# An Exploration Into How Year Six Children Engage With Mathematical Problem Solving

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# Abstract

This thesis provides some new insight into children's strategies and behaviours relating to problem solving. Problem solving is one of the main aims in the renewed mathematics National Curriculum 2014 and has appeared in the Using and Applying strands of previous National Curriculums. A review of the literature provided some analysis of the types of published problem solving activities and attempted to construct a definition of problem solving activities. The literature review also demonstrated this study's relevance. It is embedded in the fact that at the time of this study there was very little current research on problem solving and in particular practitioner research.

This research was conducted through practitioner research in a focus institution. The motivation for this research was, centred round the curiosity as to whether the children (Year Six, aged 10 -11 years old) in the focus institution could apply their mathematics to problem solving activities. There was some concern that these children were learning mathematics in such a way as to pass examinations and were not appreciating the subject.

A case study approach was adopted using in-depth observations in one focus institution. The observations of a sample of Year Six children engaged in mathematical problem solving activities generated rich data in the form of audio, video recordings, field notes and work samples. The data was analysed using the method of thematic analysis utilising Nvivo 10 to code the data. These codes were further condensed to final overarching themes. Further discussion of the data shows both mathematical and non-mathematical overarching themes. These themes are discussed in more depth within this study. It is hoped that this study provides some new insights into children's strategies and behaviours relating to problem solving in mathematics.

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# Chapter One - Introduction

This chapter aims to establish my motivation for conducting the current research project. In addition, I will also explore the research that has inspired me and the way my professional background has influenced this research project. Within this chapter I also discuss the purpose of my research which includes my research question and how I have organised my dissertation.

## 1.1 The Inspiration For My Research

I begin my introductory chapter by discussing the motivational factors which led me to carry out my research. These factors are important and ultimately had a bearing on the research choices I made. I will elaborate on these further within Chapter Three, my Methodology Chapter. Some influences are professional and linked to my roles in educational establishments, whilst others were personal and connected to my professional development as a primary school teacher. To that end I have included the following section explaining my teaching background and how this has inspired my research.

At the commencement of my doctoral studies I was a primary school class teacher working in a Greater London state school. During my teaching career I have taught in both the independent and state sectors of the English education system. The experience of teaching in these schools had an influence on both my choice of subject and the focus of my research. It was whilst I was teaching that I started to question how children applied their learning from one subject to other classroom situations.

My curiosity to carry out research was prompted by the mathematics curriculum I was teaching. I perceived this curriculum to be fractured and compartmentalised. Through various government strategies the mathematics curriculum has been structured into several discrete units. There appears to be a reliance on skills learning and very little time put aside for consolidation and application of these skills. Although, the subsection of the mathematics programme of study 'Using and Applying' of skills appeared in the original 1988 National Curriculum and remained constant throughout the evolution of the curriculum, it has become lost amongst the varying foci of these strategies. Ernest (1996) suggested that the interpretation of the curriculum has led to a perceived compartmentalization of the curriculum and this is apparent in many commercially produced mathematics schemes. This discussion is continued later in the review of the literature.

My interest in problem solving was initially due to my teaching experiences in primary schools. Whilst I taught mathematics I had anecdotal evidence of children struggling when they were completing problem solving questions. These children appeared to confidently complete pages of algorithms quickly and had developed excellent mental strategies. However, they appeared to struggle to apply their knowledge of mathematics to new situations such as problem solving tasks. There was also an apparent culture within these classes where some of the children and parents thought of mathematics as just learning calculations. They also felt that speed positively correlated with achievement, Boaler (2016) noted this as a serious problem in current mathematics education. I was also fascinated by comments made by Wu and Adams (2006) they found that 'student's proficiency on traditional school mathematics topics does not necessarily reflect their ability to solve real-world (real-life) problems' (p. 99). This encouraged me to continue my research and investigate how the children in the focus institution were completing problem solving tasks and the mathematics they used. I will discuss the focus institution in more detail later in this chapter.

Boaler's (2009) research into mathematics education was another motivational factor inspiring this research. Within her research Boaler (2009) described her belief of mathematics coming alive for children noting that it is more than just facts to be memorized. I found her comment very thought provoking as I reflected on my own memories of mathematics education and the facts I had to learn. I wondered if the joy of mathematics was becoming lost in the classroom. For example, with the teaching of methods as routine processes, was their use and application lost? I was also influenced by my own views of mathematics. My philosophy is one that sees mathematics as an organic subject. I perceive it as a subject that has a knowledge base which is constantly evolving. Ernest (1988) described several philosophies of mathematics, I favoured one. My beliefs resonate with his philosophy, termed as 'problem solving view' which sees mathematics as 'a process of enquiry and coming to know, not a finished product, for its results remain open to revision' (Ernest, 1988, para. 5). I also feel that mathematics is exciting and ubiquitous in everyday life. I believe that it is a vast subject, with the occurrence of mathematics in so many everyday tasks not just numbers and facts.

There have been several other studies which have been inspirational. Lesh (1981) proposed that the fact a person knows how to compute does not mean that they know when to apply it. It is the ability to integrate new ideas with previous knowledge and recognizing the situation in which these new ideas are relevant. Putnam's (1987) commented that 'helping children to learn mathematics in a way which enables them to apply their knowledge has been an enduring problem in Mathematical education' (p. 687). The implication being that children are not able to use the mathematics they learn in other situations. There have been some studies which have compared the performance of students completing real-life problems in a classroom setting (Jurdak, 2006) and also those that have looked at problem solving tasks within textbooks (Vincent & Stacey, 2008). I was curious to observe the processes and strategies children would employ in attempting to solve mathematical problem solving tasks.

I was also curious what impact pressures examinations had made on the mathematics curriculum. The final years of primary school education (Key Stage Two) can be one focused towards passing exams, either the Key Stage Two SATs (The English National Tests for children age 10-11 years old), entrance exams for admission into either private secondary schools or grammar school. It was noted by previous researchers that 'high stakes testing is even more likely to restrict curriculum opportunities' (Galton, Hargreaves & Pell, 2009, p. 119). My interests included the perceived culture surrounding these examinations. I was troubled that some children might be learning calculation techniques and strategies without actually understanding *what was going on* in the mathematics. I wondered whether the children in these situations developed a deep, entrenched understanding or just the memorization of facts to pass exams. Previous studies (Cooper & Harries, 2003) have looked at children who, in words of the authors 'played the game' in answering examination type questions correctly. Cooper and Harries (2003) investigated whether their participants could reason realistically. I therefore wondered if the children in the focus institution would be able to answer questions realistically and apply common sense to their answers.

## 1.2 Research Question

Previous problem solving studies have included very little evidence of classroom based investigations observing primary aged classes attempting problem solving activities. As a result I chose to examine the mathematical strategies that a cohort of Year Six children (aged 10 - 11 years) employed. Therefore, my research aimed to answer the following question;

How do Year 6 children engage in mathematical problem solving activities?

As I have mentioned my curiosity regarding children’s application of knowledge was the main motivational factor influencing my research question. I have noted my thoughts about how many children only know mathematic methods as steps they blindly follow. Both Polya (1957) and Skemp (1989) stated that the importance of knowing how to apply mathematics to situations beyond the execution of mechanical processes of solving algorithms is well known. Several other researchers have highlighted the incidence of children learning algorithms where they have little understanding of the actual mathematical concepts imbedded within them (Putnam, 1987; Boaler, 1993b). My interest was further piqued as when I was teaching; I was repeatedly asked by the children ‘is this problem an addition or subtraction question?’ I wondered whether these children could select the correct method and apply it effectively. I will continue this discussion of the mathematics curriculum in my review of the literature.

After examining the mathematics curriculum I decided to use mathematical problem solving as the focus for the practical work in class. Solving problems is an important life skill and ‘today’s workplaces demand people who can solve problems’ (OECD, 2014b, p. 26). The PISA 2012 (2014) assessment measured ‘the extent to which 15-year-old students have acquired key knowledge and skills that are essential for full participation in modern societies’ (OECD, 2014, p. 3). Its assessment tool included using non-routine problems in real-life contexts. Their results showed that children in Singapore and Korea scored the highest in problem solving activities and England was ranked 11$^{th}$ out of the 44 participating industrialised countries. In the problem solving section England out performed other western countries with similar mathematics, reading and science results and although this seems fairly impressive, the PISA 2012 (2013) study discovered that ‘on average across OECD countries about one in five students was only able to solve very straightforward problems’ (OECD, 2014, p. 32). The PISA 2012 (2014) results were interesting as it suggested that the participants where less successful when solving more complex problems. This further directed my research to investigate and discuss what actually happens when children are working on mathematical problem solving tasks.

An aim of my literature review was to find a robust practical definition of mathematical problem solving. This was important as this research was based on problem solving activities. A good starting point for my research was the rather poetic description proposed by Burton (1980);

> Problem solving is 'an area of study which is thoughtful, provocative, beautiful, curious, a means invented by man to help him in his search to better understand and explain the world about him' (p. 570).

I have expanded on this definition within chapter two to include the tasks and participants involving in problem solving activities.

## 1.3 Overview of My Research

My literature review forms the basis of Chapter Two. This contains a review of the current literature including research and reports from peer reviewed journals surrounding the topic of problem solving in mathematics education. I focused on the primary school curriculum and education policy. The literature review sets the context for my research and is fulfilling the role of the Institution Focused Study (IFS). This chapter also has the additional aim of situating my research within the current educational climate at the time of my research.

In Chapter Three I discuss my theoretical framework. These include the ontological, epistemological and methodological approaches I adopted when I explored my research question. I also explored several of the common methodological approaches that are available to qualitative researchers. I furthermore discuss the rationale for my method of data collection and look at methods of analysis focusing on thematic analysis.

It was my aim for the children in the Year Six class to undertake several problem solving tasks during the data collection period. In Chapter Three I describe these activities and how they were administrated. I felt the inclusion of this was necessary because these activities were an essential part of my research. The data collection and analysis was the product of the children's reactions to the problem solving tasks as without these stimulating and thought-provoking activities it might prove difficult to answer the research questions. Finally in this chapter I considered the ethical implications of my research.

The data from the lesson observations was transcribed and uploaded to Nvivo 10 (computer assisted analysis program). The analysis of this data is the basis of Chapter Four. Within this chapter I explored the data using thematic analysis and use examples in the data to illustrate the preliminary codes and overarching themes.

Chapter Five, the discussion, contains a further examination of my results. The codes that I assigned to the raw data have been collated and now form overarching themes. It was these themes that I discussed in relation to my research question. This chapter also highlighted the main findings from my study.

In Chapter Six I present my conclusions, including a discussion centred on how the findings relate to my initial research questions. I also explore the limitations of my research and its generalisability. I furthermore comment on how this research may contribute to knowledge on the subject of mathematics and problem solving and its implications for practice including the education of teachers. In addition I explored my personal learning whilst I conducted this research. I finally conclude with additional questions which have arisen from the research as potential areas for further research into problem solving and the curriculum.

## 1.4 Study Context And The Role Of The Researcher

Up to this point I have discussed my background in education and the motivational factors that influenced my research. I will now start to explore the case studied and my role as I conducted this research.

Within the introduction to this chapter I referred to the establishment where the research was conducted as the *focus institution.* I will refer to this establishment as *the school* throughout the remaining sections and chapters. The school was a mixed gender, non-selective, fee paying private primary school, universally known in England as a preparatory school. The school was situated in one of the Home Counties with many of the children living in the neighbouring Greater London boroughs. The pupil intake represented a wide range of backgrounds, ethnicity and culture partially due to its lack of selectivity and also the affordability of its fees. The school was ethnically diverse with approximately a quarter of the children having English as an additional language. Although the school is non-selective the ability profile of the children is above the national average. However, some children classed as having learning difficulties or disabilities attend the school and receive additional support outside of their regular timetabled lessons. Within the school I selected the Year Six cohort as my ‘case’, the rationale for their selection from the school population is discussed in more detail in my methodology chapter.

During the research process I assigned myself several labels. However, the first role I took was as a teacher researcher. At the time of conducting my research I was a practising teacher within this school and had taught the participants. I chose to research in my own school as I was curious regarding how the children in this school were applying mathematics to situations other than completing calculations. I wish to note that this was not limited to this school but as I was now the lead mathematics teacher I had the opportunity to learn more by conducting a study. This also directed my research towards my final methodological approach.

The possibility exists that conducting research within your own school can be challenging. Nonetheless, there are advantages linked to conducting teacher led research in schools. Obviously there are ethical considerations when carrying out any kind of research. These issues are discussed in more detail in the methodological chapter. One concern was the power differential when working with children. There was a potential for coercion and the Hawthorne effect due to my position in the school and previous profession relationship with the participants. However, the advantages of teacher research are many; this includes the reflective element of looking in detail at classroom activities from the perspective of the teacher at the ‘chalk face’. There is also the reportedly bigger impact of interpretivist research conducted by teachers being a response to a need for a ‘more inductive, qualitative approach to explore the complexities of social phenomena such as classroom practice’ (Taylor, 2015, para. 42). This adds some validity to my further discussions regarding the practical implications of my research in chapter six.

My role continued to develop during the data collection phase of my research to that of participant observer. The dual role of class teacher and observer did pose some difficulties. This was due to the fact that I was seeking the collection of observation data from the lessons I taught and was reliant on recording devices to capture the data. Recording devices, audio recorders and video cameras were set up in the classroom and these were stationed near the children's work areas to collect the data. After delivery of the initial lesson introduction, the recording devices were monitored whilst the children attempted the activities I maintained my role as the children's teacher throughout. I did make some field notes whilst the children were attempting their problem solving activities.

A further concern in relation to my research was the issue of objective analysis. Therefore, to negate this issue I have been transparent in acknowledging my relationship with the participants as their teacher. Possible complications resulting from this situation are mainly concerned with the process of data review therefore to counteract this I ensured that I was not adding my values when interpreting the data. I also incorporated field notes alongside the transcription data. Furthermore the discussion of the data and subsequent conclusions included how my role as participant researcher might influence my research.

# Chapter Two - Literature Review

## 2.1 Introduction

This chapter is focused on identifying and reviewing some of the current research and reports from peer reviewed journals, books, government reports and other relevant publications. I have included international literature and those focused on secondary school students. The decision to include these was partly due to the lack of current studies within the field of problem solving in English primary schools which will be elaborated on within this chapter. In the main, the reviewed materials are limited to a twenty five year time period (between 1990 and 2015). However, I have cited some of the early work in this field such as those of Polya (1957). His work has been very influential in the field of problem solving as purportedly a lot of problem solving work 'stands largely on the foundations of his work (Schoenfeld, 1992, p. 16).

The chapter is structured as follows. I begin with an introduction to problem solving including a definition developed from the literature. I then focus on the mathematics curriculum, taking account of how the curriculum has developed since the introduction of the National Curriculum in 1988 and various other policies. I endeavour to explore in what way problem solving fits into the modern day primary mathematics curriculum of England. I conclude with the justifications for why I felt the need to conduct my research into problem solving in the primary curriculum focusing on mathematics.

## 2.2 Defining Problem Solving

In order to answer my research question and contextualise my work a robust description of problem solving incorporating mathematics was essential. A concise definition would add to the relevance of my research for others and the implications for future research. Therefore, my initial task was to define the term mathematical 'real-life' problem solving and include the associated activities.

Schoenfeld (1992) noted how 'problems and problem solving have had multiple and often contradictory meanings' over time. There was even a suggestion that problem solving tasks are ‘ill-defined and understood differently by different authors’ (Ernest, 1991, p. 202). As this comment was made twenty years ago there was some hope that some progress might have been made. It is interesting to consider that there are so many interpretations of the terms problems and problem solving tasks. It is for these reasons defining problem solving has been a very challenging task. From my review of the literature I noticed that there was no absolute definition of problem solving.

It is possible to simplify problem solving, it appears that problem solving is something people do all the time, possibly without even realising it. As early as the early 1960’s Polya (1962) and more recently PISA (2014b) discussed a general problem solving approach to life such as the problem the basic need to find food. PISA (2014b) even implied that ‘in modern societies, all of life is problem solving’ (p. 26). I could surmise that problem solving is similar to evolution as from solving problems new knowledge is developed. This goes as far back as Darwinian thinking with problem solving as an active adaptation to the environment for survival of a species.

When searching for a definition I initially looked at the individual words which make up the phrase ‘problem solving’. This echoed the work of Barmby, Bolden & Thompson (2014) who partitioned problem solving into ‘problem’ and ‘solving’ before defining each independently. They suggest that the problem element is concerned with moving from a ‘starting state’, the question and changing this to find a solution (Barmby *et al*., 2014). Another similar definition proposed that problem solving is a process of trying to reach a goal when the path is blocked (Bayazit, 2013). Skemp (1989) elaborated on this and proposed that it is a process whereby the solver is ‘achieving a goal from a given starting point when they do not have a ready-made plan’ (p. 167). As the solution is not obvious it requires the learner to work to attain the answer and this forms a cognitive struggle. In recognising problems Polya (1962) noted that ‘to have a problem means, to search consciously for some action appropriate to attain a clearly conceived, but not immediately attainable aim’ (p. 117). This implies that the act of solving is not a passive action. Polya (1962) goes further and suggests that the solver has to ‘devise a well-conceived coherent scheme of operations’ to move ‘from the things we have to the things we want’ (p. 123). There is a common thread running through these definitions that the problems are perceived obstacles that is impeding progress and requires cognitive effort find a solution.

When describing the 'solving' element of problem solving Barmby *et al*., (2014) cited the earlier works of Polya (1957). They referred to Polya's (1957) four phases of problem solving, where he suggests that the solvers;

> *'First - Understand the problem*
>
> *Second – See how the various items are connected …to make a plan*
>
> *Third – Carry out the plan*
>
> *Fourth – Look back at the completed solution, review and discuss it'* (Polya, 1957, p. 5*)*

These stages demonstrate how the act of solving problems is an active process with the solver engaged in the process at each stage.

To this point, I have focused on a generic definition of problem solving we now need to consider the mathematical element of real-life problems. Some, such as Ollerton (2011) cite problem solving within mathematics as the 'basis for the existence and construction of mathematics' (p. 84). This comment helps to affirm the importance of problem solving within the mathematics curriculum but does not particularly define what constitutes mathematical problem solving. Although, it has been suggested in the literature that it is the context that the problem is situated in which defines whether it is mathematical or not (Polya, 1962). The context is also linked to whether a problem could be considered as real-life. Context in relation to problems is discussed later in section 2.6 in more detail. The task and those solving the problems had an impact on my final definition of what is problem solving. The following section will take a closer look at the tasks and the solvers.

### 2.2.1 Problem Solving Tasks

Whilst acknowledging the complex and dynamic nature of problem solving we have to consider how the solvers influence what constitutes a problem solving task. Both Polya (1962) and Ernest (1991) suggest that the thought processes and an individual's ability play a part in what one might consider as a problem. If the solution to the proposed problem is found immediately without any difficulties then it is not a 'problem'. Therefore a problem posed that can be solved with ease it is only an exercise and the cognitive demand for the solver is low.

In regards to mathematical problem solving tasks there is a suggestion that 'suitable problems require and facilitate meaningful engagement with the relevant mathematics' (Beswick, 2011, p. 382). This is similar to the works of Wu and Adams (2006) who described that 'to teach mathematical problem solving skills effectively, one needs to link the problem solving tasks to the cognitive processes involved' (p. 94). Along with that of Vincent and Stacey (2008) who suggested that cognitive complexity is dependent upon 'whether the problem requires the solver to make connections across different mathematical concepts' (p. 86). There are other considerations which may impact the cognitive demand which included the problem context and the solver's life experiences.

Up to now my discussion has centred round the perception of problem solving tasks from the point of view of the participants. Previous researchers and authors have also categorised these tasks, Polya (1962) proposed that problem tasks can be sorted into those which are 'problems to find and those which are problems to prove' (p. 119). Gilfeather and del Regato (1999) also divided problem solving activities into two categories, routine and non-routine problems. Routine problems are those where the solution is found by using known facts or by following an example. Within these problems the solver has 'no opportunity to use his judgement or inventive faculties' (Polya, 1957, p. 171). Non-routine problems require the solver to 'synthesize ideas and procedures' (Garelick, 2013, p. 1341) and they do not necessarily use algorithms but do utilise heuristics. These problems can be further differentiated as active or static which differ by the end goals and strategies needed to find the solutions (Gilfeather & del Regato, 1999). Static problems have a known goal whereas active problems can vary with combinations of fixed, changing and alternative goals and strategies. Due to these features the active non-routine problem activities are closely related to investigative activities and provide the opportunity for participants to think creatively.

In the previous section I discussed problem solving tasks in relation to the cognitive demand. In a continued effort to better understand problem solving I have scrutinised some of the associated activities that children are asked to complete in classroom situations. I will begin my discussion with word problem type activities.

It has been suggested that word problems are a subdivision of problem solving (Barmby *et al.,* 2104). As a practising mathematics teacher I noticed that the mathematics in word problems was intertwined in structured stories or scenarios. This opinion was formed from my experience of various published mathematics schemes and textbooks. These tasks do require some mathematical dexterity from the learner as they try to apply their knowledge and understanding to unfamiliar questions.

As the name suggests word problems are language based mainly appearing in print. Unfortunately, due to the fact they contain a lot of language they may be partially inaccessible to learners who cannot decode text easily. In his study De Lange (1981) mentioned that his participants had difficulties translating the text of a problem into mathematics. He described the process of reading and finding the maths as mathematising (De Lange, 1981). Word problem activities can be complex and have been described as a diverse group which contain open or closed and multi-step questions (Barmby *et al.,* 2014). In some commercially published schemes of work word problems appear at the end of a unit of work with an expectation that the children will use the calculation method they have just finished learning. This was also commented on by other researchers (Gravemijer & Doorman, 1999) who noted that context or real life problems was 'limited to the application that would be addressed at the end of a learning experience' (p. 112). An additional criticism levelled at textbooks word problems was how they only 'serve to illustrate one rule and offer some practice in its application' (Polya, 1962a, p. 157). As can be seen from these comments there is a concern regarding how and when word problems are used in mathematics teaching.

We should also be mindful of the benefits of using word problems. They do have an important role in education notably in the assessment process in the Key Stage Two SATs (The English education system, end of Key Stage Two National Tests). Where, according to Wild (2011) assessment using word problems reaches its zenith. The relevance of this to my research is that from 2015 the revised tests will contain two papers focusing on mathematical reasoning which incorporates problem solving questions. The audience for these tests are children in Year Six and compulsory for those in the maintained education sector in England.

As well as word problems investigation type activities have also been defined as problem solving activities. Jaworski characterised investigation tasks as 'loosely-defined problems' (1994, p. 2). From Jaworski's comment it can be inferred that there are some subtle differences between investigations and other problem solving activities. Investigations differ from other problem types as they are characterised as being open ended activities, with more problems posed during the process before conclusions are drawn.

Until now I have only looked at word problems and investigations but 'some problems arise from pure mathematical contexts' (Ollerton, 2011, p. 95). Ollerton's (2011) problem solving activities are those concerned with the exploration of elements within mathematics. These types of problems can be focused on proving theorems and investigations of pure mathematics. I have limited my review of the literature to real-life type problems as these are the focus activities in my research.

In conclusion my study would focus on the solving of non-routine problems in a mathematical context. Importantly the review of literature has demonstrated that this investigative task must be somewhat unfamiliar to the solver and require them to think creatively to attain a solution. Also the significance of the actual solver is integral to the choice of activity as the task has to require the solver to use their knowledge to come up with a solution otherwise it is not a problem. Therefore, careful consideration of the audience is required when choosing problem solving activities.

## 2.3 Educational Policy And The Mathematics Curriculum

The following section has been included so as to describe some of the past Educational policies and how they have had an impact on the mathematics curriculum of England. As my research was concentrated on the current mathematics programme of study my discussion is limited to those policies directly related to the National Curriculum from the original 1988 document to the current 2014 version. This leads to discussion regarding some of these political interventions and their impact on the primary school curriculum.

From my observations as a practising teacher in the English education system I concluded that educational policy appeared to be quite a dynamic concept. The introduction of many new initiatives and the policy changes rolled out by successive governments meant that I was being asked to implement strategies or even discontinue strategies. These constant changes were described by Brown (2011), as a roundabout which turns 'depending on social and economic contexts' (p. 3). Within the next sections I have included some key examples of policies relevant to my study. The first I wish to note is the English National Curriculum for Key Stages One and Two.

The 1988 Education Reform Act introduced the National Curriculum into all state schools in England. The primary phase was split into Key Stages (known as KS). KS1 for children aged

5 -7 years and KS2 for children 7 – 11 years old. For each subject there were Programmes of Study and an assessment guide. Early Years and Foundation Stage education covered Nursery and Reception aged children. Initially the mathematics curriculum was organised into 14 Attainment Targets which were later reduced to five main targets with the titles; Using and Applying, Number, Algebra, Shape and Space, and Data Handling (Jaworski, 1994). This was subsequently superseded by the National Strategies in 1999. At this point the original curriculum from 1988 was rather broad and conceptually based (Brown, 2011). In the 1988 version of the National Curriculum, problem solving appears in the strand Using and Applying. In 1995 the curriculum was revised, it now included a reference to solving real-life problems within the Using and Applying strand of the Programme of Study for mathematics.

A further development was the introduction of National Numeracy Strategy (DfEE, 1999) into primary schools. This contained five strands one of which was Solving Problems. It has been implied that the 'emphasis of this strategy was on calculating and number work and not problem solving, which was seen by many as an add-on' (Barmby *et al.,* 2014, p. 3). This devaluing of problem solving was rather concerning as many other researchers have noted the importance of problem solving as being at the heart of the subject (Skemp 1993; Barmby *et al.,* 2014). It is therefore disturbing to discover that problem solving was not the focus of the Numeracy Strategy. I have anecdotal evidence of teachers adding problem solving to the end of a unit of work only if they had enough curriculum time. In my experience as the mathematics coordinator in several primary schools the acquisitions of processes is considered more important. Consequently more teaching time is given over to learning skills and processes, this is somewhat linked to examination preparation. This is similar to the findings of Galton *et al.,* (2009) who argue that 'the final year of primary school is where high-stakes testing is even more likely to restrict curriculum opportunities' (p. 119). Possibly the curriculum has become more focused towards exam success, a topic which I will revisit later in this chapter.

In 2003, as part of the Primary Strategies the mathematics curriculum was reorganised into seven strands and renamed the Mathematics Framework (DFE, 2003). These strands were known as; Using and Applying Mathematics, Counting and Understanding Number, Knowing and Using Number Facts, Calculating, Understanding Shape, Measuring, Handling Data (DFE, 2003). Problem solving was incorporated into the Using and Applying strand, embedding it into the teaching and learning of mathematics. It is appears as the first learning objective in this strand for each year group, which has possibly raised its profile.

The PISA 2012 (2014b) note that globally there has been a shift in education. Educators are aiming to provide students with skills to ‘confront and overcome complex non-routine cognitive challenges’ (OECD, 2014b, p 26). The National Curriculum was reviewed again in 2013 and a new Programme of Study (DFE 2013a) was launched in September 2014. Problem solving was explicit in the aims for the Programme of Study which state that all pupils;

> ‘can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions’.
>
> (DFE 2013b, p. 3)

The quote above, DFE 2013b (p. 3) demonstrates the importance of problem solving within the mathematics curriculum. The subsequent aims were concerned with reasoning and the applicability of mathematics across the whole primary school.

To this point the main policy changes that impacted the mathematics curriculum of relevance to my study have been discussed. Brown (2011) pointed out that although there have been changes made to the format of the National Curriculum ‘the content has remained substantially constant between 1989 and the present day’ (p. 14). The present day that Brown was referring to was 2011. However, the present curriculum (DfE 2013b) differs from the earlier curricula as shown above. The new programme of study includes knowledge and ideas earlier than in the previously discussed versions. This means that there is an expectation that children will encounter concepts earlier than in previous versions of the curriculum. An example of this is the inclusion of using simple formulae in Year Six and an emphasis on fractions. There now follows a continuation of the discussion regarding the curriculum concentrating on the teaching of mathematical problem solving within the curriculum.

## 2.4 Problem Solving In The Mathematics Curriculum

In the previous section I located problem solving within the aims of the 2014 National Curriculum for England. This section now focuses on how problem solving has been incorporated into the taught curriculum. As long ago as 1982, the Cockcroft report (1982) was asserting the need for children to 'experience applying the mathematics they are learning to familiar everyday situations and to problems' (p. 95). They advocated the development of children as independent mathematical thinkers. The Department of Children and Families produced the publication, 'Mathematics in the Primary Curriculum - So what is Mathematics?' (DCSF, 2008) Within this they highlighted the features of mathematics including the observation that 'mathematics is creative subject in which ideas can be generated, tested and refined' (DCSF, 2008, p. 3).

