

# Theoretical modeling and experimental investigations for the improvement of the mechanical efficiency in sliding vane rotary compressors

Giuseppe Bianchi, Roberto Cipollone\*

*University of L'Aquila, Department of Industrial and Information Engineering and Economics, Via Giovanni Gronchi 18, 67100 L'Aquila, Italy*

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## Abstract

Positive displacement compressors lead the market of compressed air production for industrial applications. Among them, sliding vane rotary compressors represent an energetically virtuous alternative to the current compression technologies. In the present work, the effects of compressor design parameters were investigated through a comprehensive approach that aimed at addressing more efficient machines to promote sliding vane compressors as the key enabling technology in compressed air systems. A comprehensive mathematical model was developed to study the main phenomena occurring in this kind of compressors. The model provides the cell volume evolution over a whole rotation during which filling, compression and discharge processes occur. The first and latter phases are described by the quasi-propagatory approach that represents the inertial, capacitive and resistive features of one-dimensional unsteady flows. The dynamics of the compressor blades led to four different arrangements inside the rotor slots while an analysis of the hydrodynamic lubrication established between blade tip and stator wall focused on the oil film thickness evolution to prevent dry contacts. An extensive experimental campaign on a mid-size industrial compressor allowed the model validation at different outlet pressure levels and revolution speeds using a direct measurement of mechanical power and the reconstruction of the indicator diagram from piezoelectric pressure transducers. The friction coefficient at the contact points between blades with stator and rotor was estimated in 0.065 and further improvements of the mechanical efficiency were eventually addressed considering the roles of compressor aspect ratio, revolution speed, and blade tilt. The first two theoretical optimizations might lead to an increase of the compressor efficiency of 2 and 9 percentage points respectively. On the other hand, acting on the blade tilt would not produce relevant improvements.

*Keywords:* sliding vane rotary compressor, positive displacement compressor, compressed air systems, indicator diagram, piezoelectric pressure transducer, mechanical efficiency

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\*Corresponding author

*Email addresses:* [giuseppe.bianchi@graduate.univaq.it](mailto:giuseppe.bianchi@graduate.univaq.it) (Giuseppe Bianchi), [roberto.cipollone@univaq.it](mailto:roberto.cipollone@univaq.it) (Roberto Cipollone)

## 1. Introduction

Energy saving is today recognized as the primary action to accomplish energetic and environmental commitments of all the Countries in the World.  $CO_2$  concentration in atmosphere is a crucial concern, universally recognized as the tomorrow's main challenge. Therefore, a new energy paradigm based on a sharp reduction of the energy consumptions and a renewable-based energy production is sought by many scientific and institutional contexts.

Compressed air accounts for a mean 10 % of the global industrial electricity consumptions (7850 TWh in 2012) which reaches a share close to 20 % if commercial and residential needs are included [1, 2, 3]. Facing this issue implies the employment of many saving measures with actions upstream and downstream of the compressed air production: pipeline leakages reduction, adjustable speed drives, optimization of the end use devices, etc. In addition, with reference to electricity consumptions in compressed air systems, the saving potential related to compressor technology has been estimated 25-30 % [1]. From an economic point of view, although energy costs for compressed air are predominant with respect to the capital ones (70-75 %) [2], an investment increase of 10 % in a 10 year operating period would be feasible only if the compressor efficiency increase was 4 % greater than the former technology [3].

In industrial applications, rotary volumetric compressors are able to match flow rate and pressure level requirements with an electrical power range from a few to several hundred kilowatts. Among them, Sliding Vane Rotary Compressors (SVRC) revealed a better energetic behavior whether on/off load conditions were taken into account [3]. This operating regime is a common process to match the line pressure needs in terms of flow rate: when the air demand decreases, thanks to the automatic depressurization of the machine and some intrinsic features related to the sealing among vanes (e.g. absence of the so called "blow hole line"), SVRCs accomplish in an efficient way a process that is usually energy wasteful.

In the literature, sliding vane compressors have been investigated both theoretically and experimentally. The machine geometry and vane kinematics were modeled following a trigonometric approach [4, 5]. Circular and elliptical stator configurations to achieve higher volumetric ratios or even dual stage compressions were considered [6, 7] as well as slanted blade arrangements inside the rotor slots [8, 9]. Preliminary studies assumed suction and discharge processes as isobaric while the compression was modeled using semi-empirical formulations [6]. However, zero dimensional models for the cell thermodynamics were also developed based on the energy conservation to predict the pressure evolution over the whole cell rotation [5, 9, 10, 11]. To detail the friction power, the contribution due to the blade dynamics was widely investigated [12, 13]. Comprehensive analyses that involve secondary contributions such as friction at the bearings and on the side covers of the machine [14, 15] or the blade tip profile [16, 17] were also developed. Leakage paths models were presented assuming clearances as orifices [18]. As concerns the experimental activities, tests at different steady conditions [19, 20, 21] and on unconventional configurations like the blade tilting [22] were

35 carried out. Experimental methodologies have been set up to measure the pressure inside the compressor  
36 cells [23, 24, 25] while theoretical approaches as the Helmholtz's resonator one supported the discussion of the  
37 results [26, 27].

38 Although it deeply focused on specific aspects of the sliding vane compressor technology, the above  
39 mentioned literature does not strictly aim at maximizing the overall machine efficiency. This goal is of  
40 major concern for sliding vane rotary compressors since the benefits that would come out from an energy  
41 performance enhancement might make them as the key enabling technology in compressed air systems. At  
42 the moment, there are air flow ranges in which SVRCs are characterized by specific energy consumptions  
43 lower than other compressors [3]. The reinforcement of this issue and the broadening of their behavior at  
44 wider flow ranges appear the way to refer to these machines as premium ones concerning the energy con-  
45 sumptions. Furthermore, compared to other compression technologies, SVRCs have a relevant improvement  
46 potential whose development is referred to friction power reduction and to the internal air cooling during  
47 the compression phase [28, 29]. Despite similar methodologies are not currently available in literature, in  
48 order to identify the energy saving potential, a wide-ranging approach becomes necessary to understand how  
49 the different design parameters can contribute to the machine efficiency. A comprehensive model appears  
50 to be the best way to observe cross coupled effects among different design parameters. Once experimentally  
51 validated, it behaves as a SVRC virtual platform allowing the addressment of efficiency improvements and,  
52 at the same time, the reduction of prototype construction.

53 The model presented in the current work is composed of a geometrical section that is able to provide the  
54 cell volume evolution for any compressor layout in terms of aspect ratio, blade tilt, intake and exhaust ports  
55 arrangements. A quasi-propagatory approach described the flow dynamics during the vanes filling and emp-  
56 tying while a lumped parameter model was adopted to predict the pressure evolution over a whole rotation  
57 of the compressor cells. The study of the blade dynamics coupled with an analysis of the hydrodynamic  
58 lubrication at the blade tip provided the oil film thickness evolution at the interface with the stator wall.  
59 The role of reaction forces and pressure distribution along this oil layer is of major importance to account  
60 for losses by friction, thus to a proper investigation of the mechanical efficiency of the machine.

61 An experimental campaign on a mid-size industrial sliding vane compressor was performed at different outlet  
62 pressure levels and revolution speed regimes. The direct measurement of mechanical power, as well as the  
63 reconstruction of the indicator diagram from pressure data provided by piezoelectric transducers, allowed to  
64 validate the model and led to an estimation of the friction coefficient. The pressure distribution along the oil  
65 layer at the blade tip was further calculated. Parametric analyses on the effects of aspect ratio, revolution  
66 speed, blade tilt and mass on the mechanical efficiency of the compressor, were eventually carried out in  
67 order to identify improved configurations that would reduce the energy consumptions.

## 2. Mathematical model

The main phenomena occurring inside a sliding vane rotary compressor were modeled through an essential formulation: the sliding vane compressor from the suction port till the compressed air line inlet was represented as a sequence of ducts and capacities that exchange mass and energy according to a 1D quasi-propagatory formulation. The compressor core, i.e. the stator rotor and blades assembly, was modeled with a lumped parameter approach. Hence, the compressor cell acted as a volume of variable capacity that interacted with the adjacent cells and with the rest of the compressor through the suction and discharge ports. The solution of the momentum equation for the compressor blades provided an estimation of the friction power that directly affects the machine efficiency. The lubrication circuit modeling and the calculation of the load distribution at the blade tip eventually allowed to estimate the pressure distribution and the oil film thickness evolution along contact surface with the stator. Since the oil sealing in sliding vane compressors is effective, leakage flows modeling (for instance using semi-empirical correlations) was not pursued; this assumption allowed to keep the model platform independent from the compressor tested, thus able to simulate the performances of different machines.

