MGF based energy detection of unknown signals in $\kappa-\mu$ shadowed fading channel and diversity reception

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The performance of an energy detector over $\kappa-\mu$ shadowed fading is evaluated using the moment generating function of the received instantaneous signal to noise ratio. The study is then extended to be included various diversity reception schemes over independent and identically $(i.i.d)$ $\kappa-\mu$ shadowed fading channels. To this end, both the average detection probability and the average area under the receiver operating characteristics curve are derived.

Introduction: Among many spectrum sensing techniques, energy detection (ED) has been greatly employed. Because the secondary user (SU) can detect any type of primary user (PU)'s signal including unknown with low computational and implementation complexities [1]. Hence, the performance of ED has been extensively analysed in the open technical literature. In [1, 2], the average probability of detection ($\overline{P_d}$) and the probability of false alarm (P_f) over Rayleigh, Nakagami-m and Rician fading channels with different diversity schemes are derived. The behaviour of ED over different generalised fading channels that provide better fitting to the practical measurements than the traditional models is investigated in [3-5]. In [3, 4], the performance of ED over $\eta-\mu$ which models the non-line-of-sight (NLoS) communication scenarios is studied. The analysis in LoS scenarios using $\kappa-\mu$ fading is presented in [5].

The wireless signals may undergo shadowing and multipath fading simultaneously. In [6, 7], the performance of ED in K and K_G fading channels which are mixed of Rayleigh/gamma and Nakagami-m/gamma respectively is given for no diversity and diversity reception. The behaviour of ED in gamma shadowed Rician fading with different diversity combining is recently investigated by [8]. The performance of ED over $\kappa-\mu$ shadowed fading which is newly proposed by [9] as a composite $\kappa-\mu$/gamma fading is evaluated in [10].

Unlike [10] where the probability density function (PDF) of the received instantaneous signal to noise ratio (SNR) is employed, this paper provides comprehensive analysis of ED's behaviour over $\kappa-\mu$ shadowed fading using the moment generating function (MGF) approach. In spite of this approach leads to limited expressions by some conditions such as μ should be an integer number, it gives tractable closed-form analytic expressions. The analysis is then extended to maximal ratio combining (MRC), square law combining (SLC) and square law selection (SLS) diversity schemes over independent and identically $(i.i.d)$ $\kappa-\mu$ shadowed fading channels.

Energy detection model: The principle work of the ED is based on filtering the received signal with an ideal band-pass filter with bandwidth W. Then, the filtered signal is squared and integrated over T time interval to compute the decision statistic Λ. The distribution of Λ is either central chi-square or non-central chi-square. The former represents the hypothesis $\mathcal{H}_0$ which means the received signal is noise only whereas the latter represents the hypothesis $\mathcal{H}_1$ which means the received signal is the PU's signal plus noise. Thereafter, Λ is compared with a threshold value, λ, to decide whether the PU is present or absent. Accordingly, in additive white Gaussian noise (AWGN), this yields [2]

$$P_d = Pr(\Lambda > \lambda|\mathcal{H}_1) = Q_u(\sqrt{2\gamma}, \sqrt{\lambda}) \quad (1)$$

$$P_f = Pr(\Lambda > \lambda|\mathcal{H}_0) = \frac{\Gamma(u, \lambda/2)}{\Gamma(u)} \quad (2)$$

where $u = TW$ and γ, $\Gamma(.,.)$, $\Gamma(.)$, and $Q_u(.,.)$ are the received instantaneous SNR, the upper incomplete Gamma function, the Gamma function and the uth order generalized Marcum-Q function respectively.

The $\kappa-\mu$ shadowed fading: The $\kappa-\mu$ shadowed fading contains κ, μ and m as fading parameters. Here, κ represents the ratio between the total power of the dominant components and the total power of the scattered waves, μ indicates the number of the multipath clusters and m stands for the shadowing severity index.