Some criticism has been levelled at the organisation of the teaching elements in the mathematics curriculum (Ernest, 1996). Boaler (2009) suggests that in some classes mathematics is taught as a narrow subject. She describes these lessons as those where the children are taught methods which they then reproduce 'accurately, over and over again' (p. 2). There is a concern that if children were taught a narrow and perhaps test-driven curriculum, they may not actually have the opportunity to be creative and generate their own mathematical theories and hypothesises. This also led to my thoughts regarding the learning of methods without understanding the mathematics embedded in them.

My earlier discussions have focused on the evolution of the curriculum. There is an insinuation that over time the reportedly 'broad and conceptually based' curriculum shifts to one that favours 'abstract number knowledge and procedures, down playing application and problem solving' (Brown, 2011, p. 22). This was reiterated by Pound and Lee (2011) who alluded to the fact that the end of Key Stage tests and the Numeracy Strategy have a deleterious effect on the mathematics curriculum. They note a shifting of the curriculum back to a narrower perception of mathematics (Pound and Lee, 2011). It should be noted that Pound and Lee's (2011) comments are focused on the pre-2015 curriculum but as my research was conducted in June 2014, before the introduction of the new mathematics curriculum these comments are still relevant to my research.

An additional concern was levelled at the compartmentalisation of the curriculum and its eventual impact on the teaching of mathematics. Within the 2012 PISA report they observed how problem solving is 'compartmentalised by subject' (OECD, 2014b, p. 120), the teaching of problem solving is not cross curricula and skills are not transferred between subjects. However, Burton (1980) noted how the skill set and procedures used to solve problems are not exclusive to mathematics but are transferable and science was one such subject. Coulter (2004) also described how science and mathematics although closely related in the real word are often separated in the school curriculum. Even though these comments are true mathematics does appear in the science curriculum in the guise of measuring, data handling and calculations.

Skemp (1993) noted that one of the qualities of mathematics is its 'applicability' to other disciplines (p. 1). Venville, Wallace, Rennie and Malone (1998) investigated the integration of mathematics with both the science and technology curricula. Integration across the curriculum was also noted in the Cockcroft (1982) report. Ginsburg and Golbeck (2004) found that there is a tendency for us to 'study mathematics and science learning in isolation from the child's everyday world' (p. 192). Interestingly the Programme of Study for Mathematics (DFE, 2013b) highlights the interconnectedness of mathematics and states that;

> 'The programmes of study are, by necessity, organised into apparently distinct domains, but pupils should make rich connections across mathematical ideas.'
>
> (DFE, 2013b, p. 3)

The mathematics curriculum itself has been described by several researchers (Bean 1991; Ernest 1996) using some interesting analogies. Bean (1991) uses the analogy of the 'jigsaw' curriculum (p. 9) and Ernest (1996) describes the structure as spiracle. Bean (1991) suggests that the curriculum can be seen as a series of facts and skills with no obvious relationship between them. This is observed in the Numeracy Strategy (DfEE, 1999) and the strands of the Primary Strategy (DfES, 2006).

The mathematics curriculum is somewhat shaped by how it is taught in school. Ernest (1996) builds upon the idea of the fragmented curriculum by describing the spiral approach. He describes how the curriculum is 'treated in several booklets or chapters spread out over years of study' (Ernest, 1996, p. 7). The curriculum content becomes progressively more difficult as schooling continues as it spirals up through the academic years. From my own teaching experience I have witnessed how mathematics becomes more abstract as it rises through the year groups from the younger children in Reception class (ages 4-5 years) onwards. As children grow older they move away from using concrete resources (counting cubes, number lines etc.). This is a natural maturing of mathematical learning as children gain confidence and understanding. Eventually they progress to perform more structured 'pencil and paper' activities and use recognised formal written methods. Reportedly the spontaneity and enthusiasm observed in a pre-school child's learning patterns dissipate fairly rapidly once they experience these more formal teaching methods (Burton, 1980). Ernest (1996) indicated that the spiralling nature of the curriculum was intended to promote integration and continuity in learning. However, he concluded that this was not the case and 'that the student rarely manages to link up the different bits that are studied' (p. 7). This links back to the previous quote from the DFE (2013b) regarding how pupils should be making these connections, whereas Ernest (1996) does not believe that they can.

Returning to the earlier topic of examinations, formal calculation methods have become very important in the end of Key Stage Two exams. From May 2016 Year Six children are required to complete one examination paper on their use of formal methods and additional papers focusing on the application of mathematics to problems and their use of reasoning skills (Standards and Testing Agency, 2014). There is an implication that this might impact on attitudes towards the teaching of Mathematics that force teachers to only teach the calculation methods needed for these examinations. The guidance document stated how 'pupils will be expected to use formal methods to solve specific arithmetic questions', the appropriate method will be worth one mark (Standards and Testing Agency, 2014, p. 2). There is a place for the learning of procedures with Mathematics. Nevertheless, children also need to develop this knowledge by learning how to do calculations and apply these to problem solving and other tasks (Putnam, 1987). This relates back to my comments in section 2.3, regarding the National Numeracy Strategy (DfEE, 1999) and how it prompted an emphasis on number work and not problem solving (Barmby *et al.,* 2014). The Cockcroft report (1982) warned that independent thinking should not be dismissed in favour of learning methods. This is especially pertinent now with the development of a problem solving focus in the 2016 SATs examinations.

My research has been particularly influenced by my experiences of teaching primary aged children in preparation for examinations. Some teachers have developed teaching strategies to gain the best examination results, the so called ‘teaching to test' phenomenon (Styron and Styron, 2012). Although these researchers note that this method of teaching had the potential to improve scores, it can narrow the curriculum to just the areas being tested. Yet, the Williams review concluded that the mathematics curriculum taught at the time of their report was well balanced (Williams, 2008). This was however, published before Styron and Styron’s (2012) research.

In addition to the SATs end of year examinations, some children sit the Eleven Plus examination for entry into state-funded Grammar school or the Common Entrance examinations set by selective fee paying Independent Secondary schools. These examinations are highly competitive and some parents turn to private tuition in an effort to help their child succeed. This tuition includes, for example, Kumon classes as well as one to one lessons. The Kumon method has been criticised by some as rote-style learning as it stresses computation and overlearning (Ukai, 1994). This teaching method utilises worksheet based activities focusing on calculation methods that the children are encouraged to complete quickly. Kumon is not taught as part of the National Curriculum but as parents buy into it, it does have an influence on the progress and achievement of some of the children taught in English schools. The influence of tuition becomes apparent in the discussion section of my research.

My previous discussion was centred on how the mathematics curriculum is organised I now return to problem solving. There is a suggestion that children can have their problem solving ability metaphorically ‘knocked out' of them (Boaler, 2009, p. 38). One possible reason for this occurrence is through the use of passive teaching methods. These include the rote learning of number facts and written calculation methods. When children learn facts in isolation they only memorise the information (Cockcroft, 1982). There is evidence that some children are actively using the ‘passive learning approach’ described by Boaler (2009, p. 35). These children believe that success occurs if they just memorise facts. Some children are very good at learning facts by rote although previous research discovered that it is much harder for them to apply what they have learnt from one situation to another (Pepperell, Hopkins, Gifford & Tallant, 2009; Putnam, 1987). It is these children which triggered my initial interest in completing this research. I was curious as to whether these children would be able to apply their skills and knowledge to unfamiliar tasks. The William’s report (2008) acknowledged that;

> ‘fostering children's natural interest in numeracy, problem solving, reasoning, shapes and measures is central to effective mathematical pedagogy in Early Years Education’ (p. 36).

It could be surmised that this would be applicable throughout all the Key Stages including the participants of my study.

School mathematics has a somewhat negative reputation amongst some adults and there is a belief that it is ‘socially acceptable to profess an inability to cope with mathematics' without feeling embarrassed’ (Williams, 2008, p. 3). It is claimed that parents may even model a ‘fear' of mathematics, and it is possible that these feelings and attitudes are then projected onto their children (Ginsberg & Golbeck, 2004). Wedege (1999) revealed that adults have a ‘complicated and often emotional relationship with mathematics’ (p. 211). Attitudes towards mathematics are powerful, they can crush confidence, deterring learning by making them feel both stupid and helpless (Boaler, 2009). Yet conversely, parents also ‘exert inordinate pressure on children to learn' (Ginsberg & Golbeck, 2004, p. 198). This is a confusing set of messages for children to assimilate. However, it does fit with those parents who pressurise their children to pass examinations, which I mentioned in the previous section. That said it is not only parents who influence children's attitudes teachers can also have an impact. Williams (2008, p. 62) noted that;

> ‘If children's interests are not kindled through using and applying Mathematics in interesting and engaging ways, and through learning across the full mathematical curriculum, they are unlikely to develop good attitudes to the subject'.

This reiterates Burton’s (1980) suggestion that problem solving activities have to be ‘engaging enough to ensure that mathematics is seen as an enjoyable and applicable activity’ (p. 56) and therefore change attitudes. These quotes also bring to light one of the possible benefits of using problem solving which is to engage reluctant learners. It is pertinent to remind ourselves that the ability and experiences of the learner need to match the demands of the problem. I discussed this in detail whilst attempting to define problem solving earlier in this chapter.

Attitudes towards Mathematics were a feature of the TIMSS 2011 survey (Sturman, Burge, Cook & Weaving, 2012). As part of the survey they collected data regarding children's attitudes towards mathematics. The children responded by stating that they somewhat liked the subject, this lies between liking and not liking the subject. The PISA 2012 (2014a) study reported that the female participants were underachieving and exhibited anxiety when attempting mathematical problem solving tasks as compared to the boys in their study (OECD, 2014a). This was rather interesting although gender differences were not considered within my study as they are not within the scope of the present study but might be of interest for future research.

Burton (1980) noted that for certain learners, problem solving provides a satisfaction that traditional lessons lack. This learner, known as the 'divergent thinker', has the opportunity to metaphorical 'spread their wings' during problem solving by asking their own meaningful questions (Burton, 1980, p. 50). As they are free from the confines of their textbook. As well as success problem solving activities can also help children gain the valuable experience of failure. Pepperell *et al.,* (2009) proposed 'to think mathematically is to pursue lines of enquiry, explore and be open to possibilities' (p. 2). Within a problem solving approach children have to learn from their mistakes and accept that an incorrect answer can be as useful as a correct one. This is especially important when restructuring their attitude towards a difficult problem. It is the old adage of learning from our mistakes and developing a positive mind set.

One of the factors inspiring my research was whether children fully appreciate the links between different mathematical concepts and topics for example those connecting division and fractions. As previously noted mathematics is not merely about learning facts and solving calculations. The Programmes of Study from the National Curriculum (DFE, 2013b) and Numeracy Strategy (DfEE, 1999) both indicate that problem solving should be taught as part of the curriculum. My research question developed from my need to investigate whether the Year Six children in the school could use their mathematical tool kit of strategies and techniques to solve problems. There is need for caution as Boaler (2009) suggests children that 'try to memorise hundreds of methods find it hard to use any method in any new situation’ (p. 36). Therefore, this tool kit of strategies and techniques should be viewed as methods and concepts the children using them fully understand and are comfortable using.

Whilst writing this review two questions emerged: why teach problem solving and is it a useful skill for adulthood? Lesh (1981) notes that ‘applied mathematical problem solving processes constitute an important part of the basic skills required for mathematical literacy amongst average citizens’ (p. 235)  Earlier I cited the trend of social acceptability, that is to admit a mathematical weakness. There is a place for mathematics in everyday life and it would be remiss to leave the subject at the classroom door at the end of formal education. It could be considered presumptuous to assume that every single skill learnt at school is consistently employed by individuals into adulthood. There are many anecdotes of learners professing 'what's the point in learning this and when am I going to use it?' Especially when faced with topics such as simultaneous equations or calculus.  It is true that  it is possible to be successful in a career with only the basic mathematical training but research has described arithmetic problems where it does make a difference whether someone attended school, and gained the specific skill set needed to find a mathematical solution (Greiffenhagen and Sharrock, 2008). The works of Lave (1988) demonstrate the use of mathematics by adults in everyday situations including problem solving in the supermarket and following Weight Watchers healthy eating plans. This began reaffirm the importance of problem solving in the mathematics curriculum.

For the following section I explore more aspects of problem solving. As the reading for this review progressed, the importance of problem solving in mathematics curriculum became more evident. For the children the benefits include a more satisfying learning experience, rather than being a passive learner they are engaged learners and ‘active constructors of knowledge' (Putnam, 1987, p. 688). As an approach to mathematics education, problem solving is a move away from rote and fact learning and starts to link different mathematical concepts for the learner. Within the next section I investigate the literature surrounding the strategies children employ to answer problem solving activities.

## 2.5 Problem Solving Strategies

Problem solving activities are not simple endeavours as they include a high cognitive demand on the solver to construct a solution employing strategies to find the answer. Edens and Potter (2008) explained that problem solving 'involves schematic and strategic knowledge and a number of cognitive processes' (p. 185). Therefore, if problem solving stages are not defined then the 'problem solving activities carried out in the classrooms are somewhat ad hoc and disorganised' (Wu and Adams, p 95 2006). In his seminal work 'How to Solve it', Polya (1957) proposed a four phase structure to solving problems this begins with the initial step where the solver has to understand the problem. He recommends that the solver demonstrates the ability to verbally 'state the problem fluently' and 'point out the principle parts of the problem' (p. 6) before moving on to the next stage. In addition Polya (1957) also suggests that at this stage the solver can discuss whether they have sufficient information to answer the problem. The second phase involves the solver making sense of the problem and formulating a plan, which entails deciding how the 'unknown (the problem) is linked to the data' (p. 5). Planning also includes knowing what procedures the solver will need to employ in order to find the answer. Polya (1957) highlighted the importance of this phase when he stated that 'the main achievement in the solution of a problem is to conceive the idea of a plan' (p. 8). In a later publication Polya (1962a) described the role of the problem solver was 'to mobilise the relevant elements of his knowledge and connect them with the elements of this problem' (p94). The third phase is for the solver to carry out their plan and find the solution. Once the solution has been found the final phase is for the solver to 'look back at the completed solution, review and discuss it' (Polya, 1957, p. 5*)*. Within Polya's description there is a clear path through which solvers can progress when solving problems. In the following discussion I continue to explore some of the strategies that are employed by children to solve non-routine problems.

The ability to solve problems has been described as one 'which can be developed and has associated with it, a set of techniques' (Burton, 1980, p. 52). Previously it has even been suggested that if children are to be successful problem solvers metacognitive thinking strategies need to be taught (Muir, Beswick & Williamson, 2008). There have been other studies such as Voutsina (2012) who investigated the development of their participant's abilities when repeating a problem solving task. Their findings showed that repeating tasks allowed their participants the opportunity to reflect on their learning which enhanced these children’s procedural and conceptual development (Voutsina, 2012) which is similar Polya’s (1957) last phase of reflection. Polya (1957) did suggest that problem solving was akin to swimming where, to make progress the learner needs to have opportunities to practice and develop their skills. Therefore, we could infer that problem solving has a knowledge base where it is possible to teach and develop solving techniques.

Successful problem solvers ‘identify the pieces of information available in the context that would be most useful for solving the problem’ (OECD, 2014b, p. 29). However, the PISA 2012 (2014b) assessment found that those who were ‘highly proficient problem solvers in one context may act as novices when confronted with problems outside of their field of expertise’ (p. 29). This suggested that their participants were not able to transfer skills between contexts. A strategy for successful problem solving requires the solver to make links between different pieces of knowledge. The term ‘relational understanding’ was devised by Skemp (1993) to describe the ability by children to link and relate concepts. Without this understanding the solver would not know which methods to use. The connections made between fragments of knowledge are a 'critical aspect of the mathematical understanding needed for transferring knowledge' in essence the ability to apply the skills they know to a new or different situation (Putnam, 1987, p. 692). Mason (2004) noted that ‘psychologically, knowing to use a technique is quite different from knowing-how to use it, or, knowing when to use it’ (p. 2) and ‘knowing your mathematics is a far cry from using it' (de Lange, 1981, p. 577). This relates to Wild’s (2011) work on issue of ambiguity within problems where children can be confused as to what they are being asked to solve and also the comments of Polya (1957) regarding the first of his phases. At the core of these comments are the problems of mathematising and eventually the transference of skills.

An alternative view would be to consider whether children do need to understand the mathematics they are doing. However, if we want children to confidently apply mathematics to new situations they should have an understanding of the process. As Muir *et al.,* (2008) found in their study, the below average students relied on the numbers in the question as a starting point and then ‘applied operations to them’ (p. 239). Without having a detailed background of their education, we can only assume that these children might not have the skills to select the operation but appeared to employ a trial and error approach to solving problems. Trial and error is a recognised problem solving strategy but the starting point has to be sensible in the context of the problem and not nonsensical, some logic and reasoning has been used. Elia, Van de Heuvel-Panhuizen and Kolovou, (2009) also found that trial and error was the most successful strategy in their study.

In their study Elia *et al.,* (2009) explored the strategies high achievers used to solve non-routine problems administered as exam questions. Apart from trial and error other commonly used strategies were ‘giving a proof or checking results’ (Elia *et al.,* 2009, p. 611). Due to their method of data collection the examples of strategies observed was low. However, they did comment on the inflexibility of their participants to reflect on the ‘appropriateness of their chosen strategy and to use an alternative or complementary strategy’ (Elia *et al.,* p. 616). Skemp (1993) noted that children who only memorise facts lack adaptability because this lacks creativity and relies on memory and recall. Reportedly a characteristic of being a functional mathematician is the ability to select appropriate strategies rather than relying on just one (Dickinson, Eade, Gough and Hough, 2010). The PISA 2012 (2014b) assessment recommended that successful solvers should strive to be able to be creative and flexible in their thinking. Conversely there is a school of thought that advocates the memorisation of facts such as Putnam (1987). He indicated that the practising of procedures to the point of automaticity frees up other cognitive resources and thus aiding problem solving (Putnam, 1987). Therefore, we should not underestimate the role of rote learning as a scaffolding device in the mathematical problem solving tool kit as a useful strategy amongst other strategies.

In conjunction with the procedural strategies such as trial and error there is also a behavioural element to problem solving ‘determination and emotions play an important role’ (Polya, 1957, p. 93). This refers to the tenacity exhibited by individuals who will not give up until they are content in their answer. The notion of an ideal problem solving behaviour was also discussed by Burton (1980) who notes that for productive problem solving children need to be curious, responding with interest, creative delight, spontaneity and confidence. Polya (1957) noted that when solving problems children should ‘desire its solution’ (p. 6) and this desire to solve might manifest itself as perseverance. This is also echoed by the comments of Ernest (1996) in relation to what constitutes a problem. He suggested that ‘the only real applied problems are those where the problem is genuinely part of the solver’s life’ and importantly ‘if the solver has something to gain or lose from the solution’ (Ernest, 1996, p. 7). For successful problem solving the learner has to get into the problem, which could include acting it out and becoming immersed in the context of the problem. It has been claimed that reasoning is ‘one of the most important aspects of being mathematical’ (Boaler, 2009, p. 43). Consequently it could be considered that reasoning should form an important part of any lesson whereby learners are developing their problem solving skills. In contrast there are some children who are driven to find the answer because they are result orientated. I could suggest that these children see the problem solving process as a chore. These were important concerns in the selection of an activity for the children to complete in my research.

Polya (1957) described a series of strategies employed by solvers to solve problems these included;

- Relating ideas to previous knowledge
- Speculation
- Solving part of the problem
- Graphical representations
- Recall previously solved problems
- Conjecture
- Generalising

Another problem solving strategy was how the solvers check their numerical results against common sense estimates. The PISA 2012 (2014b) assessment noted that the strategies solvers employ are related to their familiarity with the context the problem is embedded in. There is a query that this strategy is not truly mathematical thinking as it seems that there is an element of common sense being employed. Wu and Adams (2006) identified common sense as problem solving method and called it a ‘real-life and common sense approach to solving’ (p. 99). If we apply Polya’s (1957) four phase model, these strategies can be used at varying times during the problem solving process. They are mainly associated with the second stage of devising a plan such as *relating ideas to previous knowledge, recall previously solved problems* and *speculation.* In carrying the plan (Phase Three) the *solving part of the problem, graphical representations, conjecture* and *generalising* strategies are used. The final phase includes the strategies of checking work but it also includes an earlier strategy of *relating ideas to previous knowledge* where the solver evaluates their answers.

Within this section I have discussed some of the elements which I will develop into a deductive coding matrix and discuss in the conclusion of this chapter. I now continue my discussion by exploring context and real-life problems.

## 2.6 Context And A Real-life Problems

Problem solving does have the potential to afford the children the opportunity to use their learnt techniques in real-life situations. However, the context that problems are situated in has a tremendous impact on this learning. To this point I have discussed what constitutes a problem and some of the strategies employed by children but have not explored the real-life element of these problems. In the following section the role of context in relation to mathematics problem solving, its impact on learning, what this means for problem solving activities and the children solving them and how this relates to mathematisation is discussed in more detail.

De Lange (1981) found that some of his participant's struggled with mathematising word problems. At its most basic mathematisation is the process of finding the mathematics in a situation significantly, 'there is no mathematics without mathematising' (Freudenthal, 1973, p134). Menon (2013) suggests mathematisation 'captures what is at the heart of the mathematical enterprise – the thinking and the reasoning' (p260). In mathematising of a problem the solver turns 'the problem situation into a mathematical model' (Wu and Adams, 2006, p. 100). The 'situation' Freudenthal (1973) remarked upon could be considered as the context that the problem is positioned in and this can be described as the story or scenario around which that problem is based, contextualising the mathematics. Most commonly the context of a problem occurs in a hypothetical but plausible setting for the audience they are aimed at.

However, mathematisation is more complex than just finding the mathematics and contains both horizontal and vertical components. In continuing to define mathematising Freudenthal (1973) elaborates further that 'it (the problem) becomes mathematics if it is structured by logical relations'. The process of horizontal mathematisation is principally the application of skills (Treffers, 1993) whereby 'the learner develops mathematical tools or symbols that can help to solve problems situated in real-life contexts' (Dunphy, Dooley, Shiel, Butler, Corcoran, Ryan and Traver, 2014, p39). Vertical mathematisation 'stands for all kinds of reorganizations and operations done by the students within the mathematical system itself' (van den Heuvel-Panhuizen, 2003, p12). Van den Heuvel-Panhuizen (2013) continues with this discussion noting that within vertical mathematisaton the learner makes 'connections between concepts and strategies' and develops shortcuts (p12). Jurdak (2006) found that mathematisation was an obstacle to problem solving. They found that their participants had difficulties with the mathematisation of real-life problems but surmised that the delivery of the problems in school would have 'elicited the use of the tools targeted by instruction' (Jurdak, 2006, p293). They concluded that selection of tasks was critical including the context.

Researchers have questioned the role of the context itself, and if it is a 'vehicle for the learning objective, or whether it becomes the learning objective' (Wild, 2011, p. 10). Some researchers indicate that real-life settings are motivational, and arouse curiosity within learners (Boaler, 1993a; Ernest, 1996). Setting problems in real life is advantageous as it can make mathematics more meaningful and 'can provide an opportunity for appreciating the power and limitations of using mathematics in the real world' (p. 298). The Williams report (2008), noted 'that the connections between mathematical ideas should be emphasised' (p. 239). Therefore, the transference of skills is an integral feature of being a successful mathematician.

Previous studies discovered that when mathematics was studied through an 'integrated development of process and content' children develop a better understanding and can use the mathematical techniques in different situations (Boaler, 1993b, p. 372). The Programme of Study within the revised National Curriculum (DfE, 2013b) stressed the importance of mathematics for 'everyday life' (p. 3). The Realistic Mathematics Education (known as RME) approach promoted the use of realistic situations 'emphasising maths is best learnt by doing mathematics' (Van den Heuvel-Panhuizen & Drijvers, 2014, p. 522). The RME approach purports that contexts should be meaningful with the children attaching meaning to the mathematical constructs they develop (Van den Heuvel-Panhuizen & Drijvers, 2014, p. 523). An unusual feature of RME is that the context does not need to be real-life but could be from a fantasy world, it just has to be real in the mind of the learner (Van den Heuvel-Panhuizen, 2003).

The comments of Jurdak (2006), Marshall (2006) and Ollerton (2011) highlighted the importance of real-life and interconnectedness within mathematical problem solving and this reflected the aim of mathematics for everyday life. Jurdak (2006) described situated problem solving which 'simulates real-world (real-life) problem solving as a meaningful purposeful goal directed educational activity' (p. 284). Marshall (2006) indicates that to 'teach mathematics with understanding means creating experiences in which interconnections can be made' (p. 358). Ollerton (2011) indicated the need for children to work on problems which could be described as cross-curricular due to their 'real-life' context.

Moreover, the opposite might be true and there is the idea that contexts can distract leaners (Boaler, 1993a). At times some problems can appear to some individuals as contrived, for example, sharing pizza when working out fractions. There is a difficulty with problems that contain a 'non-school' context (Beswick, 2011). With the 'pizza problem' the issue is the lack of authenticity. Typically the remainder of a pizza is described as a number of slices and the only fractions used by individuals, if any, are halves and quarters. There appears to exist, a need for caution when situating problems in real-life to ensure that the context does not detract from the actual mathematics, that children are not 'bogged down' by unnecessarily wordy or overly complicated situations. This resonates with my comments regarding word problems and mathematising. Problem solving tasks in published schemes is notoriously unrealistic. Ernest (1996) noted that realistic problems are those which are closely linked to the solver's life. He continues by suggesting that in these situations 'the solver has something to gain or lose from the situation' (Ernest, 1996, p. 7) and therefore is more motivated to solve them.

Burton (1980) proposed that learning may not be internalised if the context is not meaningful, that is to say, remembered and then used again at a later date. This once again is similar to my earlier discussions on rote learning and memorisation of facts. Interestingly past research discovered that when children were exclusively taught from a published scheme their understanding of techniques was not internalized and 'their derivation, their understanding processes or their applicability in the real world was missing' (Boaler, 1993b, p. 370). With these comments it appears that meaningful maths might become another area of further research. I will now continue with my discussion regarding context in problem solving activities.

There is evidence that some children actually ignore the context. This occurred in Bayazit's (2013) research whereby he found that his participants disregarded the context completely. These children 'did not consider the factual relations between the real-life contexts elicited by the problem statements and the operations they carried out' (Bayazit, 2013, p. 1925). He indicated that the students in this study were 'result-oriented' not 'process-oriented' as they just wanted to get the answer. These types of children could be using a passive learning approach (Boaler, 2009) which shows a lack of engagement in the subject. The cause could be traced back to little understanding of the real-life situations used in the problem. It is also possible that these children are not using passive methods as they are simply driven to find the answer and do not care about the context.  Whilst solving problems some solvers are not worried when they find an anomalous result. Polya (1957) surmised that 'such neglect of the obvious does not show necessarily stupidity but rather indifference towards artificial problems' (Polya, 1957, p. 60). When discussing problems with his students, Bayazit (2013) discovered that if the context was familiar to them then they would rethink their answers coming up with a more sensible idea.

In previous sections I have discussed the importance of understanding mathematical processes before applying them. Coulter (2004) described the incidence of the *content vacuum* which is the term used to describe circumstances where skills are learnt or performed but are not applied to a worthwhile context. Initially this does seem similar to my previous comments regarding learning facts in isolation because although similar they are different. An example of an activity exhibiting a content vacuum is an exercise based on measuring amounts of liquid, with the learning intention to accurately read the scale from a measuring cylinder. This activity does not have any context because it is not situated in real-life and is being completed as part of a demonstration. These skills are not going to be internalised because the associated connections between real-life have not been made.

Boaler (1993a, 1993b) has conducted several studies which explored context in mathematics. She noted that their effects on performance have been widely underestimated. She comments that some contexts are used as motivation tools without due care and attention to their effect on procedure and performance. Cooper and Harries (2003) made comments about examinations, suggesting that the artificial context is 'merely a vehicle for introducing some mathematical task' (p. 452). Importantly, 'this strategy ignores the complexity, range, and degree of students' experiences' (Boaler, 1993a, p. 13). It is interesting to reflect on Boaler's comment; it seems that in some cases the context is pointless. There are some similarities with the pizza question but this goes further. Those posing problems need to be aware of their audience and not to just squeeze mathematics in to a real-life situation otherwise we are facing the issues of the content vacuum (Coulter, 2004). This is because the context is so irrelevant to the learner it is not helpful and perhaps restrictive. When Burton (1980) described what problem solving was, she was careful to dismiss the view that these activities were a bolt-on set of exercises at the end of a block work, they were much more.

To counteract the content vacuum, Coulter (2004) indicated a move towards 'skills being learned alongside the opportunities to use them' (p. 16). A cross curricular approach was considered for my study. This could provide the real-life element needed to ground and contextualise the mathematical processes and procedures. It might also go some way to satisfy Boaler's comment regarding their effect on performance. Other researchers have documented context in problems and their applicability to children's everyday lives. Some have emphasized the issue of realism from the child's point of view (Boaler, 1993a; Ernest 1996; Wild, 2011).