### 2.1. Compressor geometry

The compressor vane geometry was modeled tracking the trajectories of all the relevant points in the compressor core represented in Figure 1.a. To investigate the effects of the blade tilt onto the mechanical efficiency of the machine, backward and forward slanting were also implemented (Figure 1.b). In these configurations, blades are tangent to an internal circumference whose radius ( $O_{rot}A$ ) is smaller than the rotor one ( $OX$ ). As concerns the intake and exhaust ports, they are located either frontally (on the covers, with an axial air path) or circumferentially (with a radial air admission or delivery). Regardless of the configuration, the angular ports positioning has a direct influence on the volumetric compression ratio i.e. the ratio between the cell volume before it opens toward the exhaust and the one after closing the inlet port. Although the model can consider ports anyway located, in current industrial compressors the intake is usually axial while the discharge is radial, as reported in Figure 1.c.

### 2.2. Vane filling and emptying

In order to evaluate the flow dynamics during the intake and exhaust processes, a Quasi Propagatory Model (*QPM*) was adopted. Based on the 1D unsteady conservation equations, the method solves them in an analytical way taking into account the capacitive (pressure as a function of inlet and outlet mass flow rates), inertial (mass flow variation as a function of a pressure difference) and resistive features (losses by friction and heat exchange) of the flow topology [30]. QPM subdivides complex one dimensional sequence of pipes anyway interconnected into elementary parts, whose extremities are stimulated by boundary conditions (*BC*) modeled as functions of pressure and flow velocity. The intersection of the curves representing the

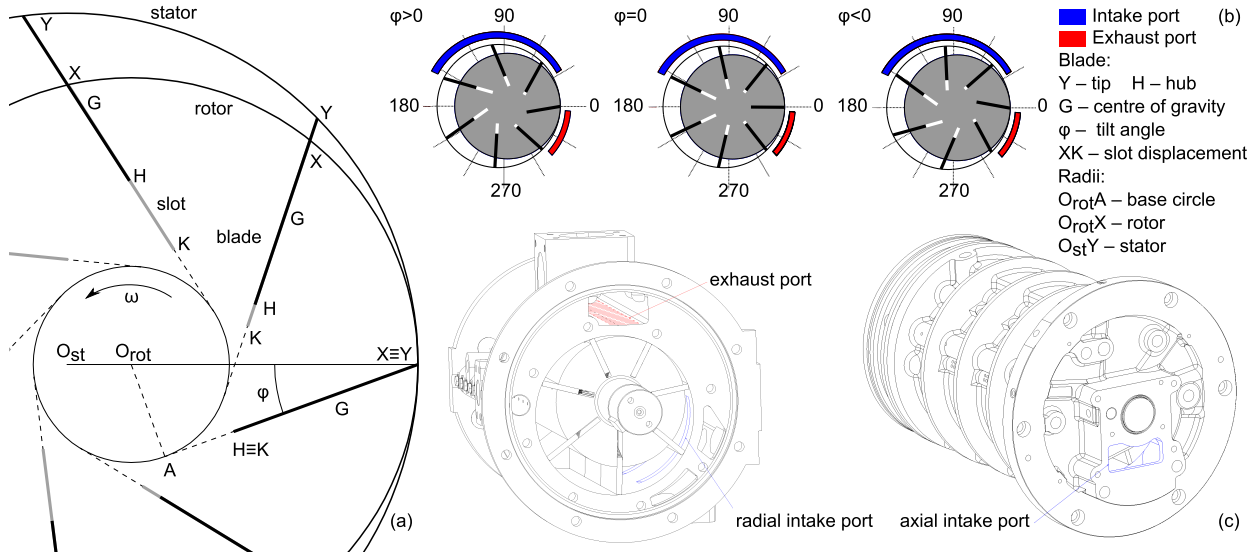


Figure 1: Compressor core - the geometrical modeling is based on the trajectories of the relevant points of the assembly (a) and may take into account forward and backward blade tilting (b) as well as axial and radial ports (c)

101 upstream and downstream BCs in the  $p - u$  plane gives the steady state ( $SS$ ) of the flow after the transient  
 102 (Figure 2.a).

103 To model the dynamics of homentropic flows only the mass and momentum conservation equations are  
 104 required. The homentropic steady state may be eventually corrected if heat exchange and friction occur  
 105 [31]. Considering the mid-point of the elementary pipe, the  $(p,u)$  solution over time is represented by the  
 106 sequence of values which result from the pressure (and speed) waves propagation calculated as intersections  
 107 between the characteristic curves entering the BCs and the BCs themselves. The novelty of the QPM lies on  
 108 the approximation of the boundary conditions with lines at constant slope (evaluated at the initial state):  
 109 this allows to linearize and speed up the calculation such that the flow evolution at a given location can be  
 110 characterized with the Equations 1:

$$p = p_u - \frac{p_u - p_\infty}{u_\infty} u = p_u - A u \quad (1a)$$

$$p = p_d - \frac{p_\infty - p_d}{u_\infty} u = p_d + B u \quad (1b)$$

111 The flow dynamics inside the pipe starts from the initial state 0 and proceeds along the characteristic curves  
 112 bounded by the BCs: 0-1 represents the propagation of the positive characteristic curve while 1-2 is related  
 113 to the propagation of the negative characteristic curve until the  $SS$ , represented by the point  $(u_\infty, p_\infty)$  in  
 114 Figure 2.a, is reached. The time base is given by the time the waves require to move between successive  
 115 intermediate states and it is equal to the ratio of the pipe length over the local speed of sound  $a$ .

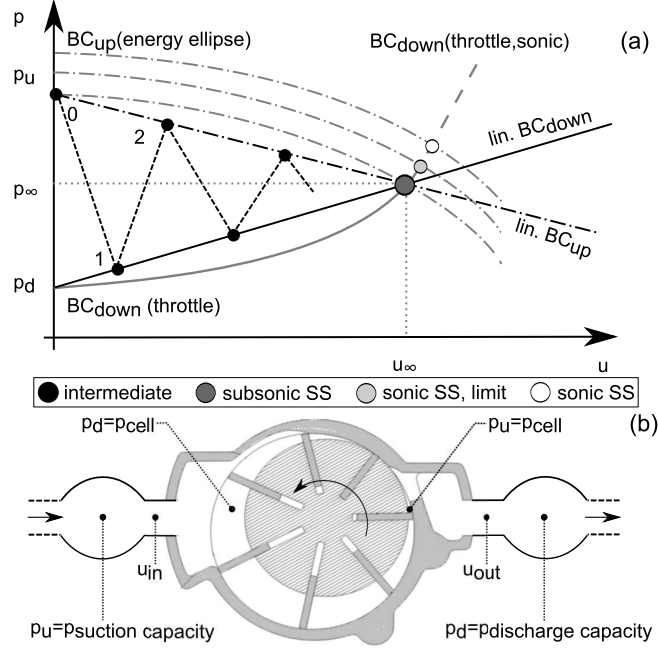


Figure 2: Quasi-propagatory flow dynamics along the suction and discharge compressor ducts

116 With reference to the initial value of pressure, the slope of the characteristic curves  $C = \gamma p'/a$  and the  
 117 linearization of boundary conditions lead to the definition of the parameter  $\Lambda$ :

$$\Lambda = \frac{(C - B)(C - A)}{(C + B)(C + A)} \quad (2)$$

118 Depending on the sign of  $\Lambda$  which is strictly related to the set of boundary conditions, the flow dynamics  
 119 follows either an asymptotical ( $\Lambda > 0$  - Eqn. 3) or an oscillatory ( $\Lambda < 0$  - Eqn. 4) trend until the steady  
 120 state is reached:

$$\dot{u} + \frac{u}{\tau} = \frac{u_\infty}{\tau} \quad (3)$$

$$\ddot{u} + \frac{2}{\tau}\dot{u} + \left(\frac{1}{\tau^2} + \Omega^2\right)u = \left(\frac{1}{\tau^2} + \Omega^2\right)u_\infty \quad (4)$$

121 The parameters  $\Omega$  and  $\tau$  depend on the boundary conditions and their slopes when linearized [30].

122 Once the midpoint velocity is evaluated at a given time step, the mass flow rate in the duct with cross  
 123 section  $S$  can be calculated according to Eqn. 5:

$$\dot{m} = \rho_\infty u S = \frac{\gamma p_\infty}{a_\infty^2} u S \quad (5)$$

124 In the compressor model, QPM decomposes the inlet and outlet sections of the compressor as a series  
 125 of ducts and reservoirs. To model the unsteadiness of vane filling and emptying, infinitesimal ducts were

126 considered between the cells and the suction and discharge capacities, as represented in Figure 2.b. The  
 127 inlet ( $u_{in}$ ) and outlet ( $u_{out}$ ) flow velocities vary with time as the result of the interaction between the  
 128 upstream and downstream BCs that characterize the intake and exhaust respectively. Both processes were  
 129 modeled as a reservoir that feeds a throttled duct: with respect to the intake pipe, during the vane filling the  
 130 environment behaves as a reservoir at ambient pressure while the motion of the compressor cell downstream  
 131 becomes a flow restriction with variable crossing area. In analytical terms, the energy conservation expressed  
 132 in terms of total pressure, leads to the energy ellipse for the upstream BC while the throttling action of  
 133 the cell considers a subsonic (parabolic) stage followed by a sonic one whose trend is linear. Similarly, the  
 134 emptying process occurs through a pipe that upstream has the discharging cell which acts as a variable flow  
 135 restriction while downstream it is connected through a plenum at the line pressure. For a given contraction  
 136 ratio  $\zeta$ , that depends on the opening of the intake and exhaust ports preliminary evaluated in the geometric  
 137 module, the steady state is analytically computable according to Eqn. 6.