The MGF of the received instantaneous SNR, γ, over $\kappa-\mu$ shadowed fading is expressed by [9]

$$\mathcal{M}_\gamma(s) = \frac{D}{(c_1+s)^{\mu-m}(c_2+s)^m} \quad (3)$$

where $D = \frac{\mu^\mu m^m (1+\kappa)^\mu}{\bar{\gamma}^\mu(\mu\kappa+m)^m}$, $c_1 = \frac{\mu(1+\kappa)}{\bar{\gamma}}$ and $c_2 = \frac{m}{\mu\kappa+m}c_1$.

Performance analysis of ED-no diversity: The performance of ED over fading channels is measured by the average probability of detection $\overline{P_d}$ and the average area under the receiver operating characteristics curve (AUC), $\bar{A}$. The former plots $\overline{P_d}$ against P_f to provide the receiver operating characteristics (ROC) curve whereas the latter measures the area under the ROC. Since (1) is for AWGN, it can not be used directly to evaluate the $\overline{P_d}$ over fading channels. Therefore, the $\overline{P_d}$ can be calculated by using the MGF of the instantaneous SNR γ as follows [1],

$$\overline{P_d} = \frac{e^{-\frac{\lambda}{2}}}{j2\pi}\oint_\Omega \mathcal{M}_\gamma\left(1-\frac{1}{z}\right)\frac{e^{\frac{\lambda}{2}z}}{z^u(1-z)}dz \quad (4)$$

where Ω is a circular contour of radius $\mathcal{R} \in [0, 1)$.

Now, substituting (3) into (4), we have

$$\overline{P_d} = \frac{De^{-\frac{\lambda}{2}}}{j2\pi(1+c_1)^{\mu-m}(1+c_2)^m}\oint_\Omega g_1(z)dz \quad (5)$$

where $g_1(z) = \frac{e^{\frac{\lambda}{2}z}}{z^{u-\mu}(1-z)(z-\frac{1}{1+c_1})^{\mu-m}(z-\frac{1}{1+c_2})^m}$.

The contour integral in (5) can be computed by applying the Residue theorem. In this theorem, the residue of a general function $g(z)$ with a pole at $z = z_0$ ($\text{Res}(g; z_0)$) in radius $\mathcal{R} \in [0, 1)$ is given by

$$\text{Res}(g; z_0) = \frac{1}{(a-1)!}\left[\frac{d(g(z)(z-z_0)^a)}{dz^{a-1}}\right]\bigg|_{z=z_0} \quad (6)$$

The residue values can be exactly computed by some mathematical software packages such as MATLAB and MATHEMATICA.

If both μ and m in (3) are assumed to be integer numbers i.e., μ and $m \in \mathbb{Z}$, the $\overline{P_d}$ of (5) is expressed in four cases follows.

$$\overline{P_d} = \begin{cases} \frac{De^{-\frac{\lambda}{2}}}{(1+c_1)^{\mu-m}(1+c_2)^m}[\chi_0+\chi_1+\chi_2]: & u>\mu, \mu>m \\ \frac{De^{-\frac{\lambda}{2}}}{(1+c_1)^{\mu-m}(1+c_2)^m}[\chi_0+\chi_2]: & u>\mu, \mu\le m \\ \frac{De^{-\frac{\lambda}{2}}}{(1+c_1)^{\mu-m}(1+c_2)^m}[\chi_1+\chi_2]: & u\le\mu, \mu>m \\ \frac{De^{-\frac{\lambda}{2}}}{(1+c_1)^{\mu-m}(1+c_2)^m}[\chi_2]: & u\le\mu, \mu\le m \end{cases} \quad (7)$$

where $\chi_0 = \text{Res}(g_1; 0)$, $\chi_1 = \text{Res}(g_1; \frac{1}{1+c_1})$ and $\chi_2 = \text{Res}(g_1; \frac{1}{1+c_2})$ are the residue of function $g_1(z)$ at $z = 0$, $z = \frac{1}{(1+c_1)}$ and $z = \frac{1}{(1+c_2)}$ respectively.

It is noted that, when $\kappa \to 0$, $\mu = 1$ and $m \to \infty$ and with some mathematical manipulations, (7) is equivalent to [1, (8)]. In addition, (7) be identical to [1, (21)] when $m \to \infty$ and $\mu = 1$ with $\kappa = K$ i.e., Rician fading. However, (7) is expressed in closed-form.