As the PISA 2012 (2014b) noted, 'the skills that underpin successful problem solving in real-life are not specific to particular subjects, students who learn to master them in several curricular contexts will be better equipped to use them outside of school as well' (OECD, 2014b, p, 29) When engaged in mathematical problem solving, De Lange's (1981) participants found it quite a challenge to recall knowledge from other subjects (p. 577). In the work of Venville *et al.,* (1998), their participants had a better understanding of science and mathematical concepts when they applied their knowledge to practical tasks. Within this style of education students began to appreciate the links between subjects and could aid understanding from a wider perspective (Lonning & DeFranco, 1997; Wicklien & Schell, 1995). This transference of skills was described as the 'application of mathematical concepts, skills and strategies to various problem solving settings' (Putnam, 1987, p. 687).

In their 2011 study, Lyon and Bragg (2011) developed a kitchen garden with a group of children. They reported that this provided these children with 'an insight into the desirability of applying mathematical understanding to real-life situations' (Lyon & Bragg, 2011, p. 30). Pepperell *et al*., (2009) mentioned the importance for children to have the opportunity to see the mathematical similarities in various situations, which included making links between mathematics and other subjects. Wedege (1999) believed that there was no such thing as context free mathematics and regarded all mathematics as a contextualized activity. Some school projects such as Lyon and Bragg's (2011) kitchen garden elicited a positive response from their participants. The students involved in the Kitchen Garden project actually wanted to understand the mathematics they were using. In this instance the context provided the stimulus for further learning (Lyon & Bragg, 2011) and although a positive response is what teachers would hope to elicit, a gain in knowledge and understanding might be the most desirable outcome.

Within this section I have discussed context and real-life problems. Whilst investigating the importance of context it revealed some areas which could be become themes in thematic analysis,  this conversation will resume once again in the final section of this chapter.

## 2.7 Summary

The previous chapters have discussed problem solving in relation to the primary school curriculum of England which included defining problem solving, and locating it in the curriculum. The purpose of this chapter was to review the current literature in relation to my research question. During this review I noticed that there was a perceived shortage of research into problem solving within the primary school mathematics curriculum.

My readings of the literature have underlined the importance of problem solving in the mathematics curriculum, with problem solving at the heart of mathematics (Ollerton 2011; Skemp 1993). Problem solving was included in the PISA 2012 survey (2014), to demonstrate how the children apply their mathematical skills. Their findings emphasised the importance of problem solving for a students' future in the job market and financially. This adds more weight to the argument that problem solving is an important feature of the curriculum and the results also suggest that the curriculum should contain more non-routine problems.

The literature also stressed the complexity of the term 'problem solving', as both a teaching method and a learning procedure. However, I concluded that the term problem solving is an umbrella term for many activities. In an earlier section I attempted to define problem solving is in the context of the primary school. In the context of this research I have constructed the following definition of problem solving;

*the creative process the learner employs when applying mathematics to find possible solutions to non-routine problems when the direct path is impeded.*

As part of my review I discussed the opinions of both children and adults to mathematics and it was concerning to discover the negative opinions some had. Included are those attitudes they inherit from key adults and the notion that it is socially acceptable to be innumerate. From the readings conducted, there was an indication that problem solving could engage reluctant learners. It has the potential to provide positive changes in attitude (Burton, 1980). Problem solving can allow children the opportunity to experience the enjoyment of finding their own answers and take ownership, and the struggle when problems are not straight forward.

My review also demonstrated some of the issues surrounding the content and context of the problems. There is a school of thought that the problem has to be matched to the children's knowledge and understanding of the world (Ernest, 1996). It must not impact on the mathematics learnt by distracting or over complicating it. Another thought was that without a meaningful context the learning is not internalised (Burton, 1980). The works of Burton (1980), Boaler (1993) and Wild (2011) have demonstrated that the selection of tasks for the current research should try to match this criterion. These comments do actually complicate the selection of problem activities for learners. It does suggest that educators need to think carefully and that children are given problems which take them from 'the reality of their world into the world of abstract mathematics' (Marshall, 2006, p. 358). There is also the added complication that the children's personality and understanding of the world outside the classroom also have a bearing on the success of problem solving.

Whilst researching the definition and the structure of word problems I started to collect together some of the descriptive terms assigned to problem solving. I am not exclusively utilising deductive coding as I am encouraged that more themes (codes) will become apparent in my observation data as analysis begins. However, from the readings I noticed some themes such as mathematising and the seminal work of Polya (1957) and *ignoring context* and *using common sense* (Bayazit, 2013) were apparent.

The justification for this research is centred on the observation of what activities children are engaged whilst they carry out problem solving type activities. My review of literature has brought to the foreground some of the issues surrounding problem solving activities. In designing my research I was aware that previous studies had conducted comparison studies of children in various schools attempting problem solving activities. These included the many works of Boaler (1993b, 2009). There was also Voutsina's (2012) study investigating procedural and conceptual changes when repeating the same problem solving activity but I had not noticed many case study research projects in English primary schools. From my reading of current research I noticed that many of the studies were located in secondary school and quantitative in nature. My research study aims to be unique as it is a case study aiming to gain a better understanding of the strategies adopted by a cohort of Year Six children engaged in problem solving activities. I will describe my methodological approach in the subsequent chapters as well as the problem solving activities the children completed. The works of other researches have aided in honing my initial question into the more precise question;

How do Year 6 children engage in mathematical problem solving activities?

My motivation for my study is highlighted in this chapter but at the core is the importance in knowing at the 'grass roots', what actually happens when children attempt these activities. Within Chapter Three I will discuss why I chose to use a case study approach as my methodological approach for data collection and thematic analysis to explore my observation data.

# Chapter Three - Methodology

## 3.1 Introduction

This chapter aims to explore the methodological approaches and methods I have utilised in addressing my research question. To this end, this chapter includes an explanation of my rationale for the selection of the final methodological approach. It also contains an explanation of my ethical considerations of my study, my methods of data collection and data analysis I employed.

In addition I discuss my philosophical viewpoint, including my ontological, epistemological and methodological assumptions which underpin this study. I begin this chapter by discussing my philosophical viewpoint and theoretical perspectives before proceeding on to my research question.

## 3.2 Research Approach - Defining My Theoretical Framework

This section contains a discussion centred on my philosophical perspectives in relation to my research questions. I explored the impact my viewpoints had on my methodological choices I made when arriving at the research questions and the choice of data collection and methods of analysis. I felt that the inclusion of this section in the methodology chapter was necessary as it is ‘crucial for researchers to think about their own political and theoretical perspectives on their topic before considering methodologies’ (Lee, 2009, p. 71). Consequently, the assumptions a researcher holds regarding the nature of reality impacts on what they report. Waring (2012) argued that, ‘researchers should acknowledge the fundamental relationship between the ontological, epistemological and methodological assumptions which underpin their research and inform their choice of methods’ (p. 17). Therefore, there is a belief that researchers should be transparent about their assumptions and personal values before the onset of a study. This included how these frame their research. Creswell (2009) used the term ‘worldview’ to group together the terms ontology and epistemology as he saw these as a ‘general orientation about the world and the nature of research the researcher holds’ (p. 6).

On reflection, I have noticed how my ideas and perceptions have developed, from those I held during my initial undergraduate research to the beliefs I have now. My philosophical worldview now lies within constructivist and interpretivist traditions, a move away from my initial positivistic views. My initial degree was in the biological sciences and grounded in quantitative traditions, using experimentation and a belief in one reality. My subsequent studies in education prior to the current study were once again quantitative. They relied on statistical analysis to measure levels of stress in practicing teachers using a questionnaire. In undertaking this research I changed my philosophical perspective and moved towards qualitative research and its associated methodologies. This change was due to the nature of the research I wanted to conduct. I intended investigating the approaches children adopt when attempting problem solving activities. Currently my epistemological position lies within constructivism paradigm as I am aiming to construct knowledge through my interpretations of the situations that occur when children attempt to solve real-life problems. This included developing an ontological belief in multiple realities instead of the belief in just one reality.

My role as teacher has also influenced my world view. My work with children had an impact on the type of research I wanted to conduct. I had moved from the mind set of experimentation and testing hypotheses to a more investigative approach, I wanted to know what was occurring during problem solving mathematics lessons. With this research I aimed to start from an inductive position, developing my theories of what is happening as the data collection process progresses, I was not trying to prove a hypothesis or evaluate an intervention from my research.

I have mentioned my previous studies using quantitative methods but I decided not to employ quantitative research for this study. This was due in part to the nature of my research question. It is apparent that the quantitative research tradition does not suit the theoretical assumptions underpinning my study.

The main aim of my study was echoed in a comment made by Jaworski's (1994). She proposed that it is possible for researchers to either look at an aspect of a theory or to 'observe and describe occurrences in a classroom' (Jaworski, 1994, p. 59). My interests lie in the latter of these two proposals. Other researchers (Gillham, 2000; Miles & Huberman, 1994) noted the usefulness of the qualitative research traditional, especially when the aim is to focus on naturally occurring events. It is also useful when the objective is to 'explore and understand the meaning individuals or groups ascribe to a social or human problem' (Creswell, 2009, p. 4). In addition it has been suggested that the data corpus might provide rich descriptions 'nested in real context' (Miles & Huberman 1994, p. 10). I will discuss this comment further in relation to my research question.

## 3.3 Research Questions

The research question has a significant influence on the methodological approach a researcher adopts and subsequently the course their study follows. I will now discuss my research question and how it has developed from my review of the literature. The literature review aimed to explore problem solving in the context of primary school education in England. Several themes did emerge from the readings which included studies evaluating the effectiveness of intervention programs, observation of classes engaged in problem solving, investigating transference of skills and attitudes towards mathematics, to name but a few.

My research question was initially motivated by the previous research studies focused on the mathematics curriculum (Boaler 2009; Burton 1980; Ollerton 2011) and some seminal educational reports and reviews (Cockcroft 1993; Williams 2008). These research reports and government reviews stressed the importance of problem solving as an element of the mathematics curriculum as well as its potential as a teaching approach. Within my review of the literature I did identify a gap in the research surrounding problem solving in primary school mathematics. Many of the studies were focused on older secondary school students but no comparable studies focusing on one cohort and how these children approached problem solving activities. The literature review noted the occurrences of problem solving as the subject of research studies including the works of Boaler (2009), Burton (1980), Putnam (1987) and Voutsina (2012). These were discussed in more detail in the literature review, Chapter Two. In my opinion these educational reports and studies have made some progress towards validating the need to further research problem solving. For example, the Williams report (2008) suggested a need for improvement in the Using and Applying of mathematics, to develop an 'appreciation of mathematics in their (children's) lives' (p. 3), to include activities in the curriculum which emphasis links between concepts. I also drew inspiration from Putnam's (1987) comment, regarding the difficulties children have in applying their mathematical knowledge to mathematics tasks. Within my study I wanted to explore some of these areas further. The review of the literature also brought to the forefront the complexity of the problem solving process. This was described by Edens and Potter (2008) as a complex cognitive process. This comment had a profound effect on the structure of the research question. Problem solving is a vast area of mathematics and there are many facets I could focus on including, problem solving behaviours, strategies and techniques.

My research aims to address the following question;

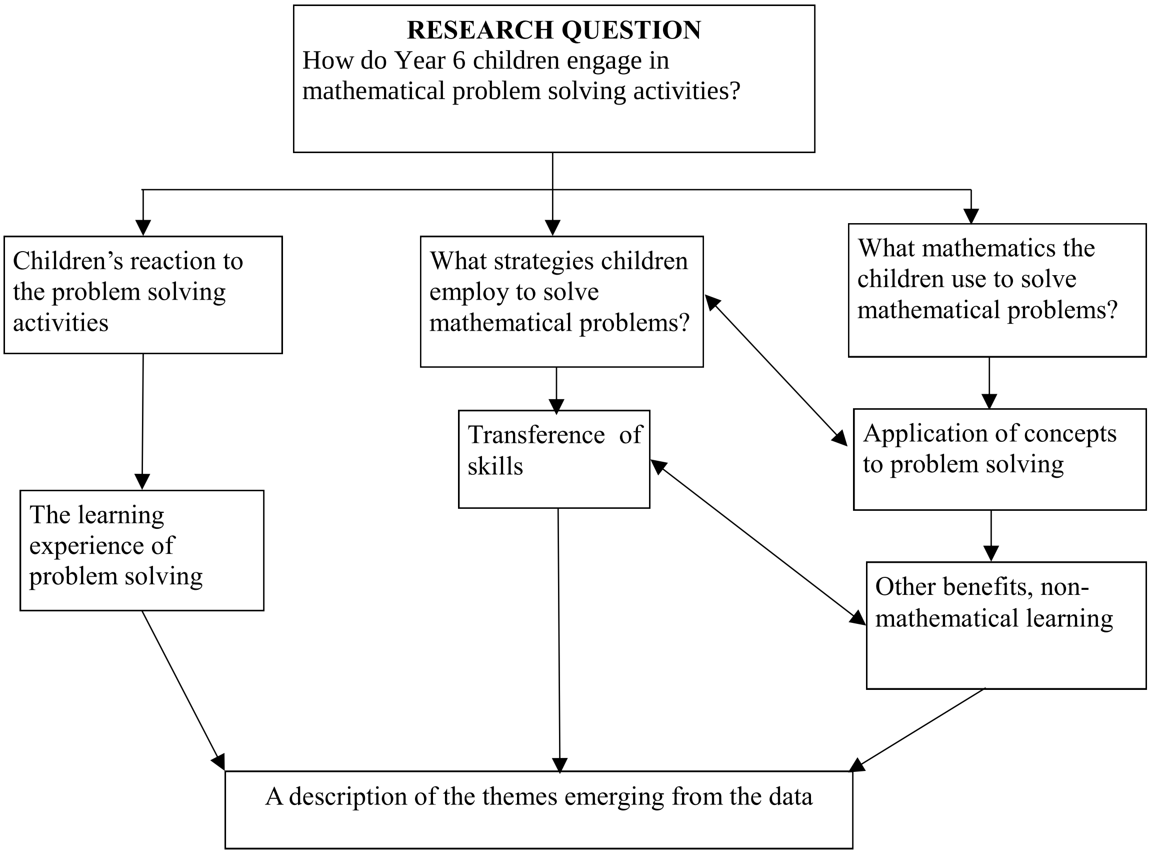
How do Year 6 children engage in mathematical problem solving activities?

Subsequent questions arose from some of the areas of problem solving described in Chapter Two. I was curious to see whether the skills children learn from completing textbook style exercises could be applied to real-life problems. Problem solving facilitates the exploration of relationships between different topics in mathematics (Pepperell *et al*, 2009). This led me to the notion of 'relational understanding' (Skemp, 1993). This is the ability to link and relate concepts. The Cockcroft report (1982) recommended that all children 'need to experience applying the mathematics they are learning to everyday situations and problems' (p. 95). The subsequent questions my research aimed to address were;

- What strategies do the children employ to solve mathematical problems?
- What mathematics do the children use to solve mathematical problems?

I employed 'progressive focusing' (Gomm, 2004) to develop further questions as the data was collected and initially analysed. This is consistent with a constructivist approach since this espouses multiple realities 'the full research question cannot be fully established' (Robson, 2002, p. 27) and therefore the research question can be refocused as the data is collected. Any additional questions will be discussed in Chapter Five, the Discussion.

### 3.3.1 Conceptual Framework




(Figure 3.1 Conceptual Framework)

The conceptual frameworks *figure 3.1* is schematic diagram which highlights the main strands of my study. Some of the links between topics are shown but this is not exclusive. Baxter and Jack (2008) have warned that a framework may limit inductive research approaches and to minimise this they suggest keeping a record of thoughts and decisions (ibid). Through the writing of my thesis I have maintained a research journal and revisited my conceptual framework several times.

As noted in the previous section my theoretical assumptions have had an impact on the methodological approaches I adopted to address the research question. Scott and Usher (1999) claim philosophical issues are integral to the research process because this is what researchers silently think about research. In the following section I will discuss some methodological approaches I considered for my research.

## 3.4 Methodological Approaches

The following section contains a discussion focused on a selection of the methodological approaches considered for this study. Understanding the advantages and disadvantages of each approach aided the selection of the most appropriate methodology for my research. There are many choices of methodologies and it is not possible to explore each one. Therefore, the following is a discussion of a limited list of popular qualitative methodologies as advocated by both Creswell (2009) and Mertens (2005). Within this section I have included a description of these approaches and explored their application in educational research and their suitability for my research.

### 3.4.1 Grounded Theory

Grounded Theory was chosen as it is one of the methodological approaches which suit the ethos of my research. In the literature, Grounded Theory has been described as both an approach and a method of data analysis but for this section I am focusing on its use as a methodological approach. Grounded Theory was developed as an alternative to other positivistic paradigms (Waring, 2012). Originally it was described as a positivist approach. However, Grounded Theory exists in several forms differentiated by the level of interaction between participant and researcher. The form of Grounded Theory that I considered was Constructivist Grounded Theory. This differs from the many other varieties as there is more emphasis on how the methodological strategies are constructed (Bryant & Charmaz, 2007). It also has the 'researcher as the author of a reconstruction of experience and meaning' (Mills, Bonner & Francis, 2006, p. 2). That is to say, the researcher is not neutral and their position, views, expectations and interactions are considered as features of both the data collection and analysis processes.

A key feature of Grounded Theory is that the researcher starts with an area and then 'evolves an appropriate theory from the relevant data specific to the situation under investigation' (Waring, 2012, p. 299). Grounded Theory is also useful when investigating 'what happens in everyday classroom life' and 'what goes on while students are doing group work' (Thornberg, 2012, p. 85). These arguments all fit well with the overarching research question of my study and its investigative nature.

A possible disadvantage of Grounded Theory is its lack of rigour. This is due to its unique approach to data gathering that 'emphasises inductive, open ended, intuitive approaches' (Waring, 2012, p. 306). Theories and conclusions are developed as questions are asked of the data. There is some flexibility within this method and Robson (2002) described it as 'systematic and co-ordinated' (p. 192), giving it an end point. This is important when considering that data collection will stop once saturation is reached. Reaching the saturation point can be very time consuming due to the need to revisit the field to collect data between analyses. Therefore, it is essential that the time allowed to complete the data analysis is factored into the research design.

### 3.4.2 Phenomenological Research

There were some tenets within the phenomenological approach that suited my research study. Within this approach there is the need for the researcher to identify 'the essence of human experiences about a phenomenon as described by the participants' (Creswell, 2009, p. 13). It aims to gain a participant's point of view and it is distinguishable from other methodologies in that, 'subjective experience is central of the inquiry' (Mertens, 2005, p. 240). The phenomenological researcher does not make assumptions but focuses on the individual's perceptions. Although it would be interesting to examine this approach it is not appropriate for my study. This was because I aimed to investigate a process from my perspective. For this study to follow a phenomenological approach the research question would have to state, that I was investigating problem solving from the perspective of the students. An example would be the exploration of the participant's perception of problem solving which would not answer my research question as I am aiming for an overview of the classroom experience.

Further reading of the literature also demonstrated the inappropriateness of this approach for my research. This was particularly due to its specialized nature. Robson (2002) suggests that in contains specialised vocabulary and philosophies and therefore this approach is not recommended for novice researchers. It is also situated in the interpretative paradigm and as I have mentioned my philosophical perspectives lie in constructivism paradigm.

### 3.4.3 Action Research

Another methodological approach I considered for this study was action research. I chose this as a possible methodology for this study because of its long history in education and community development (Munn-Giddings, 2012) and its ease of use for the practising teacher. Cohen *et al.,* (2001) claimed that action research is a 'powerful tool for change and improvement at the local level' (p. 226). This approach appealed to me as a practising teacher, particularly as the main purpose of action research is 'to affect some worthwhile change while studying how this comes about' (Gomm, 2004, p. 173). There is also the 'expectation that this research will lead to improvement in the classroom' (Bassey, 1986, p. 23). This form of research is useful when evaluating an intervention programme. The problem solving activities used in my research were not part of the Year Six curriculum that was dictated by the focus school's long term mathematics curriculum planning. However, the problem solving activities were being used as part of a project which the Year Six children of the school engaged in during the summer term. Their project work is not an intervention as it is not trying to affect a change as is the norm for action research. My aim was to see what strategies the children adopted when they attempted to solve these problems. Action research would be the better approach to use if I was implementing a new program of study and wanted to monitor the outcome. Therefore, I concluded that action research did not fit with what I was trying to research and I would not adopt it as my methodological approach.

### 3.4.4 Ethnography

I did consider an ethnographic approach for my research, this was due to its utilisation of naturalistic observations and participant observation as a main method of data collection. According to Mertens (2005) some authors view ethnographic research as case studies but this is an over simplistic view of case studies.  It was my intention to use observation as the main method of data collection. Also ethnography produces a lot of descriptive data, it is a useful approach when the research question is to describe and interpret 'the culture and social structure of a social group' (Robson, 2002, p. 186). In her research Jaworski (1994) employed an ethnographic approach. This included participant observations and interviews to investigate the teaching and learning of mathematics situated within a single educational establishment.

Ethnographic observations are conducted to study individuals over long periods of time in their own environment. Within the focus establishment I was limited to a three week data collection period. I was also constrained by the length of time allowed to complete my doctoral studies. Therefore, I did not have the time available to complete a longitudinal study.

A further characteristic of ethnography is its particular narrative writing style. This was noted by Robson (2002) who claimed that this writing can be challenging and requires time to master. I concluded due to the time constraints for collecting data and analysis, and my lack of experience in ethnographic methods I decided that I could not use this approach.

### 3.4.5 Case Study Approach

My research question has several characteristics of a case study. It is an investigation set in one educational establishment and focuses on a contemporary event. It has been suggested that case studies 'provide a tool for researchers to study complex phenomena within their context' (Baxter and Jack, 2008, p. 544). In exploring this statement further it has been argued that case study can be employed to 'interpret the world in terms of its actors and consequently it may be described as interpretative and subjective' (Cohen *et al.,* 2001, p. 181).

Case study can exist in several types either as multi-case or single case studies. These studies can be used for exploratory, descriptive and explanatory purposes (Yin, 2014). Miles and Huberman (1994) and Cohen *et al.,* (2001) describe case study as a specific instance which is bounded by it context. The context is very important in case study research as it explains the issues of concern that form the background to the research. It also guides the readers of the research to understand where the research questions are located. It appears that my research would suit a single case study approach as the case being studied is a Year Six cohort in one educational establishment, a non-selective private school. Yin (2014) described several potential single-case study designs. My research question fits with his description of the rationale for a 'common case' whereby the 'objective is to capture the circumstances and conditions of an everyday situation' (p. 52). A multiple-case study would only be useful if I wanted to examine several cases and then wished to compare them but I was only intending a single case.

One of the most important aims of binding the case is to limit the scope of the study. Reportedly, one common pitfall in case study research is the tendency to attempt to answer a question that is too broad (Baxter and Jack, 2008). Therefore, I have established some additional boundaries which might not be obvious from the initial question. For example, I am focusing on one year group and have a limited observation time. Baxter and Jack (2008) drew parallels with the boundary establishment in qualitative research and the development of inclusion and exclusion criteria in quantitative research. However, they acknowledge that boundaries also include breadth and depth of study (Baxter and Jack, 2008).

As with all approaches there are critics who highlight its disadvantages. Robson (2002) noted that in the past case study was considered a ‘soft option’ because of its use as an exploratory precursor to more research or to complement other approaches (p. 180). Yet, case study is a powerful tool and its strength lies in the ability it affords the researcher to investigate a case in depth (Day-Ashley, 2012). Due to the nature of the approach, findings are rarely generalisable but they do demonstrate what happened during a time period and this may be of use to others in similar situations. Cohen *et al.,* (1994) suggested that the benefit of case study is that they can establish ‘cause and effect’, and that they investigate ‘complex, dynamic and unfolding interactions’ in a bounded instance (p. 181). My research is addressing the question of what happens in a class when the children are engaged in problem solving activities and although it is not generalizable, it may be of use to other educators in similar situations.

Once again the question of rigour is mentioned. Yin (2014) claimed that the concern regarding rigor is due to ‘investigators who have allowed equivocal evidence or biased views to influence the directions of their findings and conclusions’ (p. 19). This was also mentioned by Robson (2002) and he also suggested the risk of researcher bias and selectivity. Therefore, as I will be using a case study approach I need to be transparent regarding my opinions. Baxter and Jack (2008) proposed some additional consideration for designing and implementing rigorous case studies. The first one was the development of propositions. Miles and Huberman (1994) discuss how propositions can develop during the data analysis phase. They describe the need to ‘formalise and systematise the researchers’ thinking’ by generating connected sets of statements (p. 75).

After completing my literature review I decided that a case study approach would be a suitable methodological approach for my research. This was due in part to previous studies and to its popularity as a methodology for the social sciences (Day-Ashley, 2012) and in educational studies (Bassey, 1999).

## 3.5 Methods Of Data Collection

Within the next section I will discuss my rationale for the methods of data collection I employed. Whilst I acknowledge that there are other methods of data collection, ultimately my choice of methods was dependent on the type of data that I needed to collect in answering my research question effectively (Creswell, 2009). I also will discuss how my sample was selected, this revisited once again in the ethics section of this chapter.

### 3.5.1 The Sample

In Chapter One I described the context for my study. I also mentioned that at the core of this study was my curiosity to investigate what approaches children adopted whilst they attempted problem solving activities. It was decided to invite the Year Six cohort to participate in the study. The data collection phase of the study was due to commence in June 2014 and this was an important influence on the selection of the sample group. The Year Six children were either preparing for, or sitting examinations from the start of the academic year in September until January. During this period, their curriculum was aimed towards exam success. At the end of the academic year these children were scheduled to complete mathematical projects which coincided with the start date of my study and therefore well-matched the focus of my research.

Although the focus establishment has classes from ages 5 years to 11 years, these other year groups were not considered for this study. The main reason for their ineligibility was due to the curriculum constraints in these classes as the children prepared for other important examinations. I gained permission from the children and parents of my sample after I had met with the relevant gatekeepers at the school. I will revisit this process and discuss it in greater detail later in the ethical considerations section of this chapter.

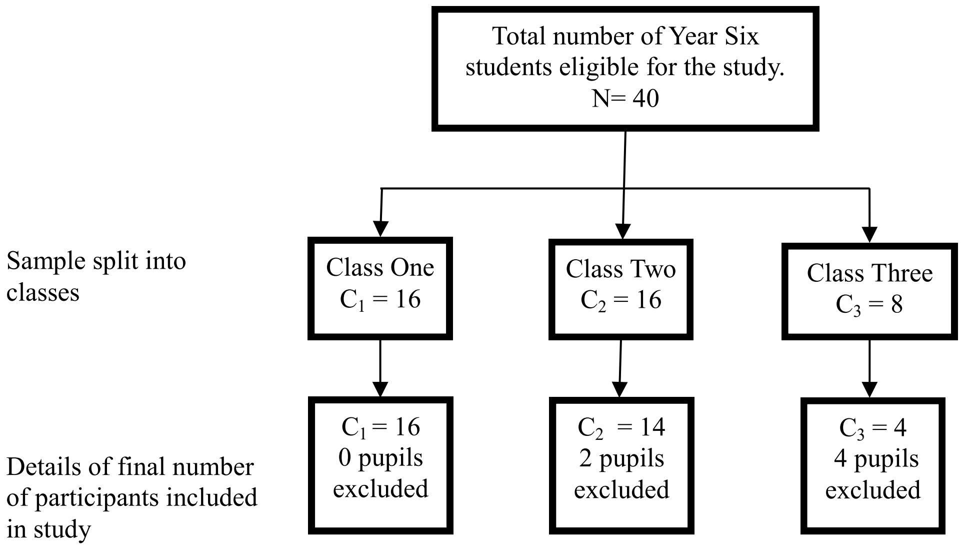



Figure 3.2: Flow chart demonstrating the final number of participants included in the data collection phase.

Consent was gained from all the members of the Year Six cohort, although I have not included all the children in the final study. The number of participants and the classes they came from is illustrated in *figure 3.2*.