$$u_{\infty}^2 = \frac{2\gamma RT_u}{\gamma - 1} \zeta^2 \left[ 1 - \left( \frac{p_d}{p_u} \right)^{\frac{\gamma-1}{\gamma}} \right] \quad (6a)$$

$$p_{\infty} = p_u \left[ 1 - \zeta^2 \left[ 1 - \left( \frac{p_d}{p_u} \right)^{\frac{\gamma-1}{\gamma}} \right] \right]^{\frac{\gamma}{\gamma-1}} \quad (6b)$$

138 Although the model scheme is the same, the boundary conditions across the suction and discharge  
 139 ducts are greatly different. Indeed, if the compressor operated at off-design conditions, at the discharge the  
 140 line pressure might be several bars different than the one in the discharging vane; a sudden pressure step  
 141 (theoretically isochoric) would thus occur because of the fixed compression ratio which is imposed by the  
 142 machine geometry. Consequently, the mismatching between pressure at the cell opening and the value of  
 143 the compressed air line would trigger pressure unsteadiness, with a magnitude proportional to the pressure  
 144 step. On the other hand, during the vane filling the pressure in the suction capacity is slightly greater than  
 145 the one in the cell. Therefore, sonic flows might occur whether the cell pressure when it opens towards the  
 146 suction port is less than 0.5 times the ambient one.

147 Using an adiabatic-isentropic approach, once the inlet and outlet velocities are known, the density cal-  
 148 culation is straightforward as well as all the thermodynamic flow properties (temperature, enthalpy, etc.).

### 149 2.3. Cell Thermodynamics

150 The model is based on a lumped parameter formulation that assumes uniform thermodynamic properties  
 151 (temperature, pressure, composition, etc.) inside the cell. Considering the lubricant injection, the working  
 152 fluid is a mixture of air and oil vapors. The cell behaves like an open system, characterized by input and  
 153 output enthalpy flows that depend on the filling and emptying processes. The first law of the thermodynamics

154 applied to an open system doing only boundary work assumes the expression in Eqn. 7:

$$\dot{E} = \dot{m}_{inl} H_{inl} - \dot{m}_{outl} H_{outl} - \dot{q}_{air-oil} - \dot{q}_{evap} - \dot{q}_{ext} - p\dot{V} \quad (7)$$

155 being  $\dot{q}_{ext}$  the thermal power exchanged with the metallic surfaces of the cells modeled according to the  
 156 approach proposed by Tan and Ooi [32].

157 The convective heat exchange between air and oil is represented by the term  $\dot{q}_{air-oil}$ . This contribution  
 158 has a great energy saving potential since it would reduce the compression work  $p\dot{V}$ : an enhanced air cooling,  
 159 for instance achieved by spraying the same quantity of oil currently injected to have a greater overall heat  
 160 exchange surface would lead towards an isothermal compression [33]. Although this technique was also  
 161 applied in screw [34] and scroll compressors [? ?], in sliding vane machines oil injections can be performed  
 162 along the axial length of the machine such that the residence time of the droplets for the heat exchange  
 163 is higher, thus the expected cooling effect on the air [35]. The mathematical approach that was used to  
 164 model the convective heat exchange between air and oil droplets relies on the Spalding low pressure film  
 165 evaporation theory [36].

$$\dot{q}_{air-oil} = \pi D_{drop} K_m Nu^* (T_{air} - T_{drop}) \quad (8)$$

166 Additional details on the thermal conductivity of the oil-air mixture  $K_m$  and the corrected Nusselt number  
 167  $Nu^*$  that are involved in Equation 8 are provided in [33, 37]. The discriminant parameter for the heat  
 168 exchange is oil droplet diameter  $D_{drop}$ . In the conventional injection technology, as the one on board of the  
 169 compressor tested for the current study, the lubricant was supplied through calibrated holes which did not  
 170 succeed to atomize the jet thus to accomplish the desired internal air cooling. This fact, together with the  
 171 rapidity of the transformation, justified the adiabaticity of the compression noticed experimentally. On the  
 172 other hand, as it was shown in [35, 37], the usage of pressure swirl nozzles led to finer oil sprays that were  
 173 able to perform an effective air cooling so reducing the compression work.

174 Even though volatility and mass diffusivity of the oil used is very low, if the oil partial pressure reached  
 175 the saturation value at the cell temperature, the lubricant would start to evaporate and gradually lower the  
 176 cell temperature. This phenomenon is taken into account in the term  $\dot{q}_{evap}$ , defined as the product of the oil  
 177 mass flow rate evaporated and the latent heat of vaporization. In this case, the additional need to compress  
 178 also the oil vapors might reverse the energy benefit. Hence, the oil vaporization should be taken under strict  
 179 control.

#### 180 2.4. Lubrication Circuit

181 In sliding vane compressors, the lubricant is injected inside the cells to mainly fulfill lubrication and  
 182 sealing purposes. At the discharge, the oil is separated from the mixture with air usually through the



183 mechanical impingement in a labyrinth chamber and a further passage through a coalescence or cyclone  
 184 separator. The oil is eventually recovered in a tank, as shown by the grey line in Figure 4. The machine  
 185 layout allows the oil circulation without an auxiliary device: it relies on the pressure difference between the  
 186 cell pressure at the discharge and the one at the injection angle as driving phenomenon ( $\alpha_{inj}$  is reported in  
 187 Table 1). Hence, the power required to pressurize the oil, that takes into account the compressor efficiency  
 188 as it was an oil pump, can be calculated according to Eqn. 9.

$$P_{oil} = \frac{Q_{oil}(p_{discharge} - p(\alpha_{inj}))}{\eta_{mech}} \quad (9)$$

189 In each injector, the oil rate was calculated as through an orifice in steady conditions. This quantity differed  
 190 from one calibrated hole to the other ones because of the sequence of pressure drops along the oil circuit,  
 191 whose extensive modeling can be found in [5].

## 192 2.5. Friction modeling and blade dynamics

193 Friction increases the overall energy expenditure for the compressed air production and it is still an open  
 194 issue for the energy saving in SVRC. Part of the mechanical power supplied is dissipated by friction at the  
 195 shaft bushes, between the rotor and the side covers of the compressor and because of the blade dynamics.  
 196 However, the first two phenomena do not produce a noticeable contribution to the overall friction power  
 197 thanks to the bush technology and to the absence of axial loads from the rotor to the covers [14]. Except  
 198 for the transient behavior during which oil must be pressurized, friction produced at the bushes is negligible  
 199 because a dry contact never appears. Hence, the blade dynamics remains the only noteworthy contribution  
 200 to the power lost by friction.

201 During its motion, the blade rotates with the rotor and slides along the slot and the inner stator surface:  
 202 consequently, there are inertial and fictitious forces (centrifugal and Coriolis) acting on it. Since blade  
 203 thickness and rotor slot width have tight tolerances to prevent leakage flows between consecutive cells, the  
 204 main degree of freedom of the blade is the translational one along the y direction of Figure 3.a. Hence, the  
 205 rotational and translational inertias along the x direction were neglected. The power dissipated by friction  
 206 depends on the reaction forces at the three contact locations, namely 1 and 2 with the side walls of the  
 207 rotor slot and 3 at blade tip. The blade arrangement inside the rotor slot may assume four configurations:  
 208 it can be tilted or pushed on the slot walls, either forward or backwards (Figure 3.b). The equilibrium  
 209 configuration is the one in which all the normal forces are of compression type. From a computational point  
 210 of view, this depends on the signs of the forces  $F_1$  and  $F_2$  with respect to the configuration shown in Figure  
 211 3.a and modeled with the Equations 10 which state the translational equilibria along the x and y directions  
 212 and the momentum equilibrium respectively.

$$F_1 = F_{pn} - F_{cor} + k_4 F_c + F_2 + k_2 F_3 \quad (10a)$$

$$k_1 F_3 = F_{in} + k_3 F_c + \lambda (F_1 + F_2) \quad (10b)$$

$$\begin{aligned} F_2 (\lambda t_{bl} - (L_{bl} - L_{out})) &= F_c (k_4 (L_{bl}/2 - L_{out}) + \dots \\ &\quad - k_3 t_{bl}/2) - F_{pn} L_{out}/2 - F_{in} t_{bl}/2 + \dots \\ &\quad - F_{cor} (L_{bl}/2 - L_{out}) - F_3 (k_2 L_{out} - k_1 t_{bl}/2) \end{aligned} \quad (10c)$$