The $\bar{A}$ can be also calculated by the MGF of the instantaneous received SNR γ as follows [4]

$$\bar{A} = \frac{1}{j2\pi}\oint_\Omega \frac{\mathcal{M}_\gamma(1-\frac{1}{z})}{z^u(2-z)^u(1-z)}dz \quad (8)$$

By plugging (3) into (8), this yields

$$\bar{A} = \frac{D}{j2\pi(1+c_1)^{\mu-m}(1+c_2)^m}\oint_\Omega g_2(z)dz \quad (9)$$

where $g_2(z) = \frac{e^{-\frac{\lambda}{2}z}}{(2-z)^u}g_1(z)$.

The residue theorem is also employed to calculate the contour integral in (9). Similar to (7), the result of (9) is four different cases

$$\bar{A} = \begin{cases} \frac{D}{(1+c_1)^{\mu-m}(1+c_2)^m}[\Upsilon_0+\Upsilon_1+\Upsilon_2]: & u>\mu, \mu>m \\ \frac{D}{(1+c_1)^{\mu-m}(1+c_2)^m}[\Upsilon_0+\Upsilon_2]: & u>\mu, \mu\le m \\ \frac{D}{(1+c_1)^{\mu-m}(1+c_2)^m}[\Upsilon_1+\Upsilon_2]: & u\le\mu, \mu>m \\ \frac{D}{(1+c_1)^{\mu-m}(1+c_2)^m}[\Upsilon_2]: & u\le\mu, \mu\le m \end{cases} \quad (10)$$

where $\Upsilon_0 = \text{Res}(g_2; 0)$, $\Upsilon_1 = \text{Res}(g_2; \frac{1}{1+c_1})$ and $\Upsilon_2 = \text{Res}(g_2; \frac{1}{1+c_2})$ are the residue of the function $g_2(z)$ at $z = 0$, $z = \frac{1}{(1+c_1)}$ and $z = \frac{1}{(1+c_2)}$ respectively.

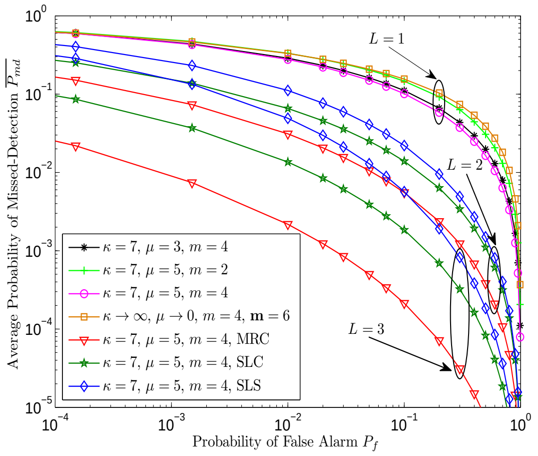

Fig. 1 *CROC curves for different diversity combining with u = 1.*

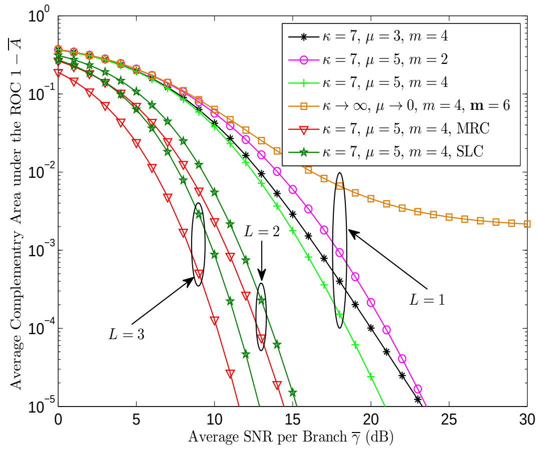

Fig. 2 *CAUC against $\bar{\gamma}$ for different diversity combining with u = 1.*

It is noted that, when $\kappa \to 0$ and $m \to \infty$ with some mathematical operations, (10) gives [4, (5)] with $\mu = m$ i.e., Nakagami-m fading.