The three classes were labelled as $C_1$, $C_2$ and $C_3$ I will use this nomenclature through the remainder of my discussions. In order to choose the most appropriate tasks for the children I made use of the assessments that had been completed by the children's' previous teachers. Their results indicated that there was a range of abilities within this cohort. The assessments used were from National Foundation for Educational Research (NFER) and relied on age standardised scores. The focus establishment used these assessments to ability stream the classes with two able ability classes ($C_1$ and $C_2$) and one average ability class ($C_3$). The average ability class, $C_3$ consisted of 8 boys. The other two classes both contained 7 boys and 9 girls for a total of 16 children. None of the children had been identified as having special educational needs. However, there was a high proportion of children with English as an additional language in these classes. In $C_1$, there were 9 children, in $C_2$, there were 8 children and in $C_3$, 6 children also spoke another language at home. The children did not receive any supplementary lessons to support them as they were considered fluent English speakers. It is pertinent to note that these children were taught with a high focus on preparation for competitive secondary school examinations. In 2014, the Key Stage Two end of year assessment's National average was 86% achieving level 4 or above and 42% achieving Level 5 (DfE, 2014). The teacher assessment of the children in the sample group was 100% at Level

4 and above. In school written assessments 26% achieved Level 4 and 74% achieved Level 5. As is evident in these assessments all of these children were of average ability or above.

Within the classes the children were sorted into working groups of three or four children for the problem solving activities. The classes were taught separately and therefore they were only mixed ability within these narrow classes. The final sample of observed children referred to as 'observation groups' was selected via a process which will be further explained in this section.

At the onset of the data collection process, there were very few barriers as to which groups I could observe. The participant groups in my study were selected after the first initial observations were completed. As the initial observations were viewed and the field notes examined I started to see groups of interest. I focused on the groups who were vocal and engaged in talk. A further factor impacting on the selection of a Year Six focus group was my ability to negotiate more time to work with these children, especially those in $C_1$ and $C_2$. This was due to the fact that I was regularly teaching these children and as their mathematics teacher I was able to set up observations during their usual lessons. Also due to the structure of the children's lesson timetable I was scheduled extra lessons with these children. The class $C_3$ was taught by another teacher and I had limited access. However, four of these children joined $C_1$ for one lesson 'School trip letter activity' and were included in that observation. I will explain why these children joined $C_1$ when I describe the activities.

### 3.5.2 Observation

I previously noted that I chose case study as my methodological approach. I was motivated by the research methods of both, Jaworski (1994) and Boaler (2002). Within their research they conducted observations of mathematics lessons. My aim was to achieve a snap-shot of the learning that was occurring during the attempts to solve of problem solving activities. I intended to capture my data in a natural setting. Gomm (2004) described this as the 'situations observed would have happened regardless of whether the children were being observed or not' (p. 217). As I mention in the ethics section below, the children would usually complete a mathematics project at this time in the academic year. Therefore, the work was not contrived and they remained in their usual groups with the exception of one observation point.

From my review of the literature, I noticed that observation was cited as a ‘frequent source of information in case study research’ (Hancock & Algozzine, 2011, p. 51). It is a useful method to ‘understand a culture’ (Silverman, 2005, p. 111). Boaler (2002) employed it to compare two different classes in her study. It can be said that observation allows for the collection of data which is not biased by the participant’s perceptions and is somewhat more objective than other methods such as participant questionnaires. The assumption is that the raw observation data is not influenced by the opinions of the children. However, I felt that I need to be a little cautious as in some of my transcriptions I have noted occasions where the children noticed that they were being observed and they have suggested that ‘they better get on’ and they have possibly altered their behaviour. I developed a coding rationale which demonstrates the how the transcribed data was selected, *irrelevant data* was data not related to the research question and therefore not included. I have previously discussed observer bias which Angrosino (2012) notes the issue of subjectivity. I will revisit the reliability and validity of the data in subsequent chapters.

Several researchers (Gillham 2000; Gomm 2004) discussed how to conduct effective observations. Gomm (2004) suggested that to yield data which could be analysed researchers need to develop rules for observation. Gillham (2000) emphasised the need to be organised during data collection and also to ensure that enough data is collected. He suggested that ‘the observer must have a clear specification of what is to be observed and a clear procedure for recording the observations’ (p. 56). Gilham (2000) implied that the observations are not random and that there should be some plan of what will be collected. This concept was quite difficult when considering the data I wished to collect. I knew what data I wanted to collect. However, due to other circumstances I had to be very flexible. As the data collection process was during the last weeks of the summer term the amount of time allowed to collect data was limited due to the availability of the children. To maximise the data collection process I decided that I would set up various recording devices in the classes enabling the capture of as much data as possible. The ethics of using recording devices with child participants is discussed later in this chapter.

My observations were focused on children completing problem solving activities with the emphasis on the actual working through the problem, predominantly how children approach the problem. An observation schedule could have offered a solution to the lack of structured observations in my study. However, I did not want to use a structured observation approach as this would limit the focus of the observations and what was recorded. I was concerned that this would result in a narrowing of the data especially if I started to reject the data before I analysed it. A Grounded Theory approach was used to collect data and negate the 'narrowing' of data. I aimed to collect 'relevant, substantial and rich data' (Waring, 2012, p. 301) and then condense it down in the data analysis phase of my study. I have mentioned that I did sort my data, this filtering occurred after transcription, once I had started coding. I will discuss this in more detail in the Data Analysis section of this chapter. Observations were conducted using video and audio taping equipment. I chose to use these devices as they are able 'to record detail without fatigue,' (Prosser, 2000, p. 129). They also allowed for a large amount of data to be collected in a short time which was especially important as there were time constraints. Field notes were included to annotate the video and audio data and copies of the children's class work were kept.

An acknowledged disadvantage of observation is observer bias. Angrosino (2012) argued that bias has been recognised by researchers 'since the method almost always involves some sort of subjectivity' (p. 168). For my research project I assumed the role of participant observer because I was using the lessons that I taught and recording sections of these lessons. I have appeared on the recordings as the class teacher. I was aiming to gain a 'fly on the wall' perspective but this would have entailed an unnoticed observer, something rather difficult in the classroom setting. Rather, the aim of a participant observer is to become an active member of the group they are observing. Angrosino (2012) purported that the 'trick is to become enough of a member to gain an insider's perspective on what was going on' (p. 166). In answering my research question I was aiming to observe the approaches children adopted when they attempted to solve mathematical problem solving activities. Therefore, my role as a teacher conducting practitioner researcher was established and I took on the role of participant observer during the data collection phase of my study.

Robson (2002) pointed out the possible ethical dilemmas of the role of participate observer as well as noting that the behaviour of the participants could be altered to please the observer. These elements of the observation have been discussed in more detail in the ethical section of this chapter and form part of the discussion of the results. Before discussing my ethical considerations I will describe the problem solving activities selected for the children to attempt.

## 3.6 Problem Solving Tasks

The tasks are essential to my project as they are the tools that elicit responses from the children. Their responses ultimately became my research data and therefore the choice of activities is, in some way as important as my methodological approaches. I have used the term *observable activity* to mean the classroom tasks the children were completing whilst I conducted my observations. I will include a synopsis of these activities and why I chose these activities in relation to the participants and my research question. I have also included an exploration of group work and why I chose this method of working for the children in my study.

Later in this chapter I will discuss the participants in terms of ethical consideration and my selection process. The choice of activity was influenced by both the participants and the school's needs and somewhat the parent's perceptions. The collection period was scheduled for the last two weeks of June 2014 as at this time of year the children would have completed all of their examinations and the curriculum as prescribed by the school. This afforded me the freedom to choose any activity, the only requirements were that the tasks matched the children's ability and were engaging.

As previously discussed I needed to consider the academic level of the task and match this to the children. Therefore, I made use of the assessments that had been completed by the school. It was also important to note that the school had adopted a secondary school format to their curriculum timetable. The children had four 40 minute and one 80 minute lesson of mathematics per week and were taught by myself, a mathematics specialist teacher for these lessons. For the majority of the observed lessons the children remained in their usual classes. There was one insistence where this did not occur and I have noted the reasons for this in the description of the lessons. Within the next section I will take a closer look at the activities the children completed.

### 3.6.1 Observation Activities

The problem solving activities were selected from three different sources. The core problem solving activities were selected from the Maths Investigator 6 scheme. From this point forward I will refer to the Maths Investigator 6 scheme as MI6 (Clissold & Pink, 2008). Due to circumstances beyond my control two observation days were disrupted by significant participant absences. It was not possible to continue the MI6 activities on these two days due to the nature of the MI6 tasks in this scheme. Unfortunately, I had not been made aware of these absences until the day and had scheduled these as data collection points. Therefore, I decided to use some additional problem solving activities. These activities had been selected as possible observation activities before I decided to use the MI6 scheme. The first substitution activity was ‘Planning a School Trip’ (Vickery & Spooner, 2004) and the second was ‘Colour Sudoku’ (Mansergh, 2007). Both of these activities were discrete self-contained problem solving activities which could be completed in a single lesson. In the subsequent sections I will describe the activities in more detail.

#### 3.6.1.1 The Case Of The Missing Suitcase - Maths Investigator 6

As I have stated the core problem solving tasks were developed from the MI6 scheme. The activities were ‘based on the concept of a spy agency using mathematics to solve problems or cases’ (Clissold & Pink, 2008a, p. 5). The school’s medium term mathematics plan noted that the children were scheduled to complete project work which coincided with the time I aimed to conduct my research. I chose the MI6 scheme as the concept of the spy agency was very appealing, I knew the children and I thought that they would enjoy the spy theme. It was also selected as the mathematical ideas and concepts were accessible for the cohort but had potential for extension.

The MI6 scheme is split into fifteen units. Each unit is taught as a continuous problem solving activity which covers several lessons. The scheme is also structured so that the activities are completed as group or paired tasks. I used the first unit of work for my study ‘The Case of The Missing Suitcase’. It was sensible to start with unit one as the children were not familiar with this scheme and the subsequent units use the narrative from the previous MI6 units.

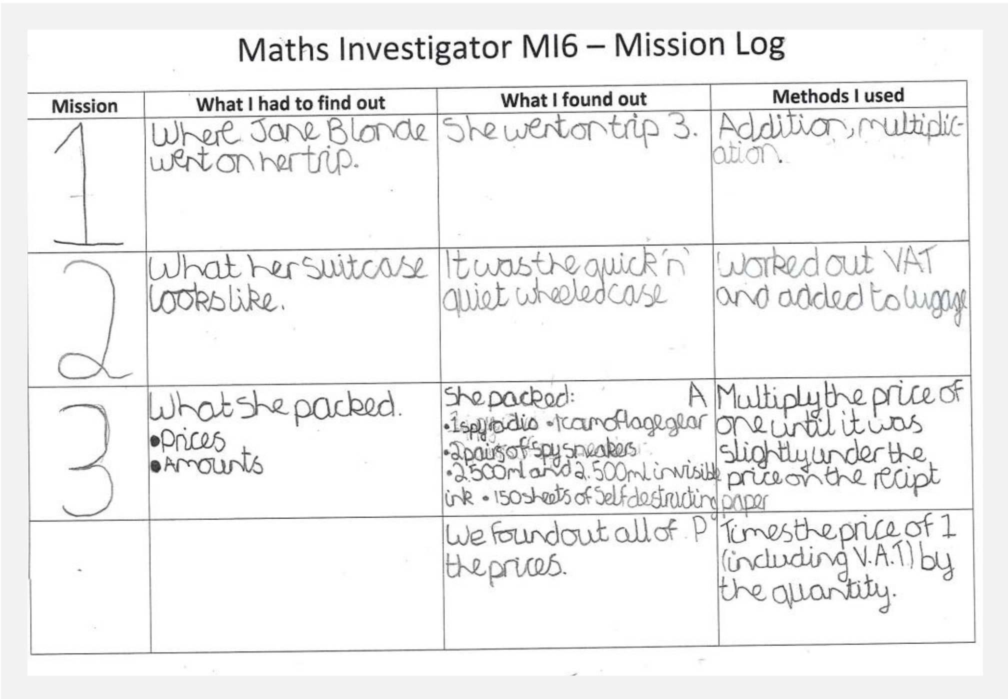
The lessons are structured with a lesson opener and then an independent activity. The children are given a mission statement from ‘M’ the head of MI6 as the lesson opener which contains the task and a double page spread in the case book as the stimulus to solve the problem. The textbook linked to the problem was unusual in that it was called ‘the casebook’ and contained pictures and information clues linked to the case (appendix 2.1). They differed from the children's usual textbooks which typically contained pages of instructions and problems with very few illustrations.

The MI6 scheme was designed around the Primary Framework (DFE, 2003). As I discussed in my literature review, the mathematics curriculum has been restructured several times since this scheme was created. However, this did not have a major impact on my choice as the school is in the English independent sector and as such they were not required under current educational legislation to follow the National Curriculum. However, the mathematics taught in the school was based on the National Curriculum (DfEE, 1988) with the children extended and taught some of the curriculum objectives from the Year 7 curriculum. The school's rationale was centred on preparing children for competitive grammar school and secondary school examinations. Therefore, the tasks I used from the MI6 scheme were altered to reflect the ranges of abilities of the children and their prior knowledge. The original MI6 Unit One was designed as a ten day module of work taught in 1 hour lessons. The following (table 3.1) is a synopsis of each task as taught and the mathematical concepts the activities contained.

| Task no. | Problem solving task | Key Mathematical idea/concept | Description of the activity |
|---|---|---|---|
| 1 | Which trip did the secret agent take? How far did she travel? | • Place value of decimal numbers<br>• Calculation using decimal numbers, including addition, multiplication, division. | Use the pedometer and the trip information in the case book and work out which destinations the secret agent visited by calculating how far she travelled |
| 2 | Which suitcase did she use? | • Percentages<br>• Calculate 17.5% of amounts including decimals | Use the receipt from the secret agent's shopping trip given in the case book and the catalogue page from the Agent's Supply Store to workout which suitcase she purchased |
| 3 | What did the secret agent pack in her luggage? Including the quantities | • Multiplication of decimal numbers (money)<br>• Calculation of VAT at 17.5%<br>• Written methods of calculations; long multiplication HTU x TU division HTU by TU | Using the receipt and the catalogue page, work out the quantities of the items she packed |
| 4 | What was the weight of the luggage? | • Weight, covert between kilograms and grams<br>• Place value of decimal numbers<br>• Addition and multiplication of decimal numbers | From the answer to Task 4, calculate the weight of the suitcase once packed |
| 5 | What was the code on the padlock? | • Order of operations – BIDMAS<br>• Calculations involving brackets<br>• Written methods of calculations; long multiplication HTU x TU division HTU by TU<br>• Square numbers and square roots | From the given scrap of paper found in the Secret agents belongings calculate the 6 digit code. |

(Table 3.1: Description of MI6 Tasks)

These children usually completed their written work in exercise books. The children completed the first task over two lessons and as they were working on A4 sheets of paper, they appeared to find organising their work challenging as sheets were misplaced and not filed correctly. I introduced the mission log (work sample 3.1) after my second round of observations.  As their class teacher I explained to the children how they could organise their work using devices such as numbering pages and adding titles to their work. There was possibility that there could be an impact on my data if I gave the children a proforma to complete. However, the mission log recording page was completed once each task ended and therefore the children would not be influenced whilst they were working through each task.

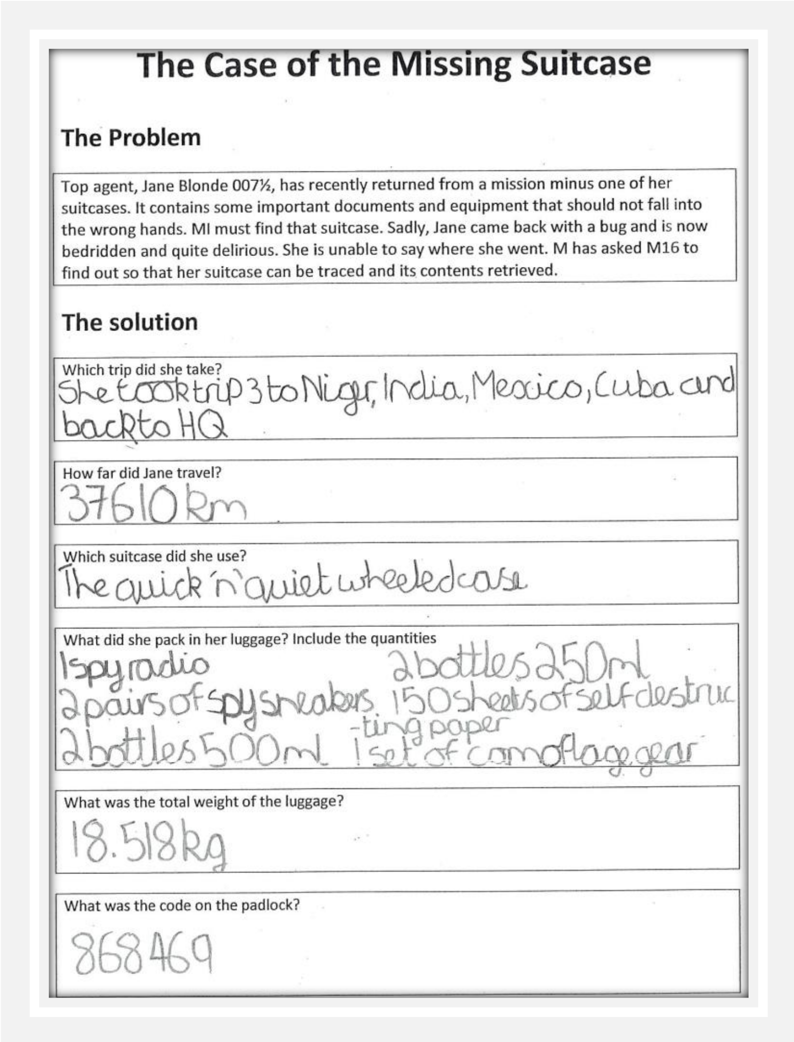


Maths Investigator MI6 – Mission Log

| Mission | What I had to find out | What I found out | Methods I used |
|---|---|---|---|
| 1 | Where Jane Blonde went on her trip. | She went on trip 3. | Addition, multiplication. |
| 2 | What her suitcase looks like. | It was the quick 'n' quiet wheeled case | Worked out VAT and added to luggage |
| 3 | What she packed. •Prices •Amounts | She packed: A •1 spy radio •1 camoflage gear •2 pairs of spy sneakers •2 500ml and 2 500ml invisible ink •150 sheets of self destructing paper | Multiply the price of one until it was slightly under the price on the recipt |
| | | We found out all of P the prices. | Times the price of 1 (including V.A.T) by the quantity. |

(Work sample 3.1)

During the data collection period the children had, had the opportunity to attempt the five tasks (table 3.1) in their groups. The children then completed a solution worksheet (work sample 3.2). The solution worksheet was used to conclude the activities and draw all their work together as a plenary. At the start of each activity they were given the task and square paper to record their workings on. I did not want to steer the children and bias my data by giving the children a structure to follow. Hence, the solutions worksheet (work sample 3.2) was not included as part of the lesson activity. As I have mentioned the activities took place over three weeks and the lessons were not always continuous. Therefore, another use of this sheet was as an aid memoire and an aid to continuity.



# The Case of the Missing Suitcase

**The Problem**

Top agent, Jane Blonde 007½, has recently returned from a mission minus one of her suitcases. It contains some important documents and equipment that should not fall into the wrong hands. MI must find that suitcase. Sadly, Jane came back with a bug and is now bedridden and quite delirious. She is unable to say where she went. M has asked M16 to find out so that her suitcase can be traced and its contents retrieved.

**The solution**

Which trip did she take?
She took trip 3 to Niger, India, Mexico, Cuba and back to HQ

How far did Jane travel?
37610 km

Which suitcase did she use?
The quick 'n' quiet wheeled case

What did she pack in her luggage? Include the quantities
1 spy radio 2 bottles 250ml
2 pairs of spy sneakers 150 sheets of self destruc-ting paper
2 bottles 500ml 1 set of camoflage gear

What was the total weight of the luggage?
18.518 kg

What was the code on the padlock?
868469

(Work sample 3.2)

### 3.6.1.2 School Trip Letter Activity

There was an absence of a considerable number of the participants on one day which provided the opportunity for a substitution activity. The first substitution activity was 'School Trip Letter Activity' taken from the Nrich website. This activity was initially published in an Association of Teachers of Mathematics publication by Vickery and Spooner (2004).

The premise for this problem is an impeding school trip. It is a group activity where the children work together to write a letter to the parents about a forthcoming trip to the zoo. The problem solving element is in the selection of the information for inclusion in their letter. Information cards (appendix 3.1) were provided which contained the instructions for the task and some of the information for the letter (completed example, appendix 3.2). The cards also contained irrelevant information which the groups had to decide whether to discard. The mathematics the children were engaged in included;

- Calculating percentages of amounts
- Addition, subtraction, multiplication and division of money
- Calculations involving time
- Organising information

This was a discrete activity, lasting for only one lesson. The end product was a letter to the parents which contained the following information;

Letter key points

- Date of trip
- Destination
- Departure time
- Arrive time back at school
- Cost of the trip per child
- Items children need to take
- Items they are not allowed to take

(Vickery and Spooner, 2004).

Nrich (2015) suggest that this activity promotes the development of organizational skills. It was similar to the MI6 tasks in that it was a non-routine problem that required the children to think creatively. However, the key features of this problem were that the children had to identify superfluous information from the information. They needed use the information selected to answer the letter key points and finally write the letter (see completed example, appendix 3.1).

**3.6.1.3 Colour Sudoku**

Due to the absence of many of the participant's on a second day another substitution activity was required. I noted in my field notes that only 7 participants from $C_2$ and 8 participants from $C_1$ attended school on this day. I chose to use the Colour Sudoku's activities devised by Mansergh (2007).

An example of a Colour Sudoku is included in the appendix (appendix 4.1). The key mathematical concepts these activities employed were the use of reasoning skills, working systematically and an element of trial and error. These activities were completed either independently or as a part of a group. The instructions for the task did not stipulate the grouping of the children whereas the other tasks did. Also the children were working on individual worksheets and did not have to share any resources unlike the other problem solving activities.

The Colour Sudoku activity was very different compared to the previous two problem solving activities. The MI6 tasks were story based activities that the children worked through. Their answers developed as they progressed through the mission, building on previous knowledge. The School Trip Letter activity was group based and relied on completing a checklist using the information on cards. These two activities contained calculation type questions and utilised logic and life skills. The Sudoku activity differed as it did not involve any calculations but utilised reasoning and logic skills. These Sudoku used colours in place of numbers in a 4 by 4 grid. Typically Sudoku are 9 by 9 grids, the rules are to place the numbers 1 to 9 once in each column, row and 3 by 3 grid. The colour Sudoku used four colours instead of the numbers 1 to 4 this simplified the process for these children which were only primary school aged (10-11 years old).

## 3.7 Group Work

My decision to use group work was influenced by both my research question and methodological approach. I have used the term group work in the context of the classroom with children working cooperatively.

The problem solving tasks from the MI6 scheme and 'Planning a School Trip' were designed as collaborative learning activities by their respective authors. The teaching guidance for the MI6 scheme encouraged group work. They suggested that the children should be working in groupings of between 4 and 5 on each task (Clissold and Pink, 2008). The teaching guidance for 'Planning a School Trip' noted how it lent itself to collaborative learning and suggested that the children work in mixed ability groups (Vickery and Spooner, 2004). The Colour Sudoku activity was flexible and could be completed as either a group or as an independent task.

Group work has been utilised by the Realistic Mathematics Education (known as RME) approach. RME 'favours whole class discussions and groups work' (Van den Heuvel-Panhuizen & Drijvers, 2014, p. 523). This is due to the opportunities for children to interact and share ideas. Group discussions allow for a certain amount of symmetry in children's interactions. Mercer and Sams (2006) argued that pupil-teacher discourse is not equal whereas pupil-pupil discourses are more symmetrical. There is a perceived equality between the children involved. Although, it is still possible for the children to feel more confident in their ability than their peers, the pupil-pupil discourse is still more equal than teacher-pupil discourse. Mercer and Sams (2006) note that pupils engaged in pupil-pupil discourse have 'opportunities for developing reasoned arguments' and 'in mathematics education this collaboration can be focused on solving problems which also have the potential value for helping children to relate their developing understanding of mathematics to the everyday world' (p. 510). Their comment is directly related to my research question focusing on real-life mathematics.

There are those who do not favor group activities in class. Some teachers are concerned by a perceived loss of control in class and an interruption to their coverage of the curriculum (Baines, Blatchford & Webster, 2015). Fortunately, for my study the children in the school had completed the Year Six curriculum and examinations. This meant that I did not have the curriculum pressures Baines *et al.,* (2015) suggested. I was not as concerned by the perceived loss of control. I was aware that the children would be mainly working independent of the class teacher which was me and I knew that I would have to relinquish control of their learning to them. This is sympathetic to the research question investigating what approaches the children adopted when attempting problem solving activities.

Another cause for concern was the structure of the groups, the mixed dynamic, children with different educational and/or behavioural requirements. Galton *et al.,* (2009) highlighted the concern that some teachers have when they place a supposedly disruptive pupil in an established group of pupils. They were anxious about the possible problematic consequences that could arise. This is an issue of classroom and behavioural management. Within my study I allowed the children to select their own groups but I did not have any behaviour concerns. I had taught these children previously and was aware of their behaviours both positive and negative.

The alternative to group work was to work independently. There are benefits to this method of working, as children can work at their own pace (Ernest, 1996). However, the virtue of group work is that children talk about the work spending time explaining their work to others. The children working is groups are exhibiting supportive behaviours ‘providing clues, reminders and encouragement in response for children’s requests for help’ (Gillies, 2003, p. 38). This also answers the question of unequal working, whereby the children collaborate and support each other. It has been argued that by working separately children ‘miss the stimulation of sharing their mathematics with others working on the same problem’ (Ernest, 1996, p. 7). There is also a suggestion that the children ‘were able to think more deeply about mathematics rather than rushing through more and more work, as typically happens in top set classrooms’ (Boaler, 2006, p. 5). I have noted in Chapter One that the pace that children complete their work was one of the motivational factors for my study and subsequent research question. I had concerns that the learning that took place was shallow and narrow.

This exploration of group work has shown amongst its many advantages how its collaborative nature is connected to problem solving. The opportunity to share ideas, explain theories and engage in reasoned arguments are all important in the solving of problems. Previous studies such as Elia,*et al.,* (2009) investigated problem solving strategies but relied on the children’s written test papers as their raw data. By using the mathematics activities chosen, I had the potential to observe the children engaged in problem solving and collaborating with their peers. This afforded me the opportunity to collect a large amount of data in several formats (audio, video and field notes).

## 3.8 Ethical Considerations

Ethics were an integral part of my study and it was important to ensure that the participants were treated with respect at every stage. It has been suggested that research that works ‘directly with young children poses ethical questions and challenges’ (Robson, 2011, p. 178). Therefore, before I was able to start the data collection phase of my research the ethical considerations connected to the study had to be made explicit. Brunel University’s ethical guidelines have been applied to all stages of this research and an application to the Research Ethics Committee was sought and granted before the commencement of this study. These considerations including safeguarding the interests and rights of any individual who could be affected by this study are discussed next.

### 3.8.1 Participants

This study contains several different participant groups, the most obvious being the children. However, other individuals indirectly involved included the parents of these children, the teachers of the participants and the Head Teacher of the school. In most research studies the participants are those directly involved but I needed to be cognizant of the impact of my research on these other stakeholders involved. Before beginning the data collection process I needed to negotiate access first with the Head Teacher which would then enable me to approach the parents and children.

The main participants of this research were Year Six children. There are several ethical issues concerned with children and most importantly is the consideration that children are vulnerable individuals. They have rights which have to be respected, 'children's participation rights entitle them to have their voices heard and taken into account, and to give and receive information' (Smith 2011, p. 11). Smith (2011) discusses how researchers should consider protection rights, including the 'assurance that the child is not discriminated, humiliated or ill-treated' (p. 12). This was considered very carefully when completing the University's Ethics Consent form, especially when I was considering who was going to give consent for the children, whether it was the parents alone or parents and child together. Another consideration was the issue of continued consent, which I will mention later in this section.

Most importantly I needed to ensure that informed consent was gained from the participants. I was aware that the failure to inform the participants of the purpose of my research could lead to deception. This occurs if the 'participants understood one purpose of the research whereas the researcher has a different purpose in mind' (Creswell, 2009, p. 89). I was therefore very explicit in my intentions and produced a participant's leaflet explaining my research (appendix 1.2). I also held a meeting for the Year Six children, with the purpose of explaining the research to them and stressing the facts that participation was voluntary. During this meeting the children were invited to ask any questions. They appeared to be very enthusiastic and did ask questions related to start dates and how they would be grouped in class. I assured the participants of anonymity and that all raw data would be kept strictly private, I discuss the measures I took in section 3.6.2. I have used pseudonyms for the children and the school was not named, nor its location or any feature which would make it identifiable. However, I can only maintain *my* confidentially, I do not have any control over the participants if they decide to disclose their involvement in this study.