213 Friction power is also related to the slip velocities of the contact points whose main influencing parameters  
214 are rotational speed and compressor aspect ratio, Eqn. 11.

$$P_{fr} = \lambda (F_3 U + v_{bl} (F_1 + F_2)) \quad (11)$$

215 Moreover, while the blade tip velocity  $U$  is always concordant with the angular velocity  $\omega$ , the sliding velocity  
216  $v_{bl}$  inverts its direction during a complete rotation since the blades come out from the rotor slot during the  
217 first half of the cycle and get in during the second half. The tilt angle  $\phi$  eventually affects the load along the  
218 directions parallel and orthogonal to the slot axis through the projection angles  $\chi$  and  $\xi$  that in Eqns. 10  
219 are grouped inside the projection coefficients  $k_{1-4}$ , whose mathematical formulations are reported in Eqns.  
220 12. Therefore, a modified friction power distribution and a different blade positioning inside the slot may  
221 be achieved.

$$k_1 = \cos\xi + \lambda \operatorname{sign}(\phi) \sin\xi + \lambda (1 - |\operatorname{sign}(\phi)|) \operatorname{sign}(\sin(\alpha)) \sin(\xi) \quad (12a)$$

$$k_2 = \lambda \cos\xi - \operatorname{sign}(\phi) \sin\xi - (1 - |\operatorname{sign}(\phi)|) \operatorname{sign}(\sin(\alpha)) \sin(\xi) \quad (12b)$$

$$k_3 = \cos\chi \quad (12c)$$

$$k_4 = \operatorname{sign}(\phi) \sin\chi \quad (12d)$$

222 Once blade loads and slip velocities were known, a deeper analysis was performed at the contact between  
223 blade tip and the inner stator wall. The lubrication regime was considered hydrodynamic [38]: indeed, the  
224 blade tip profile and its motion with respect to the stator pressurize the oil layer as it happens in a plane (or  
225 curved) hydrodynamic bearing. Neglecting the effect of eccentricity on the curvature of the stator surface  
226 and the transversal variation of oil pressure, at a given angular location of the contact the one-dimensional  
227 formulation of the Reynolds equation, that embeds both the continuity and momentum equations, assumed  
228 the following expression:

$$\frac{d}{dx} \left( \frac{h^3}{12\mu} \frac{dp}{dx} \right) = \frac{w}{2} \frac{dh}{dx} \quad (13)$$

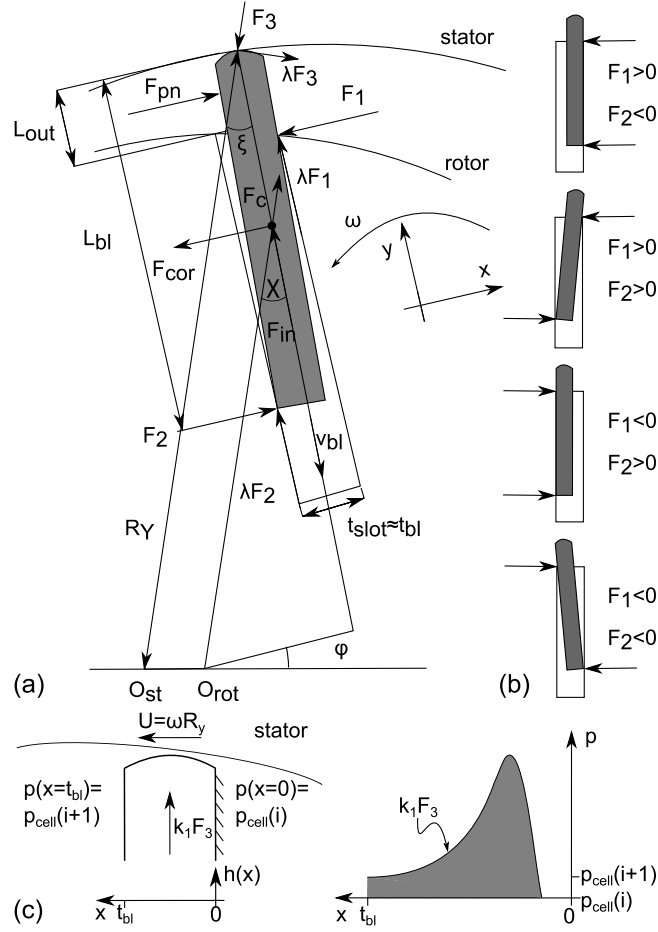


Figure 3: Blade dynamics - Free body diagram (a), possible blade arrangements inside the rotor slot (b) and reference scheme for the analysis of the hydrodynamic lubrication between blade tip and stator wall (c)

229 being  $p$  the pressure realized inside the oil layer and  $\mu$  the dynamic viscosity of the oil, a key parameter  
 230 for the lubrication phenomena which is strongly dependent on the oil temperature. However, measurements  
 231 revealed that the oil temperature from the injection angle until the discharge with the compressed air did not  
 232 increase significantly (from  $72^\circ C$  to  $83^\circ C$ ) because of the thermostatic effect of the metallic surfaces of the  
 233 machine and the high heat capacity of the lubricant. This allowed to simplify the calculation neglecting the  
 234 thermal analysis and using a value for the lubricant viscosity calculated at the mean operative temperature.

235 The oil film thickness  $h(x)$  depends on the mutual curvature between stator wall and blade tip profile at  
 236 a given angle which is known from the compressor geometry considering a unique radius of curvature for the  
 237 blade tip shape. This parameter, together with blade width, influences the oil film thickness distribution  
 238 but does not affect the blade load as long as the lubrication regime stays hydrodynamic. On the other hand,  
 239 sharp and thin blade tip profiles would ease the penetration within the the oil layer and lead to dry contacts.

240 Within the hydrodynamic assumption, the oil pressure distribution balances the blade load: the integral

241 effect of the oil pressure distribution over the whole contact surface (whose length is the stator one  $L_{st}$ ) is  
 242 indeed the orthogonal component of the reaction force at the blade tip  $k_1 F_3$  calculated with Eqn. 10.c.

$$\int_0^{t_{bl}} p(x) L_{st} dx = k_1 F_3 \quad (14)$$

243 The pressures of the consecutive cells separated by the blade act as boundary conditions for the Reynolds  
 244 equation, as shown in Figure 3.c. Equation 13 was solved considering a quasi-steady approach: for each  
 245 blade position at a given angle  $\tilde{\alpha}$  of the rotation cycle, the pressure boundary conditions imposed by the  
 246 vanes were considered constant in time as well as the normal blade load and slip velocity at the tip. The  
 247 resulting pressure distribution is reported in Eqn. 15:

$$p(x, \tilde{\alpha}) = p_{cell,i}(\tilde{\alpha}) + \int_0^x \frac{6 \mu U(\tilde{\alpha})}{h^2(x, \tilde{\alpha})} \left( 1 - \frac{h_0(\tilde{\alpha})}{h(x, \tilde{\alpha})} \right) dx \quad (15)$$

248 where  $h_0$  is the thickness at which the oil pressure reaches the maximum value. This value results from  
 249 Equation 16 knowing the pressure boundary conditions and the geometry of the contact:

$$h_0(\tilde{\alpha}) = \frac{\int_0^{t_{bl}} h^{-2}(x, \tilde{\alpha}) dx - (p_{cell,i+1}(\tilde{\alpha}) - p_{cell,i}(\tilde{\alpha})) / (6 \mu U(\tilde{\alpha}))}{\int_0^{t_{bl}} h^{-3}(x, \tilde{\alpha}) dx} \quad (16)$$

250 In this way, the pressure distribution along the contact surface and the minimum oil film thickness were  
 251 calculated over the whole rotation cycle: this value must be above a minimum threshold to prevent scuffing  
 252 or excessive wear at the stator during the machine operation.