Performance analysis of ED-MRC diversity: MRC is based on summing the received instantaneous SNR of all branches. Thus, the MGF of the received instantaneous SNR over MRC is given by $\mathcal{M}_{\gamma}^{MRC}(s) = \prod_{i=1}^{L} \mathcal{M}_{\gamma_i}(s)$ where L and $\mathcal{M}_{\gamma_i}(s)$ are the number of diversity branches and the MGF of the ith branch respectively [3]. For MRC over $i.i.d$ $\kappa - \mu$ shadowed fading channels, $\mathcal{M}_{\gamma}^{MRC}(s)$ is expressed by

$$\mathcal{M}_{\gamma}^{MRC}(s) = \frac{D^L}{(c_1+s)^{L(\mu-m)}(c_2+s)^{Lm}} \tag{11}$$

By comparing (11) with (3), one can see that the $\overline{P_d}$ and the $\bar{A}$ for MRC can be easily calculated by (7) and (10) respectively after replacing D, μ and m by D^L, $L\mu$ and Lm respectively.

Performance analysis of ED-SLC diversity: The difference between MRC and SLC which sums the calculated energy by each branch is the time-bandwidth product i.e., Lu in SLC and u in MRC [3]. Accordingly, the $\overline{P_d}$ and the $\bar{A}$ with SLC can be evaluated by (7) and (10) respectively after replacing u, D, μ and m by Lu, D^L, $L\mu$ and Lm respectively.

Performance analysis of ED-SLS diversity: SLS is based on selecting the branch of highest energy among all branches. The average probability of detection over $i.i.d$ $\kappa - \mu$ fading channels, $\overline{P}_d^{SLS}$, can be computed by inserting (4) into [2, (5)] to yield

$$\overline{P}_d^{SLS} = 1 - \left[1 - \frac{De^{-\frac{\lambda}{2}}}{j2\pi(1+c_1)^{\mu-m}(1+c_2)^m} \oint_{\Omega} g_1(z)dz\right]^L \tag{12}$$

Performance analysis of ED over $\kappa - \mu$ extreme shadowed fading: In sometimes, the total power of the dominant components is much higher than the total power of the scattered waves i.e., $\kappa \to \infty$ and the number of the multipath is too small i.e., $\mu \to 0$ [11]. In this case, the $\kappa - \mu$ shadowed fading is called the $\kappa - \mu$ extreme shadowed fading and it is utilised to model the wireless communications scenarios in enclosed areas such as buildings. The performance of ED over this channel can be deduced by inserting $\kappa \to \infty$, $\mu \to 0$ and $\kappa\mu \approx \mathbf{m}$ where $\mathbf{m}$ is the fading severity index into (7) and (10). Indeed, the condition $u > \mu$, $\mu \leq m$ is only satisfied.

Numerical results: Fig. 1 and Fig. 2 explain the complementary ROC (CROC) curve ($\overline{P_{md}}$= 1-$\overline{P_d}$ versus P_f) and the complementary AUC (1-$\bar{A}$) against SNR respectively for different scenarios and diversity combining schemes. As expected, when μ or/and m increase, the ED performance becomes better. This is because higher μ and m correspond to large number of the multipath clusters and less shadowing effects at the SU side respectively. Moreover, MRC outperforms both SLC and SLS. This refers to a higher received instantaneous SNR in MRC in comparison with SLC and SLS.

Conclusion: In this letter, we have extensively analysed the performance of ED over $\kappa - \mu$ shadowed fading channels with no diversity, MRC, SLC and SLS reception schemes. Both the $\overline{P_d}$ and the average AUC expressions were derived in closed-form using the MGF of the received instantaneous SNR. From the results, we noted that the detection capability improves when μ or/and m increase. The parameter μ has higher impacts on the detectability of ED in comparison with m. Moreover, the ED with MRC has better performance than SLC and SLS. The case of reaching κ and μ for their extreme values is also explained. These results can be employed to obtain on wide insight about the behaviour of ED over different composite fading channels.

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