I sought parental permission by means of a permission letter with a proforma (appendix 1.1) sent home via the children. Once the children returned their parental proformas I asked them to countersign them. It is easy to think of children as not having a voice assuming that if parents give consent this negates the need to gain the child's consent. However, this is disrespectful of children's participation rights. There was a dilemma as to whose permission I sought first, the parent's or the child's. My concerns were centred on who was giving consent. Smith (2011) noted that rather than assuming that children consent because their parents have consented, it was important that their informed consent was gained. I have mentioned this point several times in this section of my methodology because it was very important for the integrity of my study. In an effort to gain informed consent I had met with the children. When I asked the children to countersign their parent's permission slips, I asked them if they were happy to participate and reminded them of their right to withdrawal from the study and I did not pressurise them to sign.

Once permission was gained, there were still other ethical issues to consider including the impact of the researcher on the participants. Reportedly, 'the researcher has an impact by just intervening in the participant's lives' (Robson, 2011, p. 186). My aim was to minimise this impact by keeping the teaching as close to normal and the data collection non-intrusive. I intended using video and audio recording devices to collect my data but as Prosser (2000) suggests 'filming necessitates a close working relationship being established between the participants and the researcher' (p. 129). I had previously taught all of the participant children for a year and felt that I had a good working relationship with these individuals.

My study had an additional complication of relationships in terms of myself being the teacher and researcher. I have already touched on this and I now wish to discuss it in relation to ethics. The issue is due to the perceived power I might have in the relationship between researcher and participant. This was due to the fact that I was a teacher in the school and had taught all these children previously and I was still teaching the classes $C_1$ and $C_2$. It has been suggested that in some situations 'power can easily be abused and participants can be coerced into a project' (Creswell, 2009, p. 90). For my study this had the potential of influencing the participants in several ways. There was the possibility that they could feel compelled to participate due to not wanting to disappoint me. This leads to the phenomenon of the

Hawthorne effect, whereby participants behave in a way to please the researcher. I felt that I had a good relationship with these children and their parents. I did not expect anyone to refuse to participate due to negative personality issues and although there might have been other unknown issues, fortunately I did not encounter any in my study. I was very pleased to receive permission from every child I had approached. I had stated in the permission letter that those not wanting to participate in the study would still be able to complete the activities but I was expecting some individuals to decline filming. As an added precaution, as the study progressed, at the start of each observation lesson I asked the children if they wish to continue their participation. This is in keeping with the comments of Miller and Bell (2012) who noted how consent should be a continuous process throughout the research. The voluntary nature of the research was also reiterated to the children at the start of each session.

During the data collection process there is the potential for participants to disclose sensitive information. As my research project was not dealing with a sensitive subject, I was not expecting any disclosures. However, if this did occur the privacy of the individual would be protected. The focus establishment has procedures in place for Child Protection issues. There is also an issue of ownership of the data. Prosser (2000) noted that with video, participants should be made aware of the external interpretation of their actions. What is meant by this is the interpretation and value others who view the video will assign value to the participant's actions. The participants were invited to read the transcripts if they wished, this was included in the literature I produced for the participants (Participant Information Sheet, appendix 1.2).

Following on from this discussion regarding the participants I will discuss the ethical considerations surrounding the collection of the data and its subsequent use. A final note on the participants concerns the activities they attempted in class. As I noted in the earlier discussion on the selection of the participants, the Year Six children would normally complete a project in class at the time I wanted to do my observations. I had been asked by the Senior Management team of the school to design the Year Six project so I used this opportunity to choose to do a problem solving project. I ensured that the work set was appropriate, that is, it contained suitable learning objectives and offered challenge for the children and did not disadvantage them in any way.

### 3.8.2 Data Collection

In the following section I will discuss the ethical considerations surrounding the data collection. Angrosino (2012) proposed when using observation, researchers must pay particular attention to the ethical aspects of the project because they are dealing with real people. The raw data will be the form of video, audio recordings, copies of the children's work, field notes and lesson plans. When completing the Research Committee Ethics form I had to be very explicit in how I was collecting, storing, using and eventually destroying the data on completion of this study. When collecting the data, I attempted to be as non-intrusive as possible. To this end small camcorders, flip cameras and mp3 digital voice recorders were used. These were focused on groups although individual children might be identifiable especially with the video recording, but less so with the audio data. I decided not to use individual microphones with the participants although the advantage of using microphones is the obvious clearer audio recordings. However, I felt that they would have been intrusive and it would be time consuming to use microphones with all the participants. In addition it would have limited who I could observe as each member of the class would need a microphone. This was similar to Robson's (2011) argument against using microphones as he also felt that they would be intrusive.

Confidentiality was paramount within my research. Within my Ethical Proposal I had to maintain that data was kept private including the steps I would take to ensure this. Interestingly Heath, Hindmarsh and Luff (2010) also commented on the complexities that arise when video and audio are used as a method of data collection. They noted how Ethics committee's become concerned at the mere mention of video recording. My initial Ethical Proposal was declined and the reasons cited included the age of my participants and anonymity. Therefore, I detailed explicitly how the data would be stored, used and eventually disposed. This entailed storing the data on a removable hard drive and storing this in a locked cupboard, wiping all the data recording devices after transferring the data to this hard drive and how long the data would be kept for.

To protect the participant's identities I allocated pseudonyms. A gender specific and culturally acceptable name was assigned for each participant. This was also used on any transcripts and other documents relating to them. The raw data was kept in a secure place separate from the permission slips and after each data collection event, the data was removed from the recording device and stored on a removal hard drive in a secured cabinet. Due to the fact that the participants are young children there was some concern regarding the storage of the data after analysis by the ethics committee. This was mentioned when I submitted my initial Ethics Proposal and was revised in light of their comments. Creswell (2009) suggests that data should be 'kept for a reasonable amount of time', one of his reasons was 'so that it does not fall into the hands of other researchers who might misappropriate it' (p. 91). I did not feel this was a risk but ethically, due to the ages of these children, I decided that the raw data would be destroyed within two years of completion of the study. I have taken the extra precaution of wiping the data, emptying the computer electronic recycling bin and over-writing the storage devices to obliterate the data.

Within the next section I will explain how I analysed the raw audio and video data collected. This will include some of the reported difficulties encountered when analysing this form of observational data and how I intend performing my data analysis.

## 3.9 Data Analysis

The majority of the raw audio and video data was transcribed directly using a transcription program. The software only facilitates the process of transcription by allowing the viewing of video or listening of the audio as I typed. Therefore, the act of separating the individual voices on the recording had to be completed by me.

After the transcription process the data was analysed by looking for themes. The following section contains a discussion regarding the data and thematic analysis where I discuss the coding process and how the final themes were generated from the preliminary codes.

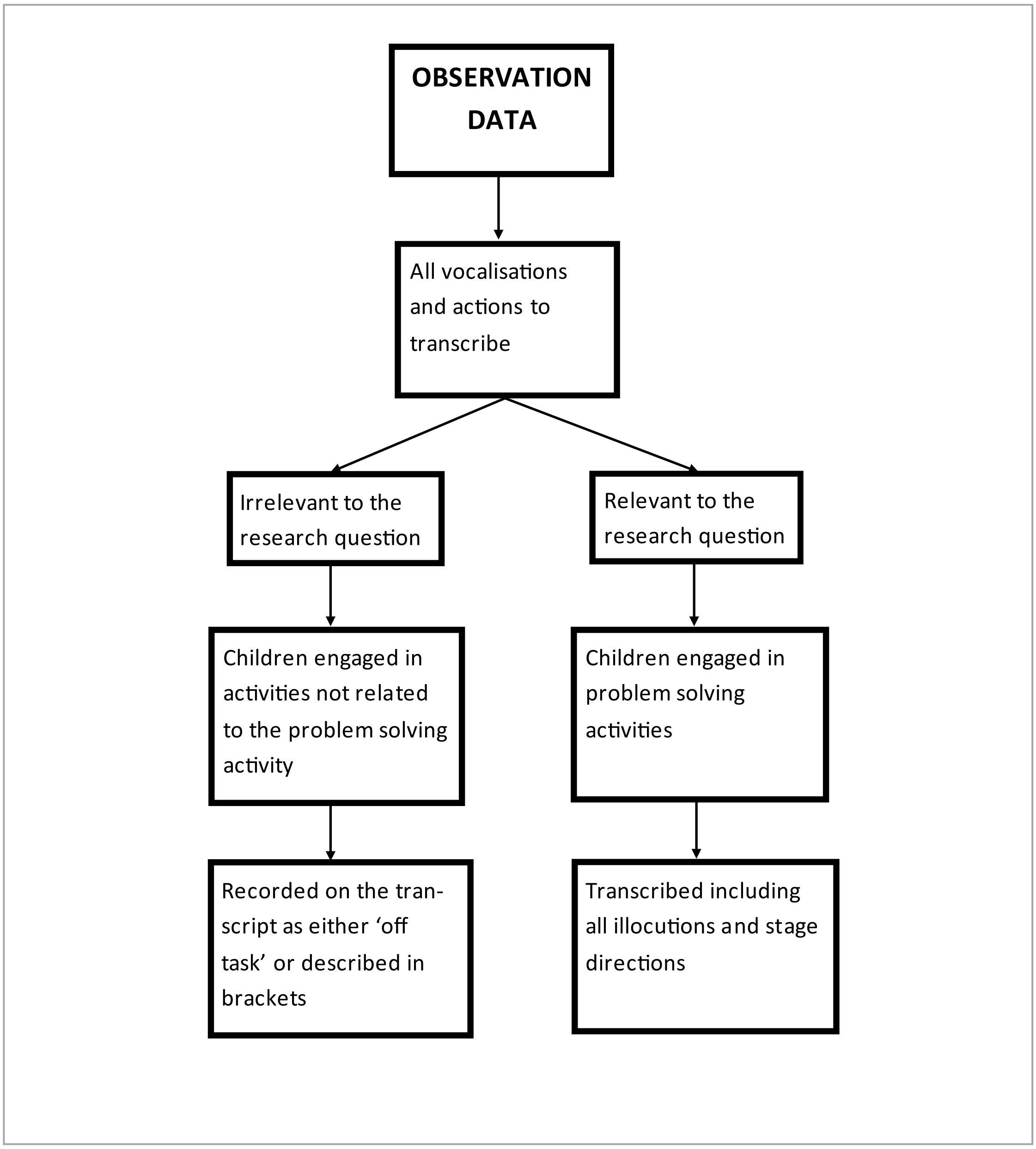
### 3.9.1 Transcription

Within this section I note how the raw data was treated and prepared in readiness for the data analysis process. It is important to note that observation is 'as much a process about what you exclude and entails a selection process at many levels' (Gomm, 2004, p. 217). There has been some comment that the reliability of audio recording is weakened by the failure of the transcriber to include 'apparently crucial, pauses and overlaps' (Silverman, 2005, p. 222). Heath *et al*., (2010) also stressed the importance of noting the visible as well as the audible aspects of the video recordings when transcribing them. Therefore, when transcribing the raw data I included all the utterances and where possible the children's actions, this was easier with the video data. The addition of field notes and the children's work was invaluable in annotating the transcripts by adding 'stage directions'. These 'stage directions' represented the actions of the participants and their 'other communicative behaviours' (Gomm, 2004, p. 217). These include such actions as pointing or using equipment, pauses and interrupting each other.

Gomm (2004) discussed what to include when transcribing data. Expanding on his comments I can infer that transcriptions of observation data are somewhat coloured by the lens we view the data through. Saldana (2013) mentioned this in reference to coding qualitative data and the impact the researcher has which included their perceptions. As the conclusions I draw from the data are interpretations, I will consider Saldana's (2013) comment in the discussion of my data. I included the field notes and the children's work in an effort to contextualise the transcripts and add reliability and validity to the results.

I have used two pieces of software in the data analysis phase of my research. The program was called Express Scribe Transcription Software Pro V5.69. This facilitated the play back of audio and video data with a foot pedal control whilst transcription took place. This software shows the video clip beside the transcription area. The transcript is typed directly it does not correct spelling or grammar. The second program was called Nvivo 10 and I discuss the merits of using Nvivo 10 in Chapter Five. As well as using Nvivo 10 for data analysis the children's work was scanned and uploaded into this program ready for analysis.

As I previously stated when transcribing the data I aimed to include all the utterances from the audio recordings. Within the video recording transcripts I attempted to include some notes on body language. Figure 3.3 was devised to aid the transcription of data. I listened to all the audio data and viewed the video data. I noticed that some children were not on task.  I used Gomm's (2004) diagram 'levels of selection in an observation study' (p. 218) as a guide for the exclusion of data. I did not want to exclude any data unless I felt that it was definitely irrelevant, caution was observed as they could be an issue of validity, for example bias occurring due to selective coding.




(Figure 3.3 Decision tree for data transcription)

### 3.9.2 Thematic Analysis

The observations resulted in a large amount of data which I decided to analyse using thematic analysis. I mentioned at the start of this chapter that my previous research was mainly concerned with quantitative research methods and I was less familiar with qualitative methods. A feature of thematic analysis was its flexibility and usefulness for researchers not familiar with qualitative methods (Clark & Braun, 2006). Thematic analysis also appealed to my constructivist epistemological assumptions as I aim to construct knowledge through interpretations of the situations that occur when children attempt to solve problems. Thematic analysis also includes a 'more personal interactive mode of data collection' (Mertens, 2005, p. 16). It is useful when 'identifying, analysing and reporting patterns' in data (Clark & Braun, 2006, p. 79).

Coding can be completed in several ways I chose to upload my transcribed data into the transcription software Nvivo 10, which facilitated this process. The software allowed for the centralisation and collation of examples of the themes found in the data. I have used the terms theme and code although they are not necessarily interchangeable. At the basic level the term code was used to represent the value I attach to sections of my transcribed data. These codes were then developed into themes which are more descriptive. It was these themes which allowed me to 'capture something important about that data relation to the research question' (Clark & Braun, 2006, p. 82).

Within Thematic Analysis codes can be described as either 'theory driven, prior data driven or inductive' (Boyatzis, 1998, p. 29). I have employed a mainly inductive coding approach, but I did start with an initial set of deductive codes that were developed from previous research including Polya's (1957) explanations of how children solve problems. These were used in the initial coding phase then inductive codes emerged almost immediately once the process of coding began. It is difficult not to describe themes as 'emerging' or 'discovered' but these terms appear rather passive. This conjures an image of themes sitting in the data waiting to be picked. Clarke and Braun (2006) refute the passive emergence of themes. Instead they and suggest coding is an active process and the researcher is fully immersed in the categorisation of the data (Clark & Braun, 2006). The researcher is cognizant of the decisions they are making and the themes they are assigning.

The coding process was a continuum moving from a set of codes to another which was then developed even further. Patterns emerge from data which allow for the creation of a coding framework where 'comparisons and contrasts between the different respondents' can be made (Gomm, 2004, p. 189). Additional cycles of coding occurred as I interrogated the data in light of these codes. These initial codes developed were then condensed and labelled as preliminary codes on the coding matrix. As I processed through the coding of the data the codes can be grouped together to form more overarching themes. These themes are discussed in more detail in the discussion and results chapters.

## 3.10 Final Remarks

It was my intention when writing this chapter to discuss the methodological choices I made. I wanted the reader to understand my decision process and to be informed of the alternatives to my chosen methodological approach. As I noted I adopted an approach incorporating both case study and Grounded Theory utilising observations and thematic analysis. This chapter also contained an exploration of the problem solving activities I employed. The Math Investigator 6 tasks were the foundation for the observations and form the basis of my data analysis. I also included discussions regarding the participants and group work in relation to the activities chosen. This was to situate these tasks within the framework of my research. With the subsequent chapters I will explore the data I collected, describing what the analysis has revealed and areas of further research.

# Chapter Four - Data Analysis

This chapter aims to describe the observation data I gathered and analysed using thematic analysis. It was the intention of my research to observe how Year Six children engage in mathematical problem solving activities and to discuss what I discovered. My curiosity was centred round the mathematics the children were engaged in, which included their strategies and other behaviours linked to problem solving. I will begin by discussing the coding process before exploring the resultant codes and themes that I developed.

## 4.1 Generating Codes

The review of the literature suggested some deductive codes as a starting point for the thematic analysis of the observation data. These were mainly developed from the works of Polya (1957) and included such strategies as;

- Relating ideas to previous knowledge
- Speculation
- Solving part of the problem
- Graphical representations
- Recall previously solved problems
- Conjecture
- Generalising

Other research also highlighted the codes of checking results and trial and error (Elia *et al.,* 2009). My definition of deductive codes acknowledges that these codes are predetermined which could be perceived as hypothesis testing. Previously, in Chapter Three I stated that my theoretical perspectives lie within both the constructivist and interpretivist traditions. I ultilised deductive codes to initiate the coding process, this followed an interpretivist approach. During the coding process I had anticipated that codes would emerge from the data, these codes are known as inductive codes. The use of both inductive and deductive coding allowed for the development of a flexible framework for my coding. The process of coding was an iterative and reflective process as I revisited the transcripts on several occasions. The data was coded and revisited and further coding applied until I decided that saturation was reached and no other codes could be applied.

I used the software QSR Nvivo 10 to support the coding of the data. I will refer to QSR Nvivo 10 as Nvivo for the remainder of this chapter. It was felt by some researchers (Gibbs, 2012) that software proves an 'over emphasis on code and retrieve approaches' (Gibbs, 2012, p. 255). Nonetheless, I decided that the software would support the analysis of my data. It would aid data management and organisation especially in my situation as a solo researcher with a time limit and a large amount of raw data. By using Nvivo, I could group the themes and explore when the events occurred. I used software for the convenience it afforded me. It was used as a repository for the raw data files from the observations and transcriptions together with scanned copies of the children's classwork. Within the Nvivo software the codes are known as nodes and were collated and visible within a dialog box, this was extremely useful as I could view the codes and how they were spread across the data sources.

More than ten years ago, Richards (2002) noted that 'qualitative computing has been marked by fear of the computational techniques and uncertainty about their effects on research' (p. 265). I used NVivo for the process of *code and retrieve* of the transcription data. I considered NVivo as a safe and reliable 'receptacle' to contain my data as I believed that with the quantity of data collected I could lose pieces of paper. Richards (2002) noted the benefits of qualitative computer analysis including its capacity to code large amounts of data quickly with unlimited storage. The computer software packages also links text with codes and have the ability to 'retrieve text according to the category or categories at which it was coded, importantly retrieving the context' (Richards, 2002, p. 268). This is linked to confidence, with Richards (2002) suggesting that as the whole data set 'can be rapidly and reliably retrieved and searched giving more confidence that the analysis was rigourous' (p. 269). Bazeley (2007) lists four issues that she suggests are the main concerns for those who doubt computer software's place in qualitative data analysis. These issues were;

- *'Potential of distancing researchers from their data*
- *Dominance of code and retrieve to the exclusion of other analytical activities*
- *The fear that computer will mechanize analysis*
- *Misperception that computers support only grounded theory methodology'*

(Bazeley, 2002, p. 8)

Reflecting on these points paraphrased from Bazeley (2002) I do not feel that I was distanced from the data. I was following a constructivist grounded theory approach. I became immersed in the data as it was coded and re-coded to saturation point and I was personally linked to the data as I collected data at my school. In addition samples of the children's work and field notes were used to contextualise the data.

In the following sections I describe my findings in relation to the codes assigned to them. I also note how these lessons were conducted, thus situating the data further. I have already noted that the participants were previously taught mathematics utilising more traditional teaching methods (Boaler, 2002). These include teacher direction from the front of class, a reliance on textbooks to support learning and very little group work with an expectation of children working independently. During the observation period the children's lessons were restructured from their normal format of an initial teacher instruction followed by an activity then a plenary. In the new format the teacher still gave instruction from the front at the start of all lessons but at times this was just a brief introduction to the lesson. The children then worked on the problem solving activities for the remainder of the lesson. They worked in the same groups for the duration of the project with activities continuing over several lessons.

As explained in my earlier chapters I had previously taught these children. I knew of their friendship groups and how they behaved so I allowed them to arrange their own groups with minimal intervention from me. I have not dwelt on the influence of their peer relationships on the work produced as this was beyond the scope of my study.

In employing thematic analysis I assigned many codes to the data which I have collated the codes into overarching themes. I will describe the overarching themes before I look in detail at the preliminary codes they contain in the following subsequent sections.

## 4.2 Overarching Themes

I had collated a large number of initial codes derived from the research observation data which I have termed as *preliminary categories* in table 4.1. The next phase in the thematic analysis of my data was the organization of codes into themes. This process involved re-analysing the codes looking for relationships and commonalities between these codes which could be developed into themes. The process of assigning the preliminary categories into broader themes occurred as the overlap between them became apparent. This was quite an organic process, not haphazard as care was taken to identify any overlap. Clark and Braun (2006) suggest that ‘you start to think about the relationship between code, between themes and between different levels of themes’ (p. 90) when creating the overarching themes. Overlap refers to similarities within these themes which allowed me to group them together into a more manageable category. These themes were then labelled as *Overarching Theme*s.

I also examined the coding completed using the Nvivo program. Within this program I was able to conduct a search of interconnected nodes (codes). This produced a table in Nvivo which demonstrated when an utterance in a transcript is assigned more than one code. For example, the excerpt from the observation on the 9th June below, was coded as both *conjecture* and *non-mathematical reasoning.*

**Transcript 1.1 - Task One, Imogen and Julia**

Julia: Maybe it’s something to do with this *(pointing to second page) (She starts to read out the information from the table).*
Imogen: No there’s nothing on there.
Julia: Wait shorts and t shirt… shorts and t shirt might be somewhere hot.
*(They look through the places commenting on which are hot).*

(June 9th/Task One/Class $C_1$ Imogen and Julia)

Reportedly careless grouping occurs, where there is too much overlap or a lack of internal coherence in the themes (Clark & Braun, 2006). Within the process of clustering the preliminary categories care was taken to limit this issue. I carefully noticed the similarities within the codes and have illustrated this in my main thematic map. I have used a thematic map as a tool to illustrate the cascading of the early preliminary themes (figure 4.1).

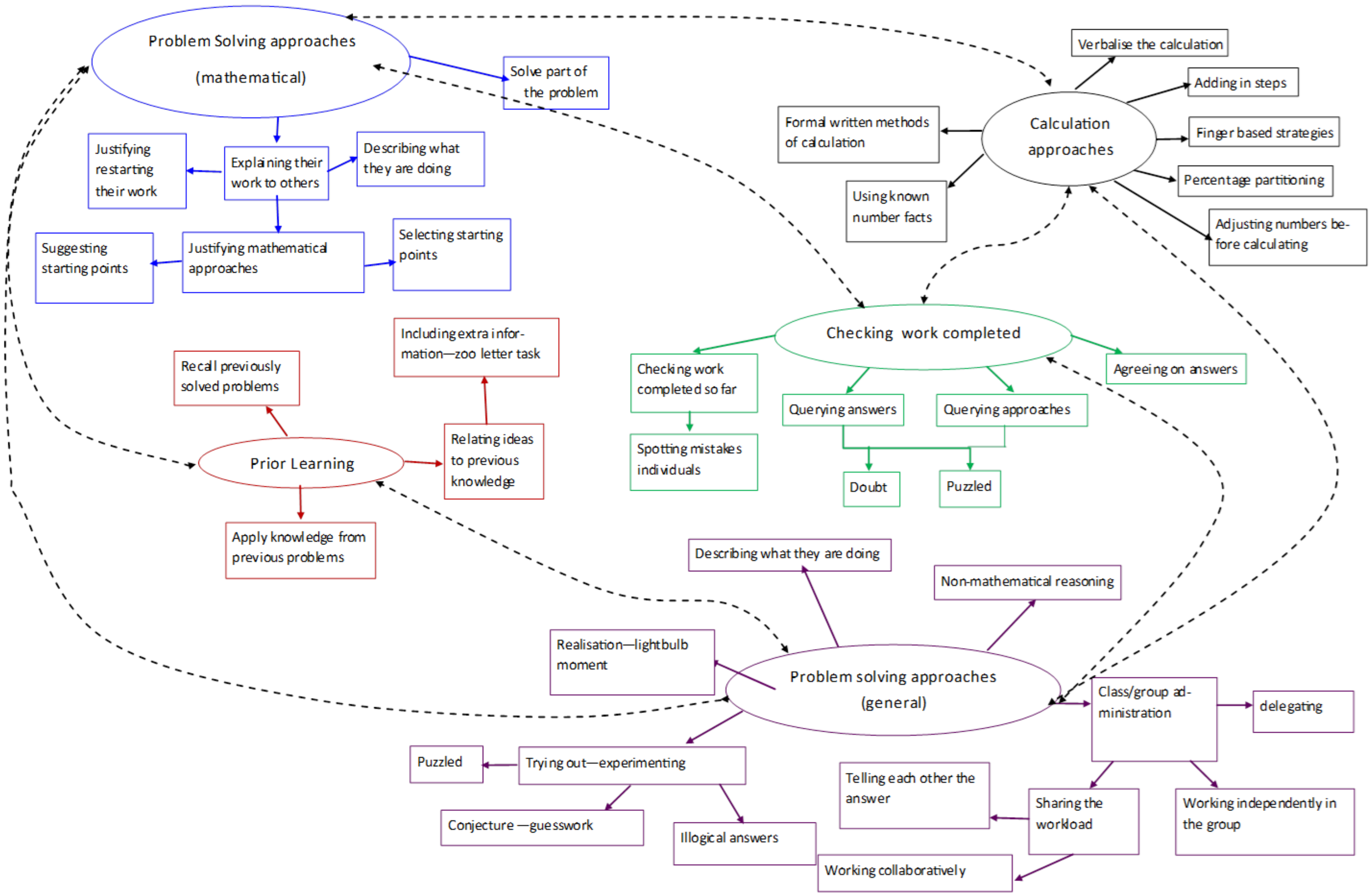
**Table 4.1 – Main Overarching Themes**

| <u>**Main overarching theme**</u> | <u>**Preliminary Categories**</u> |
|---|---|
| Calculation approaches (mathematical) | Percentage partitioning<br>Adjusting numbers before calculating<br>Using known number facts<br>Adding in steps<br>Standard method of calculation<br>Verbalise the calculation<br>Using fingers as a counting tool<br>Estimating |
| Problem solving approaches (general) | Sharing the workload<br>Delegating<br>Puzzled<br>**Conjecture – guesswork**<br>**Trying out – experimenting (trial and error)**<br>Non mathematical reasoning<br>Describing what they are doing<br>Telling each other answers<br>Working collaboratively<br>Working independently<br>Light bulb moment – realisation<br>Justifying approaches<br>Illogical answers<br>Attribute success to luck<br>Perseverance<br>Lack of self-belief |
| Problem solving approaches (mathematical) | Explaining their work to others<br>Justifying mathematical approaches<br>Agreeing on calculation methods<br>Justifying restarting work<br>Suggesting starting points<br>Selecting starting points<br>Sudoku strategies<br>**Generalising** |
| Prior learning | **Relating ideas to previous knowledge**<br>**Recall previously solved problems**<br>Apply knowledge from previous problems |
| Checking work completed | Agreeing on answers<br>Querying approaches<br>Querying answers<br>Checking work completed so far<br>Spotting mistakes |

(Table 4.1 Overarching themes and initial themes)

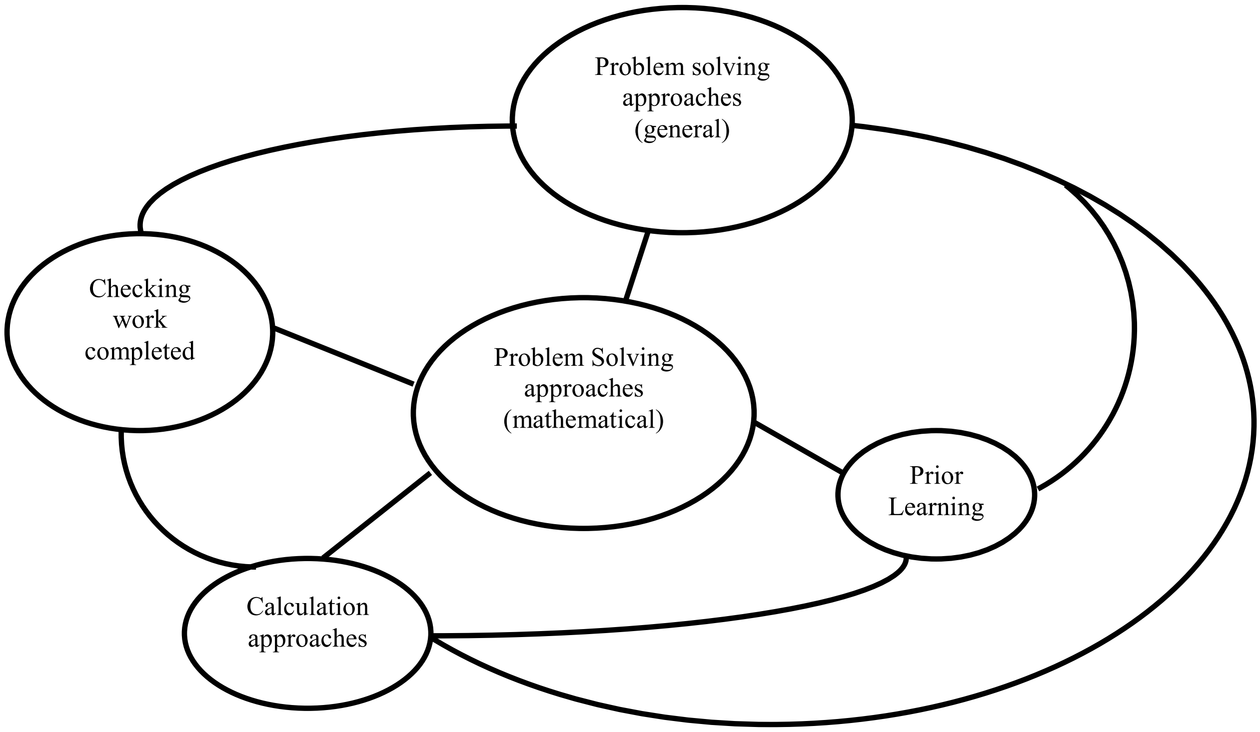
The preliminary codes shown as bold are those which were my deductive codes and come from the literature review and are based on Polya (1957) heuristics. The overarching theme of *checking work completed* is similar to the last of Polya's (1957) four phases. From the list of deductive codes (section 4.1) proposed, I did not witness the children *solving part of the problem* or *use graphical representations.* However, as can be seen in table 4.1, I did collect data reflecting many other preliminary categories. The occurrence of the codes is explored further in Chapter Five.