### 253 3. Test setup

254 An experimental campaign was performed on a mid-size industrial compressor whose geometry is syn-  
 255 thesized in Table 1. The machine was instrumented with a set of T-type thermocouples along the air and  
 256 oil paths as shown in Figure 4.a, while low frequency transducers provided pressure data at the inlet and  
 257 outlet sections of the compressor.

stator diameter	136 mm	intake port start	30°
rotor diameter	111 mm	intake port end	162°
axial length	275 mm	exhaust port start	325°
blade thickness	5 mm	exhaust port end	356°
blade height	38 mm	injection angle	248°
blade mass	0.35 kg	number of cells	7

Table 1: Geometrical features of the compressor tested

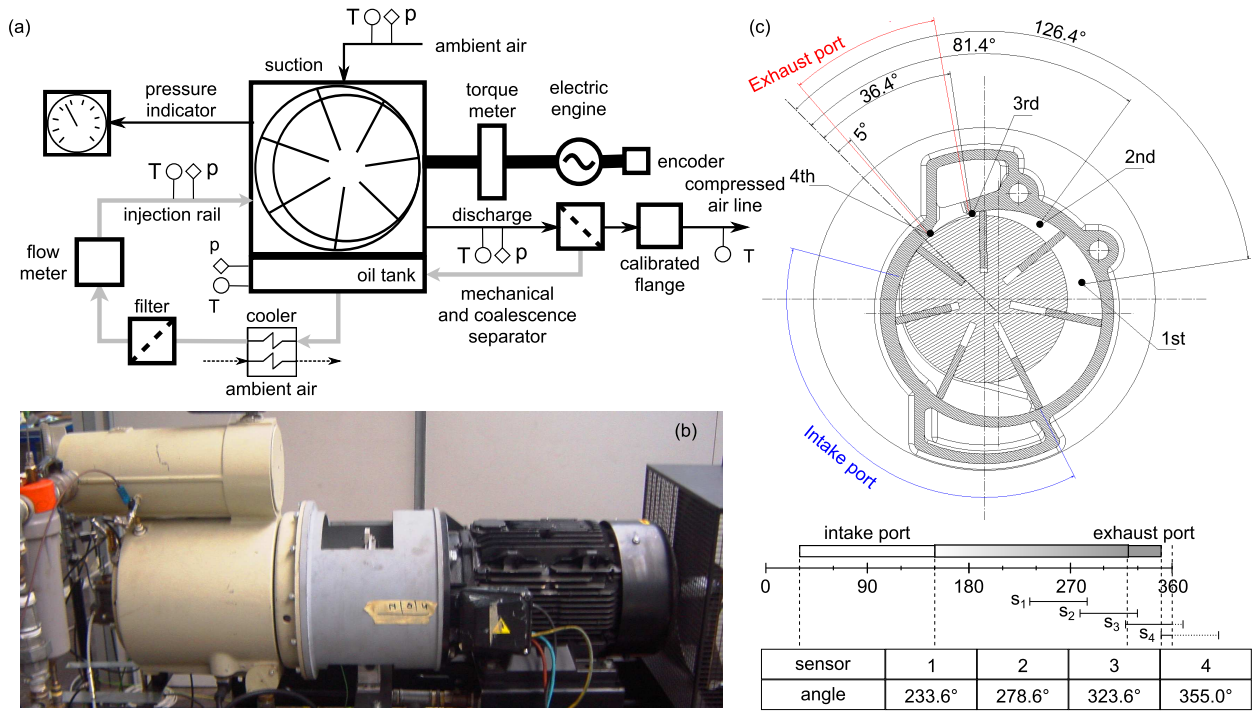


Figure 4: Experimental setup - transducers layout (a), test bench (b), piezoelectric sensors positioning (c)

258 The relevant feature of the experimental methodology lies on the accurate estimation of the mechanical  
 259 efficiency of the compressor: the mechanical power ( $P_{mech}$ ) was evaluated from the product of torque and  
 260 revolution speed measured through a flange torque meter installed between the compressor and the electric  
 261 motor (Figure 4.b). On the other hand, compression power was retrieved through the reconstruction of the  
 262 indicator diagram from the pressure traces given by a set of piezoelectric transducers.  
 263 The sensors were installed flush mounted on a side cover of the compressor, circumferentially displaced  
 264 between stator and rotor. For each of them, the measuring range is equal to the angular extent of the  
 265 compressor vane ( $\Delta\alpha = 360^\circ/N$ ): with reference to the same rotating cell, the transducer starts to measure  
 266 when the first blade with respect to the direction of rotation crosses it and ends when the second blade  
 267 leaves it. The angular position of each transducer, quoted in Figure 4.c, was carefully chosen to enhance the  
 268 unsteady phenomena that characterize the compressor. These mainly happen during the discharge phase  
 269 because of the difference between pressure realized by the volumetric ratio of the machine and the one of the  
 270 compressed air line. Since the discharge port has an angular extent ( $31^\circ$ ) lower than the measuring range  
 271  $\Delta\alpha$ , two cells discharge at the same time. For these reasons, when the process starts, during an angular  
 272 displacement equal to  $6.5^\circ$  two sensors (2, 3) measure the same pressure, so detailing the most important  
 273 unsteady phenomenon. Afterwards, sensors # 3 and # 4 measure the pressure inside the discharging cells  
 274 and inside the exhaust port. In order to reconstruct the cell pressure evolution over a whole rotation, the

275 relative pressure signals from piezoelectric transducers were referenced to the absolute values measured at  
276 the inlet and outlet of the compressor core. Different pressure offsets were applied to each signal such that  
277 a matching of the intake and discharge pressures as well as in the overlapping ranges between consecutive  
278 transducers was achieved. To further refer the reconstruction procedure to a given angle, a magnetic  
279 incremental encoder was used following the experimental methodology proposed in [23]. This device was  
280 installed on the shaft of the electric motor that was directly connected to the compressor one. The aim of  
281 the angle measurement is twofold: it allows to discriminate which part of the signals happened during the  
282 same cycle and it provides the angle evolution versus time, so taking into account the little changes of the  
283 angular velocity during a cycle that could lead to different angular displacements in the sampling period  
284 (0.1 ms). In order to compensate the effects of cyclic dispersion from one vane to the other, a phase-locked  
285 average was performed. The complete pressure trace over the whole rotation was reconstructed assuming the  
286 intake process as isobaric while the angular phase between discharge end and intake start, in which the cell  
287 crosses the tangency between rotor and stator, was considered as an adiabatic transformation because of the  
288 limited extent and the velocity of the transformation. The indicator diagram was eventually reconstructed  
289 associating the pressure-angle trace to the volume-angle graph. The reconstruction methodology was applied  
290 on the averaged signals and checking the first and second derivatives while performing the overlapping.

291 In order to evaluate the measurement uncertainty related to the indicated power, the concept of indicated  
292 mean effective pressure (IMEP) was used. This quantity is defined as the average pressure which, when  
293 multiplied by the vane swept volume, would require the same work out of the cycle as the real pressure of the  
294 indicator diagram. The uncertainty on the indicated power was defined as the ratio between the piezoelectric  
295 transducers uncertainty (0.125 bar) and the value of IMEP in each test. Hence, a conservative estimation  
296 can be safely quantified in the 5 % of the measured value. On the other hand, the direct measurement of the  
297 mechanical power allowed to reduce the uncertainty on this quantity to 0.2 % while the value related to the  
298 mass flow rate measurement through the calibrated flange (ISA 1392 nozzle) was estimated 4 % according  
299 to EN ISO 5167-3.

#### 300 4. Experimental Results and Model Validation

301 The compressor performance was investigated at different operating points listed in Table 2. The outlet  
302 pressure was varied acting on the set point of the discharge valve while a change in the revolution speed was  
303 performed by means of an inverter.

304 Figure 5 reports the comparison between experimental and simulated indicator diagrams for all the test  
305 cases. As it can be noticed from the solid lines, even at 1000 RPM the magnitude of the revolution speed  
306 narrows the cycle time for heat transfer to occur and leads to an adiabatic compression phase regardless of  
307 the outlet pressure level. On the other hand, the difference between the outlet pressure and the cell pressure

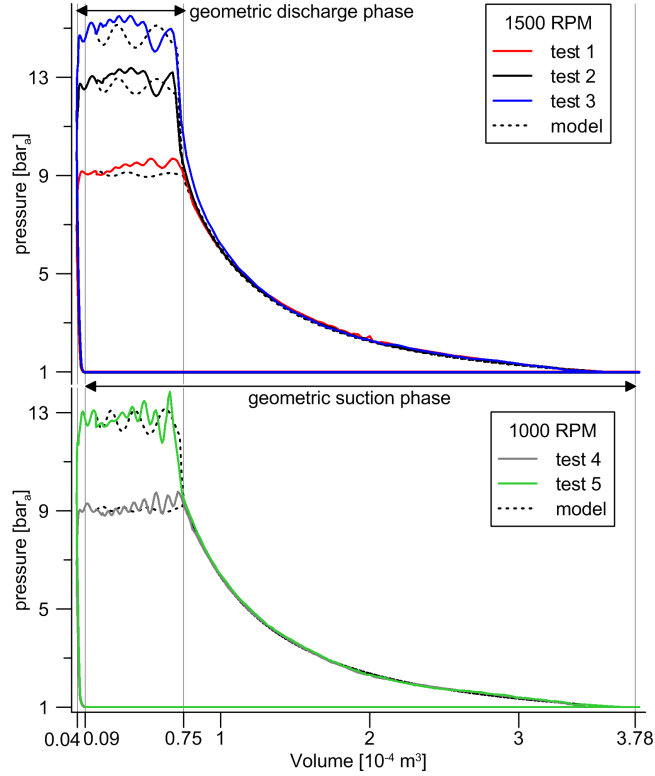


Figure 5: Experimental and simulated indicator diagrams at the operating points listed in Table 2

308 at the beginning of the discharge process affects the last part of the compression phase and triggers pressure  
 309 oscillations during the exhaust whose magnitude is proportional to the pressure gap itself. Indeed, this  
 310 phenomenon is highly remarkable at 12.5 bar and 14.5 bar. The pressure step involved at the end of the  
 311 compression phase deviates from the theoretical isochoric trend because of the dynamics of the discharge  
 312 valve. The model calculations, displayed with a dashed line in Figure 5, show a satisfactory agreement with  
 313 the experimental data although the pressure unsteadiness is not fully predicted.

