it solves  $dCS_{\delta}^L/d\bar{p} = 0$  (see (15)). Evaluating (15) at the ceiling that solves  $dCS_d/d\bar{p} = 0$ , i.e., the optimal ceiling under perfect competition (see (9)), we obtain  $-[(B - \bar{p})\mathcal{F}(n^*(\bar{p})) + \epsilon_d^L(n^*(\bar{p}))]/b = -[(B - \bar{p}) + \epsilon_d^L(n^*(\bar{p}))]/\mathcal{F}(n^*(\bar{p}))/b < 0$ . The inequality follows as  $\epsilon_d^L(n^*(\bar{p}))/\mathcal{F}(n^*(\bar{p})) = E(\eta \mid \eta < n^*(\bar{p})) \in [n_{\min}, n_{\max}]$ .

If the globally optimal ceiling lies in  $[p^{\delta}(n_{\min}, \delta), p^*(n_{\max})]$ , it solves  $dCS_{\delta}^H/d\bar{p} = 0$  (see (16)). Whenever  $CS_{\delta}^H$  has an interior maximum in the interval  $(p^{\delta}(n_{\min}, \delta), p^*(n_{\max}))$ , the optimal ceiling is increasing in  $\delta$ . This follows from the local concavity of  $CS_{\delta}^H$  at the optimal ceiling and from  $d(CS_H^{\delta})^2/d\bar{p}d\delta = \delta b(\bar{p} - C)^2\mathcal{F}'(n^{**}(\bar{p}, \delta))/c^2 > 0$ . If the optimal price ceiling with imperfect competition lies in this sub-region, it is strictly lower than the optimal ceiling with perfect competition. Let  $p^{\delta}(n_{\min}, \delta) \geq p^*(n_{\max})$ . Then, the globally optimal ceiling lies in  $[p^*(n_{\min}), p^*(n_{\max})]$  and solves  $dCS_{\delta}^L/d\bar{p} = 0$ . The above comparison with the optimal ceiling under perfect competition applies and the result follows. ■



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