## Thematic Map




(Figure 4.1 – Thematic Map)

I used the intersection of the preliminary codes to devise an additional simplified thematic map, figure 4.2 which demonstrates the interconnectedness of the data.




(Figure 4.2 Interconnectedness of the main overarching themes)

As figure 4.2 shows all the main overarching themes are linked with the exception of *Prior Learning*. Some of the codes within the theme of prior learning were linked with the overarching themes of *Checking Work* and *Problem Solving Approaches (mathematical*) only. Figure 4.2 is a simplified version of the larger thematic map which includes all the codes and their interrelatedness (figure 4.1).

In the following sections I will discuss each of the problem solving activities and the codes assigned to the data. To illustrate these themes I included relevant extracts from the transcripts and examples of the children's written work. I did have some apprehension regarding the coding process. These concerns were regarding losing sight of the original data. I was worried that by sorting the data into themes and therefore removing it from the context, some meaning might be lost. I have therefore attempted to minimise this by including of descriptions of the observed lessons in an effort to contextualise the data.

In all of the transcripts I have used punctuation to represent the natural organisation of the participant's speech, including nonstandard English and grammatically incorrect utterances. The non-verbal actions which I recorded as field notes are described and recorded in italics.

## 4.3 Analysing The Observation Activities

### 4.3.1 Activity One - Maths Investigator 6: The Case Of The Missing Suitcase

The first problem solving task was adapted from the Maths Investigator MI6 scheme (Clissold and Pink, 2008). Originally this task was the second task in the MI6 scheme, for the purpose of my study it has been renamed as Task One (appendix 2.2). As previously noted in Chapter Four I adapted this scheme to suit the school and the participants.

### 4.3.2 Task One: Where Did The Secret Agent Travel?

For the initial task the children had to decide which locations a Secret Agent, Jane Blond, had visited. The children were given a selection of possible destinations and distance in miles from the secret agent's office. To solve this problem the children had to add each leg of the journeys together and compare them with the actual distance she reportedly travelled. The distance she travelled was given as a pedometer reading which the children had to multiply by 1000. We completed the pedometer activity as a whole class which gave the children the distance travelled in kilometres.

My role during these observations was as a participant observer as I was the class teacher delivering the lessons. I started my observations once the task had been introduced to the children as my focus was on what the children did mathematically whist attempting problem solving activities. During the first day of observations I recorded the following notes regarding the introduction to the task. This incorporated some comments concerning the children's progress through this initial problem;

**Task One - Field Notes 9th June**

**Task One**

The Maths Investigator Task was introduced to all three classes on the same day during their time tabled Mathematics lessons. This was the first lesson using the resource 'Maths Investigator' MI6. It was an unfamiliar scheme to both the class teacher and the class. The following is a collection of some of the notes focused on these lessons.

Class – $C_1$

This was a mixed gender class of 16 children they worked in pairs on this task.
We completed Task one together. However, they realised that miles in Task 2 needed to be converted into kilometres. I gave them the conversion 0.62km = 1 mile. We discussed how kilometres are smaller than miles. When the children started to work they did the following;

1. They realised that they had to add to find the journey distances
2. They added all the numbers in columns, 4 or 5 4-digit numbers in each sum
3. They got very 'hung up' on the conversion of miles to km. It seemed to distract them from the task in hand, calculating the totals for the distance travelled.
4. Some children tried to convert every mile measurement into km before adding.
5. Most of the children thought they had to divide their totals by 0.62

Class – $C_2$

This is was a mixed gender class of 16 children. They worked in mainly same sex groups of no more than 3 children.
In the lesson introduction I discussed the conversion of miles to km and wrote 0.62km = 1 mile on the white board. The children appeared confused by the conversion factor. It appeared that many of the groups could not find the start point.

Class – $C_3$

This class consisted of eight boys. They worked in pairs selected by the children. I delivered the lesson of 40 minutes today. However, I am not this class's regular Maths teacher.

We discussed Task 1 which was multiplying 37.61 by 1000. We solved this together and then moved on to the main task which was trying to work out the journey taken by Jane Blond. This group did not realise that the journey measurements were measured in miles not km. It appeared that all the groups tried to add the numbers using a column method. They immediately made columns and they didn't try any other layouts. One child possibly broke the sums down into chunks, but still used columns. Interestingly by not mentioning the conversion, $C_3$ got on with the addition sums quickly. Class $C_3$ was given the conversion ratio at the end of their lesson.

(9th June/Task One)

As the field notes state class $C_3$ were not given the conversion ratio. This was the very first lesson of the three classes and I decided to focus on the addition task first because of the ability of this class as they occasionally required slightly more support. However, these children were achieving National Curriculum Level 4 and above which was the National Average for children at the end of Year 6. In subsequent classes it was decided to give the children the ratio of miles to kilometres because they were judged to be the more able classes and I felt that they might be able to complete the conversion between units. The data regarding ability streaming was provided by the school's own streaming processes. Interestingly the children in $C_3$ had not realised that the destinations were measured in miles and that they would need to convert. The units of measurement were discussed in the plenary of this lesson where the children in $C_3$ then noticed that there were different units of measurement in the problem.

**Transcript 3.1 – Task One, Hari and Jai, Class $C_3$**

| |
|---|
| Class Teacher: So, in today's lesson we were just trying to work out the distance travelled … Now some of you worked out something really interesting. I noticed something interesting, what was the really interesting thing that you noticed from this?<br>David: Ah … It shows on the places where she had been were in miles but that (*pointing to 37610 Km*) is in K. M. (*he is referring to kilometres)*. |

(June 9th/Task One/Class $C_3$ Hari and Jai)

However, it could be argued that the reason why classes $C_1$ and $C_2$ become so preoccupied with this calculation was the fact that they were given the conversion ratio for kilometres to miles. This appears to be an argument for giving children limited amounts of information at regular intervals instead of all the information at once.

The transcripts of the observations have resulted in many codes. These reflect the aim of this research to investigate the mathematics the children were employing whilst working through problem solving tasks. As well as the subsidiary questions in regard to the other strategies they employed. I will now continue to describe my findings from the observation data using work examples to illustrate some my findings.

**Commencing a task – start positions**

From the transcripts of the video data collected on the 9th June and the field notes, it was noticeable that the children in classes $C_1$ and $C_2$ were preoccupied with calculating the conversion from miles to kilometres. This meant that they were not adding the trips together first as can be seen in Transcript 1.1 where Julia suggests division. I noted that the class teacher intervened and tried to direct the children towards finding totals first. I was the class teacher and the intervention was a reminder to look at the original worksheet and to get back on task. This was the first task and it is apparent that I had some apprehension regarding how much support I gave the children.

As can be seen in transcript 1.1, Imogen and Julia then thought the solution lay in the clothing Jane Blond the secret agent was going to wear. This was possibly due to extra information given in the MI6 case book page regarding her trip (appendix 2.1). The following transcript excerpt highlights the confusion of a group from class $C_1$;

**Transcript 1.1 - Task One, Imogen and Julia, Class $C_1$**

| |
|---|
| Julia: So it might be divide, ok think about it, if you want to convert miles to kilometres would you divide it?<br>Class Teacher: *(Class teacher intervenes)* Look at the original question (*you've got to solve*) on your sheet three (*reading the sheet*) can you investigate the information so that gives us a clearer picture of where she might have gone? You've got to solve that haven't you, that step first?<br>Julia: Yeah (*reading*)… gives us a clearer picture of where she's gone… (*picks up the case book*) right let's see where she might have gone.<br>Imogen: 'Inaudible mumble'<br>Julia: Maybe it's something to do with this *(pointing to second page) (She starts to read out the information from the table).*<br>Imogen: No there's nothing on there.<br>Julia: Wait shorts and t shirt… shorts and t shirt might be somewhere hot.<br>*(They look through the places commenting on which are hot).* |

(June 9th/Task One/Class $C_1$ Imogen and Julia)

Eventually Imogen suggested that they should add up the destinations in each trip;

**Transcript 1.2 - Task One, Imogen and Julia, Class $C_1$**

| |
|---|
| Imogen: Why don't we add these up (*pointing to the trip destinations on the case book pages*) and then add all of these up, and see which one is 0.62 |

(9th June/Task One/Class $C_1$ Imogen and Julia)

Transcript 1.2 shows that Imogen wants to complete an addition calculation of each journey destination but then compare her answers to 0.62. Her last comment regarding the comparison with 0.62 appears to be related to the classes' preoccupation with the conversion of miles to kilometres. This excerpt was coded as *suggesting a starting point*. Within the other transcripts there were other incidences of the children putting forward ideas to begin the problem solving process, coded as *suggesting a starting point*.

Within the coding categories there was another instance similar to *suggesting a starting point* this was labelled *selecting a starting point*. These two codes differ by the virtue that 'suggesting' entailed the children proposing an idea. Whereas, the element of 'selecting' included the adopting of a suggested start point by the group or it could mean the implementation of the idea by the child who spoke. It wasn't until Imogen repeats her comment later that Julia listens and starts to complete the additions. As transcript excerpt 1.1 demonstrates, Imogen and Julia struggle to find the starting point for this task.

Initially Imogen and Julia did not follow the direction from the class teacher but they did eventually start to add the trips. Conversely another group from $C_2$ reacted to the task in a different way;

**Transcript 2.1 - Task One, Ana, Priya and Caroline, Class $C_2$**

Ana: We need to find out where she went first. We need to investigate the information so we look at the information and see if it gives us a clearer picture ...so… (*Ana picks up the case book and brings it closer*) so we see that she's gone ... (*mutters)* …on her first trip she's been to Edmonton Canada that's 4255… Can you jot that down for me (*directed at Caroline*).
Priya: Write Canada.
Ana: Or write C… and then 7565
Priya: Why are we adding it?
Caroline: We need to find out how many miles that is (*pointing to the trips destinations in the table)* and then when we actually do it we can work out how many miles that is?

(June 9th/Task One/Class $C_2$ Ana, Priya and Caroline)

Ana's first comment from this transcript was also coded as *suggesting a starting point* then she acts upon this idea so it also coded as *selecting a starting point*. Caroline also explains the task to Priya showing that she has also understood the task.

**Fingers as a Calculation Aid**

During Task One I noticed several examples of the children using their fingers as a calculation aid. This calculation strategy only occurred when the children were engaged in solving addition calculations. The children used their fingers in two ways;

1. Counting on individual fingers in addition calculations
2. An aide memoire

The distribution of the code, *finger based strategies* can be seen in table 4.2.

| Task | Participants | Field Notes or transcript excerpt |
|---|---|---|
| Task One<br>9th June | Hari and Jai | Hari uses his fingers on his left hand. He flicks them as if counting on them and simultaneously 'counts on' under his breath. |
| | | Hari starts to add up the columns, counting on his fingers. |
| Task One<br>10th June | Imogen and Julia | Imogen is using her fingers to add up. She is using only one hand to count on, using her thumb to tap fingers as she counts |
| | | Julia appears to be adding pairs of numbers from the columns. She has two fingers each pointing to the numbers in the columns. Her fingers appear to be indicating the numbers she is adding. |
| Task One<br>10th June | Darshana, and Dev | Darshana: Number 3 is the same as number 1. We are just going over number 4 (Darshana is using her fingers counting on the fingers on both hands – she counts 7 using her left hand and two finger from her right hand) |

(Table 4.2 Excerpts coding for finger based strategies)

It surprised me that these children were using their fingers. The children in the study were 10 or 11 years old and according to the class's records of weekly Mental Maths tests they were all proficient in mental mathematics. They had completed tests each week from Brodie's (2006) Weekly Mental Maths Tests with a range of scores between 12 out of 20 and full marks. I was not expecting these children to use their fingers for calculating.

The first task involved the children adding several 4 or 5 digit numbers together. The use of fingers for calculating was not a continuous theme. I could surmise that as this was the first task these children were completing and it was unfamiliar so they reverted back to familiar comfortable techniques. The observed children were not using both hands to count on as I have often seen with younger learners. From the transcripts (table 4.2) the children only used one hand and either touched thumb to finger on the same hand to keep count or tapped individual fingers on the table, whilst they wrote with the other hand. These children were either counting aloud or under their breath when adding on to possibly keep track of what they were doing as they moved their fingers.

I was interested to see if these mental skills they had were being transferred to other situations in mathematics or were they only using them when the activity was 'sign posted'. The term 'sign posted' to refers to instances where the teaching has directed the children towards the methods they need to use. Therefore, the child has not selected the calculation method or strategy independently. In the Mental Maths test the children are not told which methods to use. However, they are potentially going to be more focused towards using methods they have practised in class. Nevertheless, they ultimately have the choice of which methods they use. The majority of the problem solving activities in the MI6 involved the children selecting the correct method of calculation. Except in Task Five the padlock codes which did contain algorithms and this is discussed in more detail later in section 4.4.5.

The second example of *finger based strategies* I noticed was the use of fingers as an aid memoire. To explain this I have included an excerpt from my field notes attached to transcript 1.4.

**Transcript 1.4 - Task One, Imogen and Julia, Class $C_1$**

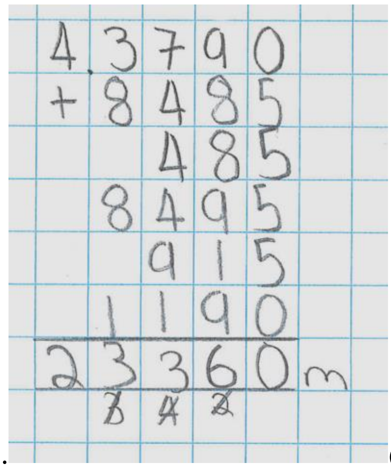
| |
|---|
| *Julia appears to be adding pairs of numbers from the columns. She has two fingers each pointing to the numbers in the columns. Her fingers appear to be indicating the numbers she is adding.'* |

(9th June/Task One/Class $C_1$ Imogen and Julia)

This excerpt describes how Julia was using her fingers to help her in the computational process. I have labelled this as *an aide memoire* because Julia was using her fingers to remind herself of the numbers she needed to use by pointing at each number in turn. It appears that she was not pointing to the numbers to inform anyone else. From the video of this observation it was noticeable that Julia was only trying to help herself, whereas the children in the first example were counting on their fingers to support their formal calculation methods. However, Julia was using a formal calculation method and was not using her fingers to scaffold the calculation process.

**Formal Written Methods of Calculation - Addition**

Following on from *finger strategies* I want to discuss the incidence of formal written methods because I as recorded in my field notes the children were very reliant on these methods. During Task One the children used the column method of addition. When faced with the addition of several large numbers, the majority of the children constructed the calculation as demonstrated in work sample 4.1. This example shows that the children arranged the numbers one on top of the other and in line by place value.



(Work sample 4.1)

A variation to this technique was demonstrated by Mary and Nitin when they were working on an addition calculation in Task One. This involved the addition of four 4-digit and one 5-digit number as can be seen in work sample 4.2, and the accompanying excerpt from transcript 4;

**Transcript 4.0 - Task One, Mary and Nitin, Class $C_2$**

Class Teacher: That's a good way of doing that Mary

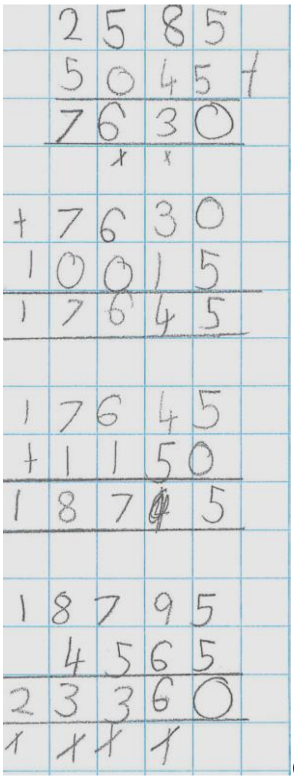
Mary: I added two at a time and then [inaudible]

Class Teacher: So you are the only person that's done it that way, that's a sensible way to do it

Nitin: Which way did you do it?

Mary: So you get two numbers, wait, so for the first one 4255 and 7565 add them and then answer another one

(10th June/ Task One/Class $C_2$ /Mary and Nitin)



(Work sample 4.2, Nitin and Mary, Class $C_2$)

When I queried why they had chosen to break down the calculation into pairs of numbers, Nitin suggested that if he added more than two numbers then he was going to get confused. Mary just said that she found this way easy as she got confused with the numbers. The numbers in the case book were displayed in the following order; 2585, 5045, 10015, 1150, 4565. This is the order in which Nitin and Mary added them they added the first two numbers in the list and then added the next number to the previous total. In this illustration they did not appear to use any other mental strategies to help them such as making ten, adding doubles or near doubles, to simplify the calculation. These strategies might have been evident if they had perhaps changed the order of the numbers they added.

In the earlier example the children had also copied the numbers directly from the book in the order given. Whilst the children were completing this part of Task One I queried their approach to the addition of the 4-digit numbers and asked if there was any way they could make this task simpler.

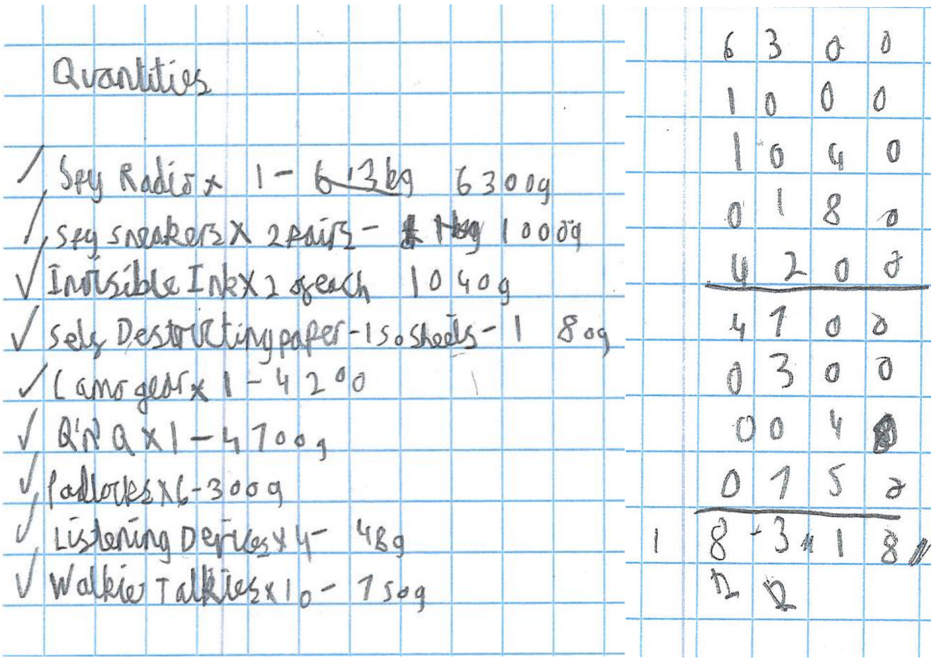
**Transcript 3.2 - Task One, Hari and Jai, Class $C_3$**

| |
|---|
| Class Teacher: How are you adding them?<br>Hari: Column by column<br>*[Class Teacher walks away and returns a little while later]*<br>Class Teacher: The same ok, is there an easier way of adding those numbers? Because at the moment you are trying to add 1, 2, 3, 4, 5, 6 numbers!<br>*[Hari and Jai look at their work]*<br>Jai: Could break it down?<br>Class Teacher: I'll leave that with you... Figure out how to make it easier.<br>*[Class Teacher walks away]*<br>(*Hari starts to suggest adding different numbers from the columns together*).<br>Jai: Does it make it any easier?<br>(*Hari starts to make tens from the numbers*).<br>Hari: (*to Jai*) did you get that (pointing at his calculation).<br>(*Jai continues with his method*). |

(June $9^{th}$/Task One/Class $C_3$/Hari and Jai)

Although Hari does try to make ten, Jai continues with his previous method which included adding all the numbers and using his fingers to count on.

Later in the problem solving activity there were other examples of column addition. These calculation methods are of interest as they satisfy one of my additional research questions regarding the mathematics used to solve problems. These other occurrences of column addition happened when the children were working through Task Four (appendix 2.7). Within this task the children were required to calculate the minimum weight of the secret agent's suitcase this involved adding nine numbers. I have included two work samples to illustrate this point. In work sample 4.3, we see Dev's work as he lists all the items with weights shown in grams. The second excerpt shows Dev's addition, he has made an error as the answer should be 18.518kg. The mistake occurs as he adds the third column (tenths) where he totals the column as 23 and has not included the two units carried forward from the addition of the previous column. This would make the total correct.



Quantities
✓ Spy Radio x 1 - 6.3kg 6300g
✓ Spy sneakers x 2 pairs - 1000g
✓ Invisible Ink x 2 of each 1040g
✓ Self Destructing paper - 150 sheets - 180g
✓ Camo gear x 1 - 4200
✓ Q'n'A x 1 - 4700g
✓ Padlocks x 6 - 300g
✓ Listening Devices x 4 - 48g
✓ Walkie Talkies x 10 - 150g

6 3 0 0
1 0 0 0
1 0 4 0
0 1 8 0
4 2 0 0
4 7 0 0
0 3 0 0
0 0 4 0
0 1 5 0
1 8 3 1 8

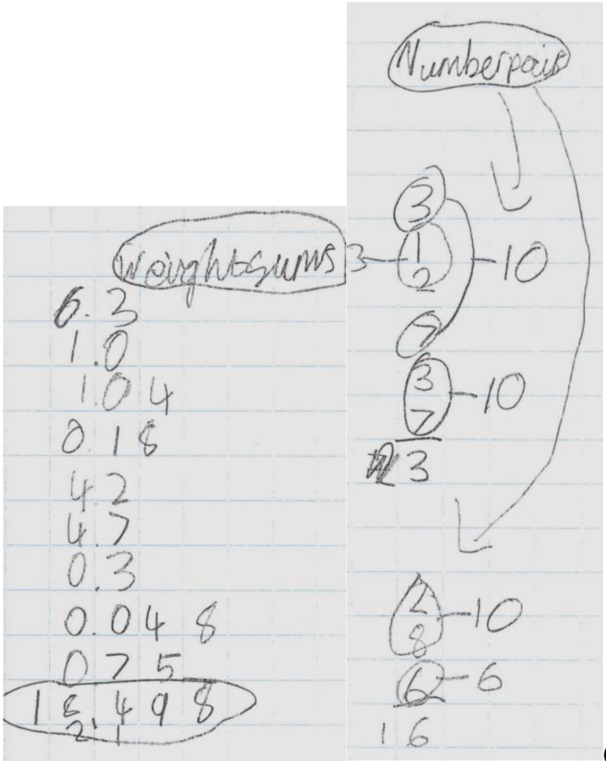
(Work sample 4.3 Dev, Class $C_2$)

After I watched these children complete the task I queried the approach Darshana, Dev and Freddie were adopting;

**Transcript 5.1 - Task Four, Darshana, Dev and Freddie, Class $C_2$**

| |
|---|
| Class Teacher: (*Moves to sit beside Dev*) 1, 2, 3, 4, 5, 6, 7, 8, 9 numbers! Now, you are going to make a mistake, wouldn't there be an easier way of doing it?<br>Freddie: Yeah, I removed all the zeros and then put them in their own column and then tried to work out the answer, thought I'd make those tens add them up and then put them there (*points to the sheet*) |

(June 25h/Task Four/Class $C_2$ Darshana, Dev and Freddie)



(Work sample 4.4, Freddie, Class $C_2$)

As can be seen in Freddie’s calculation (work sample 4.4) he finds groups of numbers that total ten and writes this working out at the side of his column addition as he describes in Transcript 5.1. I have coded this occurrence of working out as *using number facts*. There is an error in the column addition as Freddie has incorrectly added the hundredths. Interestingly when looking at how these two children attempted to add the same numbers Dev removed the decimal point from all the numbers in his addition but replaced it in the answer and Freddie kept the decimal but removed any superfluous zeroes.

I revisited Darshana, Dev and Freddie later in the same lesson, after they had solved Task Four. This conversation is transcribed in excerpt Transcript 6.1.

**Transcript 6.1 - Task Four, Darshana, Dev and Freddie, Class $C_2$**

| |
|---|
| Class Teacher: … you had an error didn't you earlier on? |
| Darshana and Dev: (*together*) Yes |
| Class Teacher: What was the error on your papers? |
| Freddie: Probably … |
| Darshana: Addition |
| Dev: With me I think I did too many at a time |
| Darshana: Too many zeros |
| Dev: Yeah, so I just broke up the chunks and then I got it |

(June 25h/Task Four/Class $C_2$/Darshana, Dev and Freddie/ 2)

The final comment made by Dev is similar to Mary and Nitin's earlier comment when they completed Task One. They also found it easier to modify a column addition which contained several numbers by making their own intermediate stages.

Within the observations of Task One I also coded the addition strategy *adding in steps*. Here I noticed examples of children adding in increments;

**Transcript 2.2 - Field notes, Ana, Priya and Caroline, Class $C_2$**

| |
|---|
| As Priya adds she says the answers in increments e.g. if she is doing 2 + 2 + 2 she would say 4, 6. Priya states the increment answers then states the final answer '37' |

(June 9th /Field notes/Class $C_2$ Ana, Priya and Caroline)

In the following transcript Anika was adding in small steps making ten and she explains her answer to her partner.

**Transcript 7.1 - Task One, Anika and Kishan, Class $C_1$**

| |
|---|
| Anika: (*Adding in steps*) |
| Kishan: Where did you get 10 from? |
| Anika: 3 and 7 (*she continues to add by counting in steps*) |

(June 10th/Task One/ Class $C_1$/ Anika and Kishan)

Julia's method differs from the example in Transcript 2.2, as she is not adding 5+5+5+5. Instead she stated aloud the number of 5's in the unit's column and then uses a multiplication fact to find the total. It appears that she knows that repeated addition is the same as multiplication and can apply this knowledge to her work.

**Transcript 1.5 - Task One, Imogen and Julia, Class $C_1$**

| Julia: (*Starts calculating*) 4 x 5 equals 20 (*adding in steps*) |
|---|

(9th June/Task One/Class $C_1$ Imogen and Julia)

There is some overlap with the strategy of *adding in steps* and that of *using known number facts*. These both are located within the overarching theme of *Calculation Approach*. The theme of *Calculation Approach* incorporated the occurrence of the children attempting a calculation whether it was written or mentally completed. These included both standard and non-standard methods of calculation as described by the National Curriculum (DfE, 2013b). The code, *using known number facts* is demonstrated in work sample 4.4, where Freddie is making tens. It is also apparent in the two examples, Transcripts 2.2 and 7.1, where Priya is using her knowledge of multiplication tables to help her add and Anika is making tens.

**Verbalising**

Another code observed was the *verbalisation* of calculations by the children. This entailed the children saying aloud the calculation they were doing. The majority of the calculations are completed without this commentary from the children. This code is very similar to the codes of *describing what they are doing* and *explaining their work to others*. They are distinguished by the virtue that the children were not necessarily explaining their work although in some cases the utterance has been coded as *explaining their work to others*. These two examples illustrate the *verbalisation* of calculations;

<u>Example 1</u>

*Whilst calculating Caroline looks up and verbalised the sum she is doing by saying X + Y = 3 (not saying letters but her numbers were inaudible).*

(June 9th/Field notes/Class $C_{2/}$Ana, Priya and Caroline)

Example 2

Julia: 23.4 then 23.4 divided by [inaudible] that's 1,1, that's 11.7 and then plus half of that so, half that too which is 11 erm …which, 5.17 zero, zero (*verbalising the calculation)* so it'll be 5.85

Imogen: 5.85

Julia: We need to add the zeros on here, ok let's work this out, so 5 plus 15, so ...24 plus ...I got 37.95 ...oh let me just check, we got 5 right? We got 9 right? Ok right?