314 The analysis of experimental data also allowed to calculate the volumetric efficiency of the machine as  
 315 the ratio of the actual mass flow rate measured and the theoretical value that depends on the volumetric  
 316 capacity of the machine and the air density at the suction process, as expressed by Eqn. 17.

$$\eta_{vol} = \frac{60 \dot{m}_{measured}}{\rho_{suc} N \omega V_{suc}} \quad (17)$$

317 where the units of the revolution speed  $\omega$  are RPM and  $V_{suc}$  is the cell volume at the end of the suction  
 318 process ( $\alpha_{suc} = 162^\circ + \Delta\alpha$  since it refers to the second blade of the cell with respect to the rotation  
 319 sense). The values reported in Table 2 show a strong influence of the suction temperature on the volumetric  
 320 efficiency of the compressor because it directly affects the inlet density  $\rho_{suc}$ : in test # 2,  $\eta_{vol}$  drops of almost  
 321 6 % compared to test # 1 that is at the same revolution speed. For similar suction temperatures (43.4°C

		test 1	test 2	test 3	test 4	test 5
discharge pressure	bar <sub>a</sub>	8.93	12.50	14.52	8.94	12.51
revolution speed	RPM	1470	1451	1455	995	981
mass flow rate	kg/s	0.067	0.057	0.063	0.046	0.044
indicated power	kW	18.86	21.82	23.71	12.90	14.79
mechanical power	kW	21.85	25.37	27.61	14.58	17.05
suction temperature	°C	43.4	62.8	49.5	51.8	57.0
volumetric efficiency	%	94.0	88.1	91.9	96.7	96.4

Table 2: Experimental compressor performance at various discharge pressures and revolution speeds

322 - 49.5°C at 1500 RPM and 51.8°C - 57°C at 1000 RPM), the volumetric efficiency slightly decreases at  
323 higher outlet pressures. This effect was expected due to the leakage flows between contiguous cells. However,  
324 in sliding vane compressors the effect of oil sealing is remarkable and led to these minor variations. This  
325 fact is in agreement with the assumption made when neglecting leakages in the compressor model. The  
326 experiments eventually state an increase of the volumetric efficiency at lower revolution speeds.

327 Figure 6 summarizes the model validation comparing experimental and simulated values concerned to  
328 synthetic parameters that quantify the compressor performance, namely mass flow rate and indicated power.  
329 The latter quantity is calculated from the area of the indicator diagram and the blade passing frequency  
330 using Eqn. 18:

$$P_{ind} = N \frac{\omega}{60} \oint p dV \quad (18)$$

331 Both simulated quantities falls within the uncertainty band of the measured data in all the test cases and  
332 state the confidence and reliability of the model.

333 From the energy balance at the compressor, whose expression is reported in Eqn. 19, only part of  
334 the mechanical power supplied is converted into indicated power and it is so used to compress the air; the  
335 remaining contribution accounts for the dissipations by friction (Eqn. 11) and the power needed to pressurize  
336 the oil (Eqn. 9).

$$P_{mech} = P_{ind} + P_{fr} + P_{oil} \quad (19)$$

337 Since  $P_{mech}$  and  $P_{oil}$  were available from the experiments, once the indicator diagrams were reconstructed,  
338 the friction coefficient  $\lambda$  in Eqn. 11 could be estimated in 0.065 for all the test cases.

339 The model calibration and validation allowed to reproduce the compressor performance map: Figure 7  
340 shows the air mass flow rate delivered at different outlet pressure levels and revolution speeds. As occurred  
341 in the experiments reported in Table 2, the suction conditions highly affect the volumetric efficiency of the



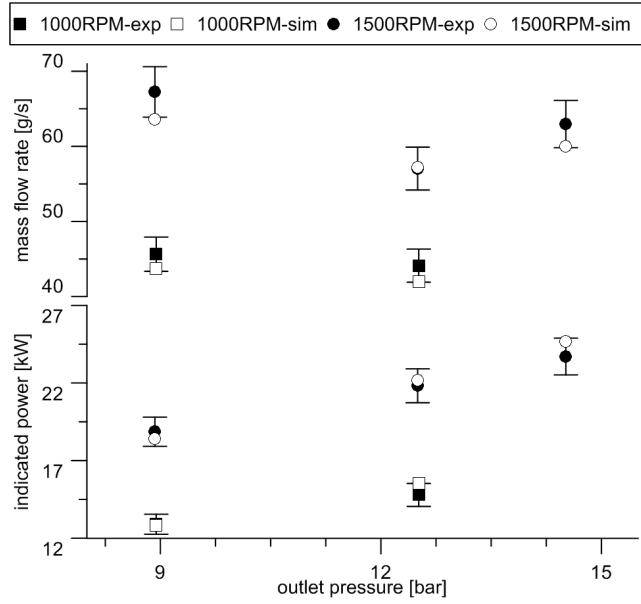


Figure 6: Model validation

342 compressor since they directly act on the inlet air density; for these reasons, the chart refers to ISO 1217  
 343 conditions (1 bar, 20°C). The volumetric nature of the machine is stated by the linear dependance of the  
 344 mass flow rate with the revolution speed. Additionally, the effect of the delivery pressure on the volumetric  
 345 efficiency of the machine can be noticed with reference to the slope of the performance curves at constant  
 346 speed: the higher is the compression ratio, the lower is the mass flow that the compressor can elaborate.

### 347 5. Parametric analysis for the performance optimization

348 In order to address future performance enhancements for the sliding vane compressor technology, a  
 349 parametric study was carried out using the simulation platform developed. The reference adopted is the  
 350 test case #1 in Table 2, a typical operating condition of the compressor in industrial applications. The  
 351 analysis aimed at maximizing the mechanical efficiency of the compressor, defined as the ratio of the power  
 352 that is actually used for the air compression and the overall mechanical power supplied (Eqn. 20).

$$\eta_{mech} = \frac{P_{ind}}{P_{mech}} \quad (20)$$

353 Figure 8 shows the effects of varying either the stator or the rotor diameter on the compressor aspect  
 354 ratio and friction power. To keep the reference mass flow rate, axial length of the compressor was varied  
 355 accordingly. The resulting machine layouts are of two categories: an elongated one, in which the axial  
 356 length is predominant with respect to the radial extent and a flat one which exhibits bigger diameters and  
 357 shorter lengths than the actual geometry reported in Table 1. A reduction of the stator diameter limits the

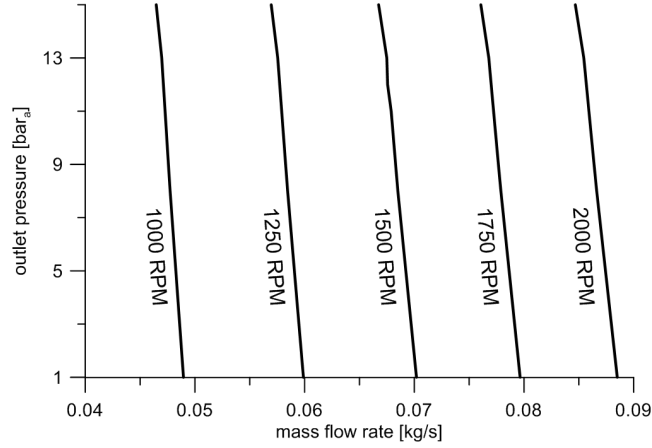


Figure 7: Simulated compressor performance map at ISO 1217 suction conditions (1 bar, 20°C)

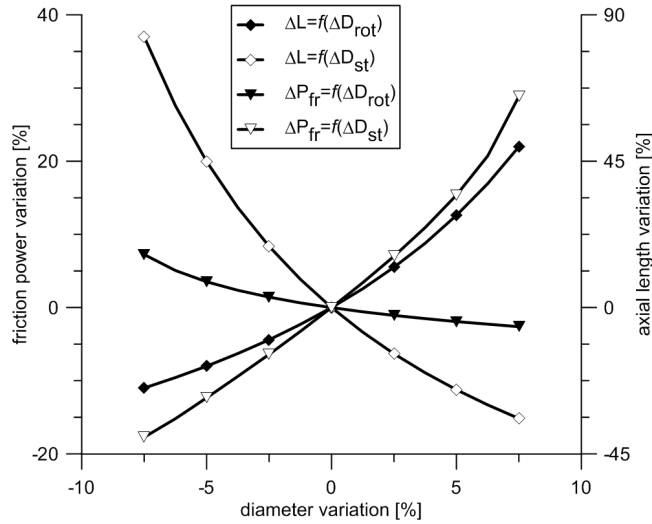


Figure 8: Influence of the aspect ratio on the compressor geometry and friction power dissipated

358 peripheral tip speed ( $U$ ) and leads to a reduction of the most significant contribution to the friction power.  
 359 On the other hand, being the revolution speed kept constant, a decrease in the rotor diameter results in an  
 360 increase of the sliding velocity  $v_{bl}$ , since the displacement that the blades have to accomplish in the same  
 361 amount of time (i.e. the revolution period) increased. Even if this contribution has a minor importance  
 362 compared to the one at the tip, a worsening of the mechanical efficiency would result from this aspect ratio.