(June $16^{th}$/Class $C_1$/Imogen and Julia)

Within the *verbalising* code it appears as if, the children were saying aloud what they were trying to calculate on paper but this was not necessarily directed to anyone as in Example 1. Although, it can be seen in Example 2 that Julia is saying aloud the calculation and then once she has finished she turns to Imogen and starts to explain their next steps for calculating a solution. She was quite dominant in the conversation and very vocal throughout.

**Group Work**

All of the tasks for the MI6 problem solving project were group activities. As I discussed in Chapter Four there were two other problem solving projects that were completed within the data collection phase by the children. These were the Planning a School Trip and Sudoku activities. Of these activities the Sudoku activity was the only one which the children could choose if they wanted to work independently or in groups. The group work aspect lead to several codes including *delegating* and *sharing the workload*. There was only one example of one child working independently but this was at the initiation of Task One, she did eventually work collaboratively within her group.

The majority of the activities were completed in a collaborative environment. The difference between the codes of *delegating* and *sharing the workload* is fairly obvious. Within *delegating* one member of the group took the role of foreman and assigned elements of the task to the other children in their group. *Sharing the workload* contains examples of the children discussing who is doing with no one child overly dominating, for example;

**Transcript 8.1 - Task One, Darshana, Dev and Freddie, Class $C_2$**

| |
|---|
| Darshana: *Yeah ok, we need to add them up (pointing to the destinations on the page).* |
| Dev: *I'll do 1 and 2; do you want to do 6? (To Freddie)* |
| Darshana: *I'll do 3 and 4, so …* |

(June 9th/Task One/Class $C_2$/Darshana, Dev and Freddie)

Within Task One there are further examples of collaborative working here the children are checking their answers and appear to be justifying their work. There were also instances of the children telling each other the answers. I revisit these codes once again in relation to the later problem solving tasks.

## Querying Answers

In my earlier description detailing how the children attempted Task One, I noted how the children where adding several multi digit numbers. The actual answers to the four calculations were interesting because the answers to the trips 1, 2 and 4 had the same total with trip 3 resulting in a smaller total. The calculations elicited the following responses from the children;

Example 1

Jai: Yeah, I think there is something not right.

Hari: Yeah, no, here not here... these have both got the same answers, 1 and 3

Jai: Hang on a minute... That can't be… Do you want to like… Do that again?

Hari: What, why, what's wrong?

Jai: I'm thinking that this one's gone wrong (*pointing at the paper*).

(We revisit them a little later in this lesson)

Hari: Now that's the thing (*mumbles some numbers*) ……so that's eliminated straight away because that's not the closest…… but three of them have got exactly the same answer…..there's something wrong? (*Jai shrugs his shoulders*).

Hari: But, we both checked it (*mumbles something*).

Jai: I think we should work that one out.

Hari: I think something's gone wrong.

Jai: I think this one's wrong (*pointing*)

(We revisit these children again later that lesson)

Hari: Turns to Class Teacher… (*He looked puzzled*) three of our answers are the same, is that possible?

(9th June/Task One/ Class $C_3$/Hari and Jai)

Example 2

Stacey: What did you get for the addition?

(*Charlotte says something inaudible to Asha*)

Asha: Yeah so it could be all three, it says (inaudible) not the only one

(10th June/Task One/Class $C_2$/Stacey, Charlotte and Asha)

Example 3

Darshana: We are chatting because we are a bit confused

Dev: The 43 is …is

Darshana: We added up 3 and that's the same so we are checking

Class Teacher: So it's the fact that they are the same that's warned you?

Darshana: Yeah

(10th June/Task One/Class $C_2$/Darshana and Dev)

These occurrences were coded as *querying answers*. There were many examples of the children querying the answers with their peers when they were completing the three problem solving projects. There was some intersection with this code and the codes of *justifying work, spotting mistakes* and *checking work*. In solving Task One the children needed some additional information to help them select the correct answer. The children knew that they had to find an answer close to 37610 km as this was the distance the Secret Agent had travelled.

**Transcript 3.3 - Task One, Hari and Jai, Class $C_3$**

| |
|---|
| Hari: Now that's the thing (*mumbles some numbers*)…so that's eliminated straight away because that's not the closest… but three of them have got exactly the same answer…there's something wrong? |

(9th June/Task One/ Class $C_3$/Hari and Jai)

The comment from Hari was coded as *conjecture*. He decided that the smaller of the answers has to be rejected in favour of one of the remaining three trip answers. Although, it appears that he has correctly rejected the smaller incorrect answer, this was possibly more by chance as his answers are still in miles. However, when looking at his answers Hari has one total of 18470 miles and three totals of 23360 miles he then rejects the smaller of the answers. Later in the transcript he starts to doubt himself after he realises that his answers are in miles and not in kilometres. He is aware that the units have to be the same to compare the distances.

The comparison of the distance travelled by the Secret Agent and the journey distances proved rather challenging. The transcript excerpts of the conversations between Darshana, Dev and Freddie (Transcript 8.2) and also the conversation between Ana, Priya and Caroline (Transcript 3.3) demonstrates the code *explaining their work to others*. There were many other examples of the children explaining their methods to their peers and the class teacher. Transcript 8.2 also included as an example of the code *querying approaches – groups*.

**Transcript 8.2 - Task One, Darshana, Dev and Freddie, Class $C_2$**

| |
|---|
| Freddie: Firstly we've got to work out how many km a mile is (*they all start writing*) let's do 10 divided by 16 ... hmm |
| Dev: 10 km is 6.2, so 100 km equals 62 |
| Freddie: (*Interrupts Dev*) Do 100 divided by 62 |
| Darshana: 100 divided by 62 |
| Dev: No Freddie, all … |
| Freddie: Then just add the decimal point. |
| Darshana: It would by 10 km would by 6.2 |
| Dev: Freddie we could do 100 km equals 62 miles and then divide the 100 |
| [They continue to calculate the distances] |
| Dev: Can you try to start converting them (pause) no…We need to try to find out what miles are first. |
| Freddie: Yeah |
| Dev: Ok, so 100km equals 62 miles, now if we divide 100 by 62…. So we just need to divide the 100 by 62 right? |
| Freddie: Yeah I think that'll get it. |

(9th June/Task One/Class $C_2$/Darshana, Dev and Freddie)

**Transcript 3.3 - Task One, Ana, Priya and Caroline, Class $C_2$**

| |
|---|
| Ana: No we times miles to get km because km is going to be a bigger number. |
| Priya: Yeah you have to times [inaudible] the kilometre. |
| Ana: Got it. |
| Priya: I think. |
| Ana: What did you do to get 0.62? (*Pause*) Oh, ok, 1 km is 0.62 so don't we have to divide (*directed to Priya*) |

(June 9th/Task One/Class $C_2$/Ana, Priya and Caroline)

### Non-mathematical Reasoning

The children also employed some strategies that I have described as non-mathematical. These strategies were characterized by the fact that the children did not use calculations or mathematical reasoning skills. I assigned these instances as *non-mathematical reasoning* because they contained an element of reasoning although it was not based in mathematics rather they drew on other knowledge to qualify their answers. I have included an example (Transcript 9.1) of a conversation between the Class Teacher and Kishan where he is using his knowledge of climate as an aid. The temperature of the secret agent was given in the doctor's report on the case book page (appendix 2.1) as 38°C. In the excerpt Kishan has made a mistake with the temperature quoting 31°C.

**Transcript 9.1 - Task One, Kishan and Anika, Class $C_1$**

| |
|---|
| Kishan: Let me see the book (*appears to study the second page with the table not the destinations*). The temperature is 31 degrees. So it's going to be a hot country. Is that a hot country?<br>(*Kishan is pointing to one of the destinations and asking Anika. He then goes through several of the countries with Anika saying which are hot)*<br>Class Teacher: (*Interrupts*) so what are you guys actually doing?<br>Kishan: Well it says the temperature is 31 degrees so we are trying to find a hot country that she went to.<br>Class Teacher: But that's her temperature because she's poorly. |

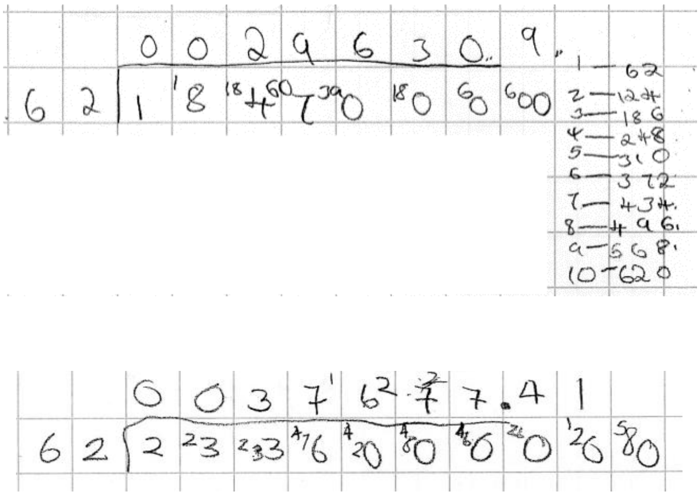
(June $9^{th}$/Task One/Class $C_1$/Kishan and Anika)

It is important to note that at this point Kishan had not calculated any of the totals. I was interested in this case in point as it illustrated that when the children were asked to find out where the secret agent had been they did not always complete the additions first. It is possible to surmise that they were either looking for extra information to support their thinking or they were trying to circumvent the need to calculate the totals.

The doctor's report in the case book also included some extra information about clothing she was 'provided with shorts and t-shirt' (Clissold and Pink, 2008b, p7). Imogen and Julia from another group thought that the t-shirt and shorts indicated that she had journeyed to hot countries (Transcript 1.1). This directed them towards looking at the individual countries in the trips before calculating the totals. The children were applying their life skills and general knowledge to the problems. This could be considered as thinking creatively, so called 'thinking outside the box'. There were other examples of *non-mathematical reasoning* centred round the items the secret agent packs in her suitcase. There is an overlap with the code of *puzzled* here as the children tried to figure out how many spy sneakers (sports shoes) she had. I have discussed this more detail in later in section 4.4.3.

**Formal Written Methods of Calculation – Division**

Some children attempted to convert between kilometres and miles using a conversion ratio of 1km = 0.62 miles. I have included the division equation from two children who worked together (work sample 4.5). They had the two answers from the addition of the trips and then performed short division.



(Work sample 4.5)

They adjusted the divisor from 0.62 to 62 and the dividend by the same factor of ten before dividing. They selected short division although long division is the recommended method when the divisor contains more than two digits. As can be seen in the first calculation (which does contain an error) Stacey includes the 62 times table in her working out. She uses this to subtract from the dividend, the subtraction calculations are not included on her papers as possibly she calculated these mentally. Completing the division calculation demonstrated that the Secret Agent had not taken Trip 2. However, the children required some extra information to complete the task.

The MI6 Task One was completed over the first two days of the observation period. For the remainder of the observation period only classes $C_1$ and $C_2$ were observed this was due to the availability of the classes. From the field notes during the second observation I noted the following;

**Field Notes – Task One 10th June**

| |
|---|
| Once the children had found the answers to the length of the trips we then discussed which trip it could not be. The children rejected trip 2 as they decided it was too short. Towards the end of the lesson they were given the clue that the Secret Agent had visited hot countries. The children surmised using their geographic knowledge and reasoning that the answer was trip 3. |

(10th June/Task One)

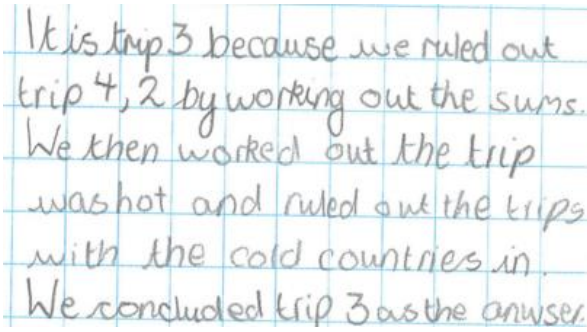
Imogen and Julia had already started to look at the temperature they appeared to have good geographical understanding of climate and applied this to the problem. This is similar to my earlier observations of Anika (Transcript 9.1).

**Transcript 1.6 - Task One, Imogen and Julia, Class $C_1$**

| |
|---|
| *(Both girls are whispering and pointing to the places they nod).* |
| Julia: We said that at the beginning, look it said shorts and t-shirt …and also the temperature |
| Julia: We think we know it |
| Imogen: We've worked it out |
| Julia: Yeah because the others got a couple places that are and that's Russia and Russia isn't hot |

(9th June/Task One/Class $C_1$/Imogen and Julia)

Julia wrote the following explanation in her working out notes;


It is trip 3 because we ruled out
trip 4, 2 by working out the sums.
We then worked out the trip
was hot and ruled out the trips
with the cold countries in.
We concluded trip 3 as the anwser.

### 4.3.2 Task Two: Which Suitcase Did She Take?

The second task focused on the identification of the Secret Agent's suitcase. The children were directed to look at the clues on pages 8 to 9, The Agents' Supplies Store (appendix 2.3), in the MI6 case book. There were given the following clues from the MI6 scheme;

> '*Fantastic work MI6! So we know which locations Jane visited – but the suitcase could be in any one of them! It should have been transferred each time she changed planes, but something must have gone wrong. Before I start contacting the airports, we need to know what the suitcase looks like. I've found a receipt for a suitcase in her office. I think that we can assume it's the one she took with her. Can you identify it from the Agent Supplies Store, so we have a description to pass on?'*
>
> (Clissold and Pink, 2008c, p. 9)

**Percentage Questions**

Within the case book the children were given the price of the suitcase as £141 including V.A.T. (Value Add Tax) at 17.5%. V.A.T. is the tax paid on the purchase price of the majority of goods in the United Kingdom. Since this scheme was written the rate of V.A.T has increased to 20%, which probably would have been an easier calculation for the children. The options given for the suitcase were;

High security trunk - £117.80
Quick 'n' quiet wheeled case - £120.00
Super attaché - £125.20
Extra hard case - £97.20
Trolley bag - £87.50 (all prices do not include VAT rated at 17.5%)

This task was not challenging for the children to complete as can be seen in the conversation (Transcript 9.1) between three children from class $C_{2.}$ The MI6 scheme suggested that this task would take a whole one hour lesson to complete this illustrates the reasons why I felt the need to adapt these tasks to suit the participants.

**Transcript 9.1 - Task Two, Darshana, Dev and Freddie, Class $C_2$**

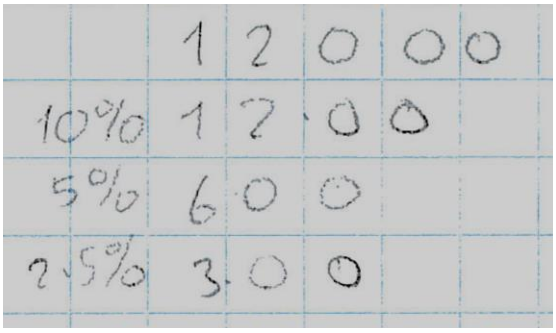
| |
|---|
| Freddie: I got it she takes the quick and quiet case<br>Dev: Really?! Really oh my...let just quickly check this, cos all we know it might be this one too<br>Freddie: No! It can only be something else that is that price (point to the suitcase price in the book)<br>Dev: Freddie found the answer first<br>Darshana: Wow, seriously! She used that. We found it out, easy! Seriously<br>Dev: I think we should all check that one<br>Darshana: Ok, ok, let's see<br>Freddie: I think I know how to half that 12<br>Darshana: £120<br>Dev: So that's 12<br>Darshana: That would be 12, no wait 10% 12, 5% is £6 and then 2.5 is £3 have you done it?<br>Dev: Apparently so |

(11th June/Task Two/Class $C_2$/Darshana, Dev and Freddie)

When Freddie was asked how he worked out the answer he stated that;

> I worked out the V.A.T. on this and then I added it on and it made the price of the luggage.

Although he has not given a lot of detail he has added the V.A.T. to the original price. He wrote on his class work that he had calculated the answer mentally. Within their discussion the children queried Freddie's answer this is an example of the codes *querying answers* and *checking work completed so far* this is related to the over-arching theme of *Checking Work Completed*.



(Work example 4.6, Darshana. Class $C_2$)

It was apparent from the observations that some of the children had developed good mental techniques. This was demonstrated by their confident division by ten and halving numbers when calculating VAT at 17.5%. From the transcript example Darshana, Dev and Freddie, were trying to find 17.5% of 120. Work sample 4.6 demonstrates Darshana checking Freddie's answers. She calculates the percentages in steps finding 10% of 120, later she uses her knowledge of halving to find 5% and 2.5% this was coded as *using known number facts*. She has partitioned 17.5% into 10% + 5% + 2.5%. There were other examples of children in other groups using the same methods. The children had previously used partitioning in their classwork. In a later task Dev suggests the following method for finding 17.5%;

> Dev: You could, you could find out 10%, quarter it, divide by 4, times 10% by 2 and then subtract the quarter

His explanation is slightly unclear but, he proposes that we initially find 10%. Then divide this answer by four which would equal 2.5% and multiply the 10% answer by two this equates to 20% and finally subtract the 2.5% to equal 17.5%.

During the plenary the children in class $C_1$ were asked to describe how they solved the problem of which suitcase the secret agent used.

**Transcript 10.1 - Task Two, Plenary, Class $C_1$**

| |
|---|
| Charlotte: We think it was the quick and quiet suitcase<br>Class Teacher: It was the quick and quiet suitcase and how did you work it out?<br>Charlotte: Ahem we did, so you get...it was £120 and then you get twelve, no you divide it by 10 that £12 and divide it again which is 6 and divide it again which is 3 then you add 12, 6 and 3 which is 21. The you do £120 add £21 which is £141 |

(11th June/Task Two/ Class $C_1$/Plenary)

Charlotte neglects to say what they are dividing by but the method is similar to the other example from Darshana, Dev and Freddie. This method was coded as *percentage partitioning*.

As I have already noted the second task was completed quicker than the first task. It could be suggested that it was a simpler task and the cognitive load was less. There is also the possibility that the children were now familiar with these tasks. Previous research by Voutsina (2012) has found that repetition of tasks produced better problem solvers. This will be explored further in Chapter Five the Discussion.

### 4.3.3 Task Three: What Did The Secret Agent Pack In Her Luggage?

#### Estimating Answers

In this task the children were asked to work out what were the contents of the secret agent's suitcase (see appendix 2.5). To solve this problem the children needed to calculate the quantities of the goods from the prices given. Interestingly Freddie starts by estimating. The code of *estimating* occurred several times in from Task Three onwards and was incorporated into the overarching theme of *Calculation Approaches*. It can be seen from this extract that the children are also explaining what they are doing.

**Transcript 11.1 - Task Three, Darshana, Dev and Freddie, Class $C_2$**

| |
|---|
| Freddie: Right so we have one of these and we need to find out how many spy sneakers<br>Darshana: I'm ...you're doing ...I'm doing the [inaudible]<br>Freddie: 12 ...I guess I am going to estimate<br>Dev: We don't estimate we do exact<br>Freddie: I'm just estimating to start off |

(13th June/Task Three/Class $C_2$/Darshana, Dev and Freddie)

**Conjecture – Spy Sneakers**

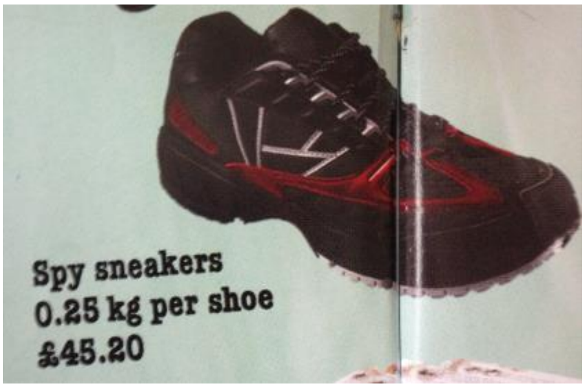
Later in the same transcript the children have to make a decision regarding how many spy sneakers (sports shoes/trainers) were packed in her suitcase. I mentioned this in relation to the code *non-mathematical reasoning.* The sneakers were shown in the case book (figure 4.3*)* as £45.20 and 0.25kg per shoe. It also states that the secret agent spent £106.22. This resulted in some discussion and conjecture amongst the children;

**Transcript 11.2 - Task Three, Darshana, Dev and Freddie, Class $C_2$**

| |
|---|
| Freddie: We've already worked out that we've got one spy radio and looking at the prices of the spy sneakers, I'm guessing that she had two but I'm going to work that out now |

(13th June/Task Three/Class $C_2$/Darshana, Dev and Freddie)

The spy sneaker question appeared to cause quite a lot of confusing amongst the children. The photo in the case book shows only one shoe and the weight is quoted per shoe but the price is per pair (figure 4.3).




(Figure 4.3 Spy Sneakers listing in the case book)

Transcript 11.2 is an example of the discussions regarding the quantity of the sneakers. Dev appears to address Freddie's concern that the answer is two spy sneakers even though conventionally shoes are sold in pairs.

**Transcript 11.2 - Task Three, Darshana, Dev and Freddie, Class $C_2$**

| |
|---|
| Dev: So we've got two, oh wait no we've got two spy radios, one spy radio |
| Freddie: We got two spy sneakers. We got a pair of them. |
| Darshana: Two pairs or one pair? |
| Dev: One pair |
| Darshana: Mmmm |
| Dev: Two shoes |

(13th June/Task Three/Class $C_2$/Darshana, Dev and Freddie)

In checking their classwork, Freddie did realise that the answer was two sneakers. Yet, when completing Task Three, this group has further doubts regarding the sneakers;

**Transcript 11.3 - Task Three, Darshana, Dev and Freddie, Class $C_2$**

| |
|---|
| Darshana: We are thinking about the sneakers...we don't know whether it is one or two |
| Freddie: The logical thing is, I can't think of anywhere that just sells one shoe, but there is only one picture of a shoe |

(13th June/Task Three/Class $C_2$/Darshana, Dev and Freddie)

Freddie is using some sound non-mathematical reasoning to justify his answer. He continues his discussion and uses the images of the other items to validate his ideas. I found the discussion regarding the sneakers fascinating and have explored this further in the next chapter.

**Transcript 5.2 - Task Four, Darshana, Dev and Freddie, Class $C_2$**

| |
|---|
| Class Teacher: Could that be because there is only one picture do you think? |
| Dev: Erm, no, but with the walkie-talkies there are two (*points to the picture*) |
| Darshana: And here there are two (*pointing to the catalogue page*) |
| Dev: And there (*pointing to the page*) |
| Class Teacher: Would a catalogue sell just one shoe? |
| Dev: No, ah yes it's a catalogue |
| Class Teacher: Think about it and decide logically which way you want to go with it |
| Dev: So that's 45. That would be twenty 'mumbles' per shoe |
| Darshana: I'm thinking it's going to be a pair |

($25^{th}$ June/Task Four/Class $C_2$/Darshana, Dev and Freddie)

The children did eventually conclude that two pairs of the spy sneakers were purchased. Another example of the codes *conjecture* and *non-mathematical reasoning* can be seen in the extract from Transcript 12.1;

**Transcript 12.1 - Task Three, Ana and Caroline, Class $C_2$**

| |
|---|
| Ana: It says 85 erm first tax I think that would be quite reasonable, so the spy radio is one, one spy radio. Spy sneakers 45 to 106, per shoe, is that 45 per shoe? |
| Caroline: Yeah, oh no, no, that's for one pair, ok, they wouldn't sell one shoe would they? |

($16^{th}$ June/Task Three/Class $C_2$/Ana and Caroline)

Further examples of *conjecture* and *non-mathematical reasoning* can be seen when the children had to decide how much self-destructing paper the secret agent packed this is discussed in the subsequent section 'Realisation – Light Bulb Moment'.

## The Children's General Approaches Towards the Tasks

I noticed many examples of team work amongst the children. The children shared answers and both described and explained what they were doing whilst undertaking the tasks.

Although the coding of the instances of *describing what they are doing* and *explaining what they are doing* they appear rather similar but there are some important differences. I coded examples of *describing what they are doing,* as those where they were only discussing their work as 'surface' retelling of events. Whereas, the code *explaining what they are doing,* included explanations of the task in a greater detail.

Occasionally the children lost focus on the task and I have coded this as *off task.* I have discussed attitudes towards mathematics in the Chapter Two including both the positive and negative attitudes. There was not a great amount of off task behaviour in comparison to the code of *perseverance*. In general the children demonstrated a positive attitude towards this work with many examples of them struggling and continuing with the task especially in the later activities, Task Three onwards. The fact that these children persevered with these tasks could be due to many other factors which I will explore further in the next chapter.

For this task the children were required to find the quantities of the spy radio, spy sneakers, invisible ink, self-destructing paper and camouflage gear (sic). Unfortunately, some of the children were preoccupied with finding the missing costs from the receipt, see figure 4.4 (Clissold and Pink, 2008b, p. 8).


(Figure 4.4 Secret Agent supply store receipt)

The preoccupation with extraneous work has occurred before when the children were completing Task One. There the children wanted to calculate the conversion of miles to kilometres.

I previously noted the codes of *off task* and *perseverance* in relation to attitude. Another code related to both attitude and also knowledge was the code *puzzled.* This occurred several times where the children professed to not knowing what to do. There is a connection to perseverance as often the *puzzled* preceded *perseverance*. The tasks of calculating the amounts of self-destructing paper and invisible ink did prove rather challenging for many of the observed groups.

**Transcript 11.4 - Task Three, Darshana, Dev and Freddie, Class $C_2$**

| |
|---|
| Freddie: Why do we need to work out how much the things are that we already know? Cos that won't help to [inaudible]<br>Dev: How do we do self-destructing paper? Should I just get an [inaudible] for that? I don't know what to do?<br>Freddie: Yeah work out the VAT, no I don't think the self-destructing paper explodes I think it just like sprays. It like melts the ink, cos you can see there is a bit splodge |

(13th June/Task Three/Class $C_2$/Darshana, Dev and Freddie)

One group tried to avoid completing it and left it until the end of the task. The fact that they had already decided that the task was difficult is once again related to attitudes.

**Transcript 12.2 - Task Three, Ana and Caroline, Class $C_2$**

| |
|---|
| Caroline: Invisible ink is going to be the hard one you know<br>Ana: Why?<br>Caroline: Because it could be 20 gram or 500 gram<br>Priya: Millilitres<br>Caroline: Millilitres, same thing<br>Ana: Let's leave that till last |

(16th June/Task Three/Class $C_2$/Ana and Caroline)

Within the task there was evidence of the children adjusting the numbers before attempting calculations this was coded as *adjusting the numbers before calculating*. An example of this occurred when the children were calculating the amount of self-destructing paper. The price of the paper was 40p per sheet. The children adjusted the price using their understanding of ratio to equate £4 to 10 sheets. This method was utilised once more as the children employed a trial and error approach which I have termed *trying out – experimenting*. These two codes are comparable with Polya's (1957) suggestion that children should think about previously solved problems and how these could help in solving the current problem. This was evident in Freddie's work who applied some sound logic when deciding the amount of camouflage gear. As the receipt price and the catalogue price were very close he concluded that it must be just the one item. This was coded as *applying knowledge from previous problems* and *relating ideas to previous knowledge*.

**Realisation - Light Bulb Moments**

As I have mentioned the children did have some difficulties calculating the amount of self-destructing paper. The excerpt below (Transcript 11.5) details an exchange between Dev and Freddie.

**Transcript 11.5 - Task Three, Darshana, Dev and Freddie, Class $C_2$**

Dev: So it's more than 100

Freddie: It's more than 150

Dev: Oh yeah, more than 150

Freddie: Less than 250, try 175

Dev: Can you try 175?

(13th June/Task Three/Class $C_2$/Darshana, Dev and Freddie)

They struggled with this task they knew that the total price of the self-destructing paper was £70.80 and that the answer was greater than 100 (possibly because this would equal £40). Freddie suggests 150 sheets as a starting point, they eventually settle on 175 as their starting number. They found that 175 sheets were too big so they decide to try 160 so they then tried 150 sheets but reject the answer and decide that the answer must be smaller;

**Transcript 11.6 - Task Three, Darshana, Dev and Freddie, Class $C_2$**

| |
|---|
| Freddie: No, do 140, do 140 |
| Dev: Ok, 140…56, £56 |
| Freddie: Ok so try 145 |
| (*They try 145…there is a little pause)* |
| Dev: (*Verbalising as he calculates*) That takes it to £64.80 |
| Freddie: So it's more than |
| Dev: So than...So at least we know is more than 140…Should we try 150 again? |
| Freddie: No, try 145. |
| Dev: £60, workout the VAT, 3.15 |
| Freddie: Add it up for me or I'll add it up myself...I think that makes it .. |
| Dev: Wait, wait, wait, waaait, what are doing with 50? |
| Freddie: **That's it yes** |
| Dev: Have we done it? |
| Freddie: We've worked it out |
| Dev: How many sheets was it again? |
| Freddie: I don't know how much £60 worth |
| Dev: Oh wait... |
| Darshana: I said 125 or hundred and [inaudible] (*Dev talking over her*) |
| Dev: No, it was 150 remember? |
| Freddie: Is that how much £60 worth is? |
| Dev: Yeah I remember |
| Freddie: Are you sure? |
| Dev: **I remember, cos that was like, I was like, erm maybe my brain popped and bypassed 150, those were my exact words** |

(13th June/Task Three/Class $C_2$/Darshana, Dev and Freddie)

I have highlighted a section of the transcript in bold which demonstrates the code *light bulb moment – realisation.* I assigned to utterances that demonstrated a moment of enlightenment, this could be a sudden realisation or a moment of inspiration. This occurred as 'wow' moments where typically the children became quite excited.

Transcript 11.6 also contained an example of children becoming confused and allowing their *non-mathematical reasoning* to affect their thinking. In this transcript the children believed that 150 could not be the correct answer. This also occurred in Transcript 13.1, where it appears that Kishan also was allowing *non-mathematical reasoning* to affect his judgement. The opposite was also observed in the other tasks whereby the children accepted illogical or nonsensical answers. I revisit these types of answers in section 4.4.6.

**Transcript 13.1 – Task Three, Kishan, Class $C_1$**

| |
|---|
| Class Teacher: Um why did you think it was too much then? |
| Kishan: Cos 150 sheets sounds like a lot |
| Class Teacher: Do you know how much there is in a pack a ream they use in a computer? |
| Kishan: 1000 |
| Class Teacher: 500 |
| Kishan: Oh |
| Class Teacher: So one of those (shows thickness of a ream) about that thick is 500, it's only a normal pack of paper you would get, I'll let you carry on |

(25th June/Task Three/Class $C_1$/Kishan)

**Justifying Approaches – Trial and Error**

I now return to the invisibility ink task once again. The children were given the option of two different size ink canisters and corresponding prices to eventually make £297.98. I have included a discussion between the class teacher and one group;

**Transcript 14.1 - Task Three, Susan and Milo, Class $C_2$**

| |
|---|
| Class Teacher: How did you come to that way of thinking? |
| Susan: [inaudible] £15.60 |
| Milo: Invisible ink is 297.98 erm if we, because there is a bottle and a canister, two of those it equals erm 240 and then add the little one in and then round it up |
| Class Teacher: So you think it's better to buy the canisters |
| Milo: Yeah |
| Class Teacher: And not just loads and loads of the little ones |
| Milo: Yeah, because then it's harder to add up |

(16th June/Task Three/Class $C_2$/Susan and Milo)

Transcript 14.1 demonstrates how the children were able to justify their approach to solving the task by explaining their ideas. It does seem that these children were motivated by finding an easy calculation. They were not considering the mathematics they were applying or problem their main concern appears to be ease of calculation. The use of trial and error, coded as *trying out – experimenting*, was another strategy used by some of the children as in Transcript 15.1;

**Transcript 15.1 – Task Three, Julia, Class $C_1$**

| |
|---|
| Class Teacher: So why are you going to times it by 2, Julia?<br>Julia: Well because you want to see if say if it was one big bottle or two small bottles<br>Class Teacher: Yes<br>Julia: Try and make that ...say four like just, just<br>Class Teacher: So you just plucked two out of thin air did you? Ok that's fine<br>Julia: It's a starting point |

(25th June/Task Three/Class $C_1$/Julia)

There were several examples of children experimenting using trial and error approaches. The excerpts below illustrate two methods utilised by the children;

<u>Example 1</u>

Nitin: Once I've worked out the two VATs I will see, I will basically add these together and see which is closer once I add the VAT on, so I add the VAT on and see, the bottles and the canisters and add them together and see which, and if I get that I then I'll … (Class Teacher interrupts)

Class Teacher: And by that you mean the answer on the receipt

Nitin: Yep

(25th June/Task Three/$C_2$ Nitin)

<u>Example 2</u>

Masoud: I keep trying numbers, I started with 10 it would be £47 so I did 13

Class Teacher: You did 13 (*repeating Masoud for the benefit of the recording*) and that was nearly close and so you went to?

Masoud: 15

Class Teacher: 15, excellent strategy Masoud

(27th June/Task Three/ Masoud)

In this final excerpt Thomas justifies his method and how he got the correct answer. He had a starting point to build upon using a combination of the strategies of *trying out – experimenting* and *estimating*.

**Transcript 16.1 - Task Three, Thomas, Class $C_2$**

| |
|---|
| Thomas: Erm, I worked, I started by working out erm the erm two big cos I knew it would probably have to be that and then I worked out the VAT on that and then sort of, I realised it must have been two so I then worked out the VAT on two bottles<br>Class Teacher: So, you said you thought it probably had to be two of the big canisters, what led you to believe that?<br>Thomas: Because if you have two big canisters it equals £140 and you can't really have three because it would go over and if you one if kind of doesn't |

(25th June/Task Three/Class $C_2$/Thomas)

Asha and Stacey tried an alternative method and attempted to divide the total cost of the canisters by £7.99 but found this too difficult. They also resorted to trial and error approaches this involves finding the V.A.T on the costs and then adding combinations.

### 4.3.4 Task Four: What Was The Weight Of The Luggage?

This was the penultimate MI6 task and built upon the answers from Task Three. The children were now asked to calculate the total weight of her suitcase.

The children found the majority of the item weights by the multiplication of one unit by the quantity utilising short multiplication, as in work sample 4.7;


(Work Sample 4.7)

The weights of all items in the Spy Store were given in either grams or kilograms. Therefore the children had to be mindful when adding. In a discussion between Freddie and Dev, Freddie asks for Dev to tell him the weights of the items so that he can write them down and construct his addition calculation. Initially he requests the weight in grams and Dev obliges and reads all the weights in grams. Freddie then changes his mind and asks for the weights in kilograms which Dev does. This conversation demonstrates Dev's secure knowledge of the conversions between kilograms and grams. They have been able to convert them to a common unit and then move between large gram weights and very small decimal kilogram weights with ease.

**Conjecture**

I have previously mentioned how the quantity of the spy sneakers confused the children. The calculation of the total weights of the walkie-talkie also caused some confusion. Within the Spy Store Supply catalogue (figure 4.3) the weight given was 75 grams each and they were sold in pairs. This resulted in many discussions surrounding the actual weights of the five walkie-talkies purchased. The children queried if the calculation was 75 grams multiplied by 5 or multiplied by 10.

**Transcript 17.1 - Task Four, Masoud and Ethan, Class $C_1$**

| |
|---|
| Class Teacher: So Ethan and Masoud why did you do two times and why do you do 375 + 375?<br>Ethan: Well we did 5 walkie-talkies on its own and 5 walkie-talkies separately and so as its 5 pairs we had to just add another 375 grams to it, 375 add 375 is 750 and our final answer is 18 kg 550 g.<br>Class Teacher: Ok, so when you did the sum you did 375 plus 375, Masoud you did 375 times 2, so why did you do times two?<br>Masoud: I found it easier |

(27th June/Task Four/Class $C_1$/Masoud and Ethan)

In Transcription 17.1 the children had been calculating the weight by multiplying by five. They had checked the addition of all their weights and found it to be wrong, that they returned to the walkie-talkie weights.

There were several examples of *delegating* and *querying approaches*. There was an interesting conversation between Dev and Freddie (Transcript 5.3) about the benefits and disadvantages of working independently. Eventually, they did all complete the calculations independently and compare their answers.

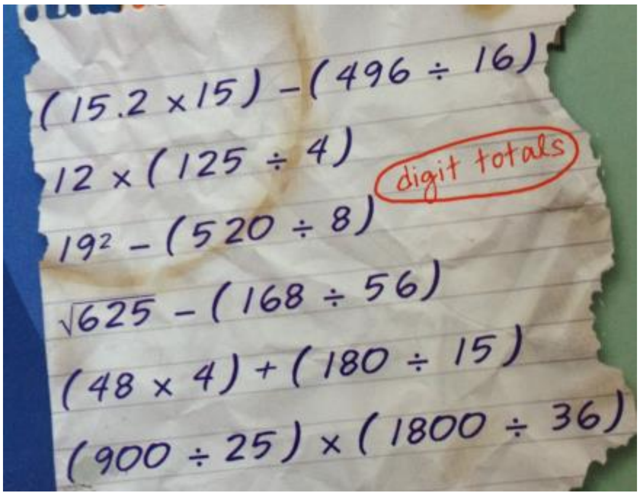
**Transcript 5.3 - Task Four, Darshana, Dev and Freddie, Class $C_2$**

| |
|---|
| Dev: No Darshana's doing it...We are meant to work them out individually<br>Freddie: I don't want to. We all get the same answer<br>Dev: [inaudible]<br>Freddie: But what if one of us gets the correct answer and the wrong then, it seems like who got the right and the wrong answer, we might as well all work together and work it out and get the right answer<br>Dev: No it lowers the possibility … (*Pause, some exasperation*) Darshana add it up and then we'll compare |

(25th June/Task Four/ Class $C_2$/Darshana, Dev and Freddie)

### 4.3.5 Task Five: What Was The Code On The Padlock?

This task was different to the others as the children were asked to solve six equations (see figure 4.5). When comparing this question to the class work the children usually completed, there were some similarities. Their class textbook contained many exercises working out similar style calculations.




(Figure 4.5 Secret Agent Padlock codes equations)

The children were familiar with the concept of brackets in equations and in their regular mathematics lessons they had covered B.I.D.M.A.S (Brackets, Indices, Division, Multiplication, Addition and Subtraction) in relation to the order of operations. This was taught in preparation for the Common Entrance examinations. Once the children had solved the equations they had to find the digit totals that would then become the secret padlock code.

**Describing strategies**

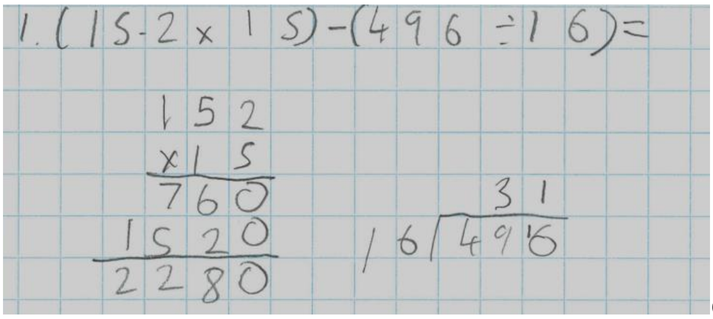
During the lesson I asked one group to explain the work they had completed so far;

**Transcript 18.1 - Task Five, Stacey and Asha, Class $C_1$**

| |
|---|
| Class Teacher: Ok, for the first one can you explain how you worked it out.<br>Stacey: Ok, so the sum is (15.2 x 15) - (496/16) so we did 15.2 x 10 and divided that by 2, oh um, which was 152 and we divided that by 2 which was 76 and added those together which was two, two eight and then we did ….<br>Class Teacher: How did you do the division, because that was quite tricky?<br>Asha: I got stuck on it<br>Stacey: Um…I did the division …I not sure I just did it<br>Class Teacher: You just did it… (*Looks at sheet*) so you just did a short division.<br>Stacey: Yeah<br>Class Teacher: OK, so Asha you said you got stuck on it<br>Asha: I wasn't sure how to do it 'cos at first I got a different answer to Stacey, I then I realized where I'd gone wrong. |

(27th June/Task Five/Class $C_1$/Stacey and Asha)

Later in the conversation Asha admits that she does not know where she made her mistake. The children were using a known number fact to calculate 15.2 times 15 as they partition the multiplier into 10 and 5. They then use their knowledge of place value and multiplication by ten to know that 15.2 multiplied by 10 would equal 152. The children were able to find half of 152. They knew that by halving the answer to 15.2 times 10 this would produce the same answer as 15.2 times 5. I have mentioned in the notes regarding Task Two how the children were able to use halving effectively to find percentages..



(Work sample 4.7)

Work sample 4.7 is an example of a method which differs from partitioning. Whereas the previous methods utilised knowledge of numbers and mental strategies the above example uses a written method. The written method to solve 15.2 multiplied by 15 was long multiplication. Children in this age group usually use the grid method of multiplication when the multiplier and multiplicand contain two or more digits. When Yash was asked why he used long multiplication his answer was;

> Yash: Well I was in tuition they did all the types we could've done and that was the quickest method they showed me

The tuition Yash stated was privately run lessons with usually one tutor per child. In this instance it is likely that these children were preparing for examinations. The syllabus covered is determined by the tutor who might be an educational professional but this is not regulated. It is interesting that Yash's calculation method was determined on the speed he could complete the task. It is possible that he is motivated by the completion of tasks perhaps using the 'passive learning approach' as described by Boaler (2009, p. 35).

I chose to informally interview two children who had both completed the Padlock activity. This was an informal discussion between Yash, Tara and I. Yash was working almost independently he interacted with the boys on his table but was happy to work alone. Tara was working in a small group of three girls and she took the lead role in the group. I considered that both children of these children were very intelligent as they had recently passed the 11+ examination for grammar school. The informal interview was conducted after the lesson. I have included a section of the interview where we discussed the six equations the children completed

Class Teacher: So …Opening the page of sums in front of you to see them in guess you haven't got them written down, which one of those sums was the most challenging?
Yash: The last one
[(900 ÷ 25) x (1800 ÷ 36)]
Tara: I thought it was the square root....cos can't really find square roots of 625

Yash continued by saying that this sum was hard as it contained a 'big multiplication' unfortunately he had made a mistake. Instead of completing 1800 ÷ 36 he had multiplied these two numbers which did result in the calculation 64800 x 36 instead of 36 x 50. The children were asked how they checked their work; they stated that they had not made any mistakes. However, Tara stated that she checked her work with a calculator and Yash repeated his calculation and then they both compared their answers. I have described how Yash made an error by substituting multiplication for division. Fortunately the digit total for Yash's incorrect answer and the correct answer were the same and thus this did not appear as incorrect when he looked at the answer sheet.

During the interview I asked these two children to describe the methods they used to solve the equations. Yash described how he completed decimal multiplication as;

Yash: I used for all of them the multiplication when there was erm decimals in it erm, you have to take out the decimals and then multiple and put the decimals back in afterwards
Class Teacher: Ok, how did you put the decimals back in?
Yash: If there was … if it had one decimal I just take it out (*Class Teacher murmurs in agreement*) And at the end of the sum I add one decimal into it (*one decimal place*)
Class Teacher: Ok

When I asked Tara, she stated that she used the same simple methods. She was asked to explain 'simple methods'. Her answers are centred on the decimal multiplication questions, she states;

Tara: I just ... most of the time if they were decimals I did it the same way

She was asked to clarify her answer. She stated that she sometimes removed the decimals and adjusted the final answers but other times she left them in and continued the calculation. She calculated 15.2 multiplied by 5 because she felt it was easy;

> Tara: Because it was only a simple number that we had to times it by and so it was easier …We didn't need to do long multiplication.

Previously I discussed the calculation methods employed by the children to solve Task One. I noticed that the children used a short division method for a calculation that they could have used long division. I was able to query the choice of short and long division with Yash, he stated that it was the easier method to use and that he does not usually use long division. The use of a method because they preferred it is becoming a recurring theme.

When I asked which questions were challenging for them Tara noted the square root of 625. The children utilised *trying out – experimenting* Yash thought it had to be a number that contained a 5 in the units column and then tested 25 and 35. Tara knew that the square of 400 was 20 and then tried a number slightly bigger. She started with 24 and found its square as this was close to 625 she tried once again using 25. These were coded as *trying out – experimenting*. The children had some knowledge and were able to select a sensible starting point they therefore were using some prior learning coded as both *estimating* and *trying out – experimenting*. It was coded as *recall previously solved problems* this was because of a comment made about completing similar questions at tuition.

Whilst completing the MI6 tasks there an additional code related to attitude known as *attributing success to luck*. There is some similarity with *describing what they are doing* although those utterances were more akin to summaries and the 'luck' comments are exclamations of wonder. I have included an exchange between the Darshana, Dev and Freddie.

**Task Three, Darshana, Dev, Freddie, Class $C_2$**

| |
|---|
| Freddie: I think I've got it |
| Dev: Freddie, why are you so lucky |
| Darshana: You know there is two different ones |
| Freddie: Yes I know ... I think I've got it |
| Dev: This is so unfair |
| Darshana: He's so lucky ... first he figured out the journey and then the wheeled suitcase and now he's figuring out the invisible ink |

($16^{th}$ June/Task Three/Class $C_2$/Darshana, Dev, Freddie)

Dev was rather positive with his comment as he was amazed that Freddie was able to complete this work so quickly. He did not attribute the success to mathematical ability, application of skills or perseverance.

### 4.3.6 Task: Planning A School Trip

The task 'planning a school trip' was the first of the substitution tasks I employed. As I noted in Chapter Four many of the children were absent on this scheduled observation day with Classes $C_2$ and $C_3$ combined, Class $C_1$ remained unchanged for this activity. The children were working in groups of three or four for this task that lasted for 80 minutes. This task differed from the MI6 tasks as the children were given 27 information cards and a task (appendix 3.1).

I noticed how the children were arranging their cards into clusters. The children in the groups decided to either make lists of the items they felt were important or collections of the cards. Transcript 19.1 demonstrates is one group's description of their collections;

**Transcript 19.1 – Trip Letter task, Sunita, Megan and Susan, Class $C_2$**

Class Teacher: (*Approaches the table and points to the table*) You've put these in groups (*points to Megan's pile of cards*) so this group is?
Megan: These are the money
Class Teacher: So this is the money side of things...And you've got three little ones here (*point to a pile of cards*)
Megan: That's like, just like how many people
Class Teacher: So that's the groups of people. That's the groups of money (*pointing to the groups*)
Class Teacher: And then what are these ones? (*To Sunita)*
Sunita: This is for the dates (*pointing to a pile*) and this one is about what time they are going to go
Class Teacher: And these ones over here Susan?
Susan: These are when they will be leaving, how they will get there and how long
Class Teacher: Ok so you've got time (*pointing to Susan*) you've got money group (*points to Megan*) and you've got the …
Sunita: Date

(18th June/Zoo Task/Class $C_2$ /Sunita, Megan and Susan)

This transcript excerpt was coded as showing examples of *sharing the workload*, *querying approaches* and *explaining their work*. It was also an example of the children selecting and sorting information. Within the observation data of this activity there were many examples of the children selecting the correct data. In photo 4.1 the groups of cards are visible for the group containing Sunita, Megan and Susan;

(Photo 4.1, Class $C_2$/ Sunita, Megan and Susan)

Another observance was the discarding of information. During this task the children discarded information that they decided was not relevant to the requirements of the task. Later in the same transcript Megan verbalises the organisation of the information cards;

| |
|---|
| Megan: (*Looking at the cards*) We've done these (*starts to move cards into piles*) this isn't really useful information but you could put this in the letter though, that's useless (*discards a card*) ...That needs to go in the letter, the coaches (*Megan has arranged her cards into a column of four*) |

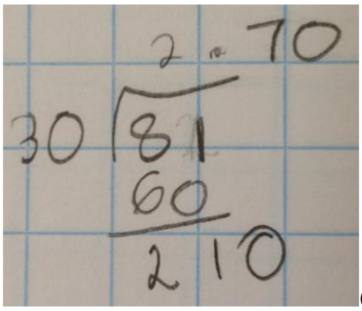
Due to the nature of this task, there was a lot of discussion and many examples of the codes *querying approaches*, *working collaboratively* and both explaining and describing their work. In one discussion between the Ana, Priya and Caroline they debated how the information should be organised and prioritised in the final zoo letter to the parents. This was coded as another example of *non-mathematical reasoning.*

**Transcript 20.1 - Zoo Task, Ana, Priya and Caroline, Class $C_2$**

| |
|---|
| Caroline: But you always do the most important things first<br>Ana: So right after that we should right the most important things<br>Caroline: Which is?<br>Ana: The date<br>Priya: Most important or they can't go the trip<br>Ana: That's annoying isn't it ...you have to put down that the trip will be exactly three weeks after the letter is sent home, we have to keep that in mind<br>Priya: Yeah, we've done that<br>Ana: Yeah but no we've said the date ...of the trip...those who have school dinners can have a packed lunch<br>Caroline: That's not really (important) it's one of the last bits |

(18th June/Zoo Task/Class $C_2$/Ana, Priya and Caroline)

I have included these children's final letter in the Appendix (appendix 3.2). In calculating the cost per child this group completed a long division. It was written very roughly but it can be seen that the child has used a multiple of 30 and subtracted this from 81. The answer 21 has been adjusted so that they could divide it by 30 and then the answer readjusted with the insertion of a decimal point.



(Work sample 4.8, Priya, Class $C_2$)

There were also further examples of ‘percentage partitioning’. To calculate 20% the children calculated 10% first and then doubled the answer. Julia had an alternative method which was to divide by five. It could be argued that she used her knowledge that 5 multiplied by 20 equals 100.

**Transcript 21.1 - Zoo Task, Julia, Imogen, Masoud and Hari**

| |
|---|
| Hari: Work out 10% first |
| Julia: And then, yeah. Isn't working out 20% divide by 5? |
| Class Teacher: Could be |
| Hari: Aww |
| Julia: Yes it is it's divide by 5 |
| Masoud: Yeah it's division |
| Julia: It's divide by 5, that's easy |
| Class Teacher: Or you could find 10% |
| Julia: And then double the answer |

(18th June/Zoo Task/ Julia, Imogen, Masoud and Hari)

There were examples of *illogical* or *nonsense reasoning* within this task. This occurred when the children accepted answers to tasks that were not logical. They did not appear to have the sense that tells an individual when an answer does not ‘feel right’ an example was accepting inflated prices for the cost of entry to the zoo.

### 4.3.7 Task: Colour Sudoku

The Colour Sudoku was the second of the substitution activities. This activity was used as once again the participant numbers were very low, only 8 ($C_1$) and 7 ($C_2$) participants were present. The rationale for analysing this data was that I was curious to see how the children would complete an activity which was the application of logic and reasoning. Also it did not have the language to wade through the text and mathematisation. Within my research question I was aiming to investigate the strategies children adopted whist attempting problem solving tasks in mathematics lessons, these were focused on real-life problems. However, the Sudoku task was an example of mathematics problem solving without the real-life context. I have noticed in the other task that the real-life context of the problem has an impact on the children and the strategies they employ to solve a problem.

In my description of the activities I noted that these were four by four grids which the children used the colours red, blue, yellow and green to complete instead of numbers. I collected the observational data for the other investigations using audio and video recording devices set up in static positions. For these observations I moved the video recording device around the room from group to group. My role of participant observer was quite apparent as I conducted some informal questioning whilst the children completed their Sudoku.

Whilst I coded the data from the Sudoku activity I noticed that there were some codes only applicable to Sudoku strategies and therefore could not be related to the other activities. However, as many these were also concerned with the starting points I subsequently included them in the code *selecting a starting point* as well. One strategy that the children used effectively during this Sudoku task was the use of elimination;

**Transcript 22.1 – Sudoku, Charlotte, Class $C_1$**

| |
|---|
| Charlotte: I worked out this was yellow, this was blue, so red or green could go here, then I looked down and this was red <u>**so this one had to**</u> be green (top left) and this one had to be red. If that was green then I randomly decided to put yellow there and blue there and then I worked from there. |

(1st July/Sudoku/Class $C_1$/Charlotte)

This excerpt from a conversation with Charlotte illustrates the process of elimination the phrase in bold and underlined is the key phase which lead me to code this as *process of elimination*.

There were also many examples of conversations that I coded as *trying out – experimenting* which were consistent with the activities from both the Zoo Letter and MI6 Tasks

When solving these Sudoku the children did make several errors. There were only two examples of the children not realising when they had made mistakes. In these cases the Class Teacher had to point out the mistakes in these children's work. Overall the children did realise that they had made a mistake in their work. At times their error was the reason why they restarted their work and was coded as *justifying restarting work*. In this example below Ethan knew he had made a mistake this it could be termed as intuition;

**Transcript 22.2 – Sudoku, Ethan, Class $C_1$**

| |
|---|
| Ethan: With the last one we got wrong, in a way it looks wrong, from the top left corner to the bottom right corner it was diagonal so there was green right it all diagonal lines so it kind of seemed a bit wrong. |

($1^{st}$ July/Sudoku/Class $C_1$/Ethan)

This task also facilitated several instances of *conjecture*. It appeared that this was a valid strategy. It was not necessarily random but these Arjun and Kishan could not explain their method (Transcript 23.1).

**Transcript 23.1 – Sudoku, Kishan and Arjun, Class $C_1$**

| |
|---|
| Arjun: Cos we put a red dot first and we assumed it was yellow and then we coloured it in yellow but then it turns out our first assumption was right. |
| Class Teacher: Ok, and what was your first assumption built on? Why did you think it was yellow to begin with? |
| Kishan: We thought it was red to begin with. |
| Class Teacher: Why did you think it was red to begin with? |
| Kishan: Because it only works like that. |
| Class Teacher: What do you mean it works like that then? |
| Kishan: Cos…Erm….Arjun |
| Arjun: You can explain it. |
| Kishan: Honestly it's fine. |
| Arjun: Go on |
| Class Teacher: Just briefly why did you think it was red? |
| Kishan: Because it seemed so right. |

(1st July/Sudoku 1/Class $C_1$/Kishan and Arjun)

## 4.4 Final Remarks

Data analysis has shown a wide selection of strategies adopted by the children when attempting problem solving activities. These strategies varied from mathematical based (using calculations e.g., long multiplication) to applying life-skills and general knowledge. I have included transcription excerpts and work examples to illustrate my preliminary codes and contextualise the data. Some of the strategies the children employed surprised me such as the use of their fingers as a calculation aid and others intrigued me, the non-mathematical reasoning the children employed. I have discussed that I started the coding process as a constructivist with codes developed from Polya (1957). As I was coding I developed my own codes and I began to emerge as an interpretivist as I explored the data. The data has also demonstrated the importance of matching the problem to the intended audience. I will discuss all of these comments regarding the preliminary codes, real-life context and calculation strategies in more detail in the following chapter.

These preliminary codes reflected the aim of this research to investigate the strategies children adopt when attempting problem solving activities. I have mentioned that I used

Nvivo to organise the data by facilitating the grouping of the preliminary codes, condensing the data down and simplifying the codes into more manageable themes. From the data I developed five overarching themes;

- Problem Solving Approaches (general)
- Calculation Approaches (mathematical)
- Problem Solving Approaches (mathematical)
- Checking Work Completed
- Prior Learning

The overarching themes were the result of collating the preliminary codes, looking for relationships and commonalities. Within the overarching themes I also discovered an interconnectedness which allowed for the development of thematic maps (figure 4.1. and figure 4.2). The thematic maps show how the strategies are not necessarily completed in isolation and are more dynamic. In the next chapter I will discuss these overarching themes further in relation to my research question and the wider educational community.

# Chapter Five - Discussion

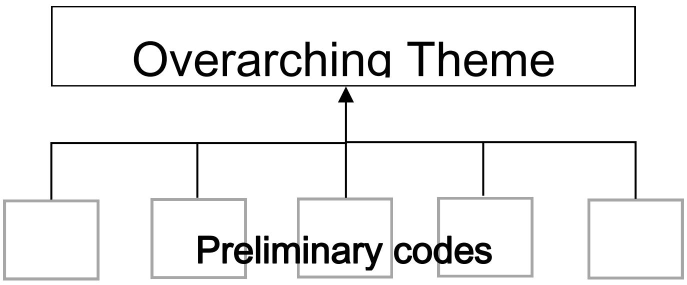
## 5.1 Introduction

My analysis of the problem solving lessons revealed preliminary codes which have been reduced to overarching themes. It is the aim of this chapter to discuss these themes in relation to my research question. The aim of my research was to examine the mathematics the children in a Year Six cohort from one school (aged 10 -11 years) were engaged in whilst attempting problem solving tasks during mathematics lessons. This also included the various strategies and techniques these children employed to reach a solution to their problems. To recap, the research question was;

How do Year 6 children engage in mathematical problem solving activities?

Additional questions;
What strategies do the children employ to solve mathematical problems?
What mathematics do the children use to solve mathematical problems?