363 A combined analysis on the effects of revolution speed and blade tilt on the mechanical efficiency of  
 364 the compressor is reported in Figure 9 in terms of specific power dissipated by friction with forward and  
 365 backward blade tilt varying the angular velocity from 1000 RPM to 2000 RPM. The reduction of revolution  
 366 speed has a direct and significant effect on friction power since it reduces both the inertial and fictitious  
 367 forces as well as all the slip velocities. This methodology cannot be applied to other rotary volumetric

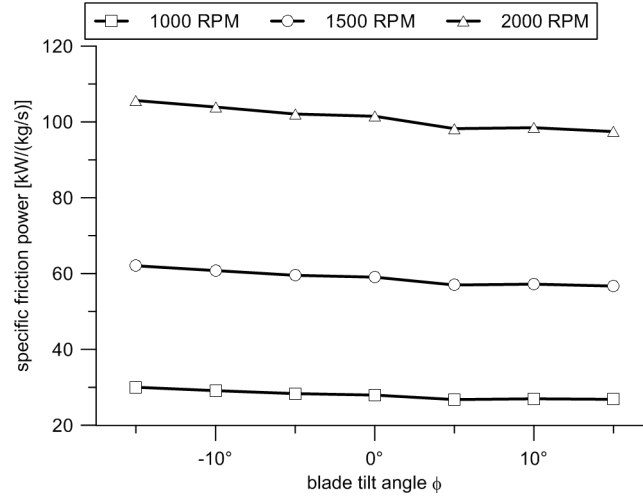


Figure 9: Variation of the friction power per unit air delivered with revolution speed and blade tilt

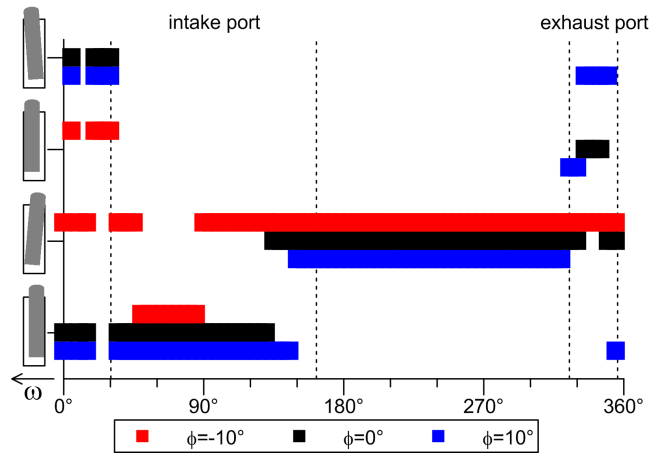


Figure 10: Blade positioning inside the rotor slot for radial and non-radial configurations

368 compression technologies (e.g. screw) and becomes a distinguishing feature of sliding vane machines. The  
 369 results presented are in agreement with the experimental data of Tramschek and Mkumbwa who tested a  
 370 circular sliding vane compressor with radial and non-radial blades [22]. The energy benefit that might be  
 371 achieved with forward-backward blade tilting is hardly noticeable, especially at low speeds.  
 372 However, the non-radial blade arrangement affects the load distribution on the blade since both the active  
 373 and reaction forces involved are differently projected along the symmetry axes of the blade, as in Figure 3.  
 374 This fact leads to another force balance, thus to a modified positioning inside the rotor slot, as displayed  
 375 in Figure 10. With reference to the radial configuration ( $\phi=0$ ), at the intake the side walls of the blade  
 376 are both exposed to the suction pressure. Therefore, inertial and fictitious forces become predominant and  
 377 push the blade backward onto the rotor slot. As the compression starts, the pressure difference between the

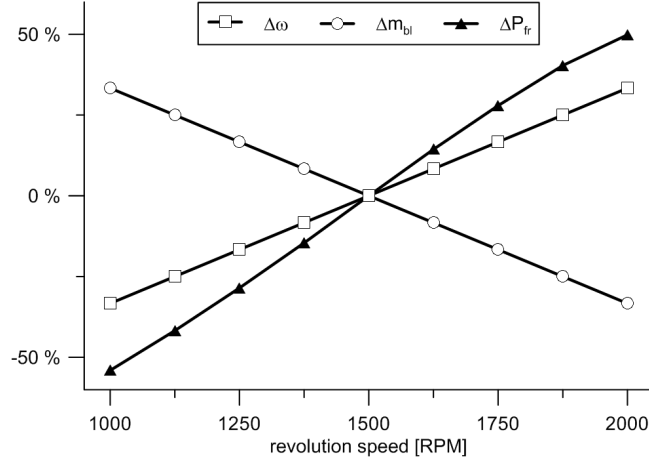


Figure 11: Effects of the revolution speed on the overall blade mass and friction power with respect to test case # 1 in Table 2

side walls tilts the blade towards the rotation sense and makes it to assume the configuration reported in Figure 3. The angular extent in which the blade assumes this position is amplified for forward blade tilt ( $\phi > 0$ ) and reduced for backward tilt ( $\phi < 0$ ). The pressure unsteadiness that characterize the discharge and the passage across the tangency between stator and rotor leads the blade to assume all the possible configurations. The combined examination of Figures 9 and 10 allowed to state that blade tilt does not effectively contribute to achieve an improvement of the mechanical efficiency of the compressor since it does modify the blade load distribution but without affecting the slip velocities at the friction locations.

Being the highest slip velocity at the contact point between blade tip and stator wall, the main contribution to the overall friction power dissipated was deeply investigated varying the mass of the compressor blade and revolution speed. Since the latter parameter directly affects the flow rate delivered by the machine, the axial length of the compressor was varied to keep the reference test conditions. Using the same linear density for the blade, the length adjustment led to heavier (longer) blades whether the revolution speed was lower than 1500 RPM and to lighter machines at revolution speeds higher than the reference value.

Although both mass and revolution speed affect the friction power, while the mass dependence is linear, the effects of revolution speed are more noticeable since they have a quadratic trend on the centrifugal force as well as a linear influence on the slip velocities at all the contact points. However, as displayed in Figure 11, if the volumetric capacity of the machine is kept constant, the blade mass varies with an opposite trend with respect to the revolution speed. Hence, the benefits of lowering the angular velocity of the machine onto the friction power become linear.

Using the simulation approach of Figure 11, a further analysis was performed at the same revolution speeds additionally aiming at addressing the benefits of a blade mass reduction. This could be achieved through a mechanical removal of some material from the current blades or using unconventional materials able to match the lubrication issues. The results displayed in Figure 12 are reported in terms of minimum

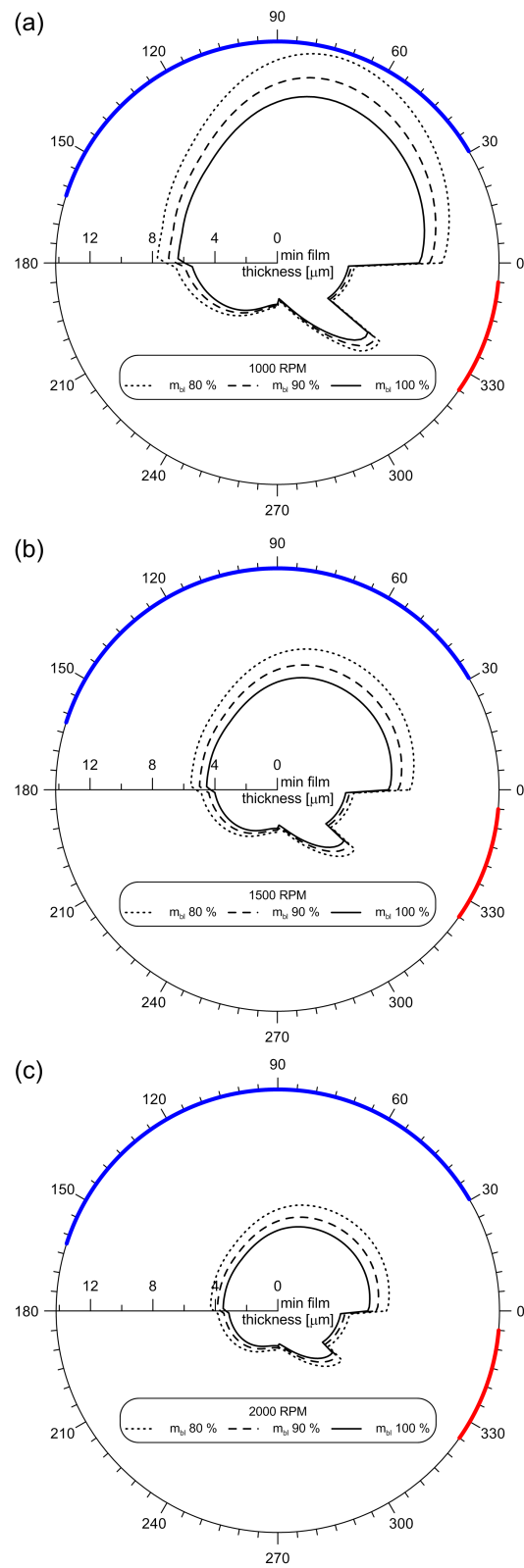


Figure 12: Effects of the blade mass reduction on the minimum thickness of the oil film established between the blade tip and the stator wall at 1000 (a), 1500 (b) and 2000 RPM (c)

401 value of the oil film thickness to highlight the hydrodynamic capabilities of the lubricant layer in withstanding  
402 the blade load. Furthermore, this parameter is inversely proportional to the maximum pressure along the  
403 contact surface. All the simulations show that the oil film thins during the second half of the rotation since  
404 the inertia force becomes concordant to the centrifugal force (as in Figure 3) increasing the blade load at  
405 the tip. The highest stress is reached when the vane pressure is maximum, i.e. before the cell opens towards  
406 the exhaust port. The location of the minimum film thickness is anticipated of  $\Delta\alpha$  in Figure 12 since the  
407 graph refers to the second blade of the cell with respect to the rotation sense. The exhaust opening produces  
408 a discontinuity, thanks to which minimum thickness increases. During this phase, indeed, pressure forces  
409 acting on the lateral blade surfaces are equal and this reduces  $k_1 F_3$  term. The blade result less loaded and  
410 oil thickness tends to increase. When the exhaust port is closed the remaining air is pressurized and the  
411 minimum thickness reassumes the values it had before the port opening. On the other hand, during the  
412 suction process, both sides of the blade are again exposed to the same pressure performing a reduction of  
413 the load at the blade tip that ends up in a thickening of the oil film.

414 When the revolution speed increases, the minimum thickness decreases being the effects of the centrifugal  
415 force more severe. Hence, critical operating conditions get closer, opening the way to possible dry contacts.  
416 Furthermore, at any revolution speed, lighter blades lead to thicker oil layers thus to a decrease in the  
417 orthogonal load between the blade tip and the stator wall. In this way, a reduction of the friction power  
418 dissipated might be achieved. However, these actions need to deal with the blade stability and sealing issues  
419 to prevent mass leakages between adjacent vanes.

## 420 6. Conclusions

421 Sliding vane rotary compressors might represent an astonishing technology to accomplish energy saving  
422 commitments in compressed air systems. The current work investigated the peculiar features of these  
423 machines from theoretical and experimental viewpoints aiming at characterizing the compressor energy  
424 performance and addressing improvements of the mechanical efficiency.

425 The comprehensive mathematical model developed considers all the relevant phenomena involved in a  
426 sliding vane rotary compressor. The geometry was modeled to explore any machine configuration: backward  
427 and forward blade tilt can be simulated as well as suction and exhaust ports with an axial or radial arrange-  
428 ments. The flow dynamics was investigated using a quasi-propagatory approach that takes into account  
429 the inertial, capacitive and resistive features of the flow during vane filling and emptying processes. On the  
430 other hand, solving the unsteady energy equation at the compressor cells provided the pressure evolution  
431 over the whole rotation cycle. The blade dynamics and the analysis of the hydrodynamic lubrication between  
432 blade tip and stator wall were eventually implemented to characterize the friction power dissipated, thus  
433 the mechanical efficiency of the compressor.

strategy	design parameter change	$\eta_{mech}$
test #1 Table 2	reference	86.3%
aspect ratio	-5% $D_{st}$ , + 45% $L_{st}$	88.0%
blade tilt	+10°	86.8%
revolution speed + blade tilt	1000 RPM, +10°	95.5%
revolution speed, constant blade mass	1000 RPM	95.2%
revolution speed, variable blade mass	1000 RPM, +50% $m_{bl}$	91.5%

Table 3: Optimization summary

434 An experimental test campaign on a mid-size industrial compressor at different revolution speeds (1000  
435 and 1500 RPM) and discharge pressures (9, 12.5 and 14.5 bar) was further carried out using a set of  
436 piezoelectric transducers to retrieve the indicator diagram. The narrow time cycle led to an adiabatic  
437 compression phase in all the test cases while the outlet pressure value influenced the unsteadiness during the  
438 discharge process. Hence, the indicated power and the mass flow rate measured validated the model results  
439 with simulated values within the uncertainty bands and provided an estimation of the friction coefficient of  
440 0.065.

441 The simulation platform was ultimately used to investigate geometrical and operating parameters of the  
442 compressor to maximize its mechanical efficiency (Table 3). Since friction power dissipated between blade  
443 tip and stator wall is the most significant contribution, a change in the compressor aspect ratio would result  
444 effective whether a reduction of the stator diameter was realized. Accordingly, a reduction of the blade mass  
445 would decrease the reaction force at the blade tip that balances the centrifugal effects while decreasing the  
446 revolution speed would also act on the slip velocities. However, these actions need to be applied without  
447 affecting the vane sealing and blade stability. Although the blade positioning inside the rotor slot during a  
448 whole rotation is affected by a forward or backward blade tilt, a noticeable effect on the overall friction power  
449 dissipated is not achieved: the blade tilt affects the load distribution but does not modify the slip velocity  
450 at the friction contact points. The oil layer at blade tip, that assures the sealing between vanes, assumes  
451 its most critical conditions when the blade is approaching the exhaust port. Operating conditions at lower  
452 revolution speed and lighter blades would favor the hydrodynamic lubrication and keep the minimum oil  
453 film thickness above a safety threshold to prevent dry contacts with the stator wall.

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## List of symbols

$\alpha$	angular coordinate	$[rad]$	$P$	power	$[W]$
$\gamma$	specific heat ratio	$[-]$	$Q$	flow rate	$[m^3/s]$
$\xi$	contraction ratio	$[-]$	$R$	gas constant	$[J/kg/K]$
$\eta$	efficiency	$[-]$	$R_Y$	tip radius	$[m]$
$\lambda$	friction coefficient	$[-]$	$S$	duct cross section area	$[m^2]$
$\mu$	oil dynamic viscosity	$[Pa\ s]$	$T$	temperature	$[K]$
$\zeta$	force projection angle	$[rad]$	$U$	blade tip speed	$[m/s]$
$\rho$	density	$[kg/m^3]$	$V$	cell volume	$[m^3]$
$\tau$	QPM parameter	$[s]$	<b>Subscripts and superscripts</b>		
$\phi$	blade tilt angle	$[rad]$	$'$	initial state	
$\chi$	force projection angle	$[rad]$	$0$	thickness at maximum pressure	
$\omega$	revolution speed	$[RPM]$	$\infty$	steady state	
$\Lambda$	QPM parameter	$[-]$	bl	blade	
$\Omega$	QPM parameter	$[s^{-1}]$	c	centrifugal	
$a$	speed of sound	$[m/s]$	cor	Coriolis	
$h$	film thickness	$[m]$	d	downstream	
$k$	force projection coefficient	$[-]$	drop	oil droplet	
$m$	mass	$[kg]$	evap	evaporation	
$p$	pressure	$[Pa]$	ext	external	
$q$	heat flux	$[J]$	fr	friction	
$t$	thickness	$[m]$	in	inertia	
$u$	flow velocity	$[m/s]$	ind	indicated	
$v$	slip velocity	$[m/s]$	inj	injection	
$w$	oil flow velocity	$[m/s]$	inl	inlet	
$A$	upstream boundary condition slope	$[N\ s/m^3]$	mech	mechanical	
$B$	downstream boundary condition slope	$[N\ s/m^3]$	out	outside the rotor slot	
$C$	characteristic curve slope	$[N\ s/m^3]$	outl	outlet	
$D$	diameter	$[m]$	pn	normal pressure	
$E$	absolute internal energy	$[J]$	st	stator	
$F$	force	$[N]$	suc	suction conditions	
$H$	specific enthalpy	$[J/kg]$	u	upstream	
$K_m$	mixture thermal conductivity	$[W/m/K]$	vol	volumetric	
$L$	length	$[m]$	<b>Mathematical operators</b>		
$N$	number of cells	$[-]$	$\dot{\Upsilon}$	first time derivative of $\Upsilon$	
$Nu^*$	corrected Nusselt number	27 $[-]$	$\ddot{\Upsilon}$	second time derivative of $\Upsilon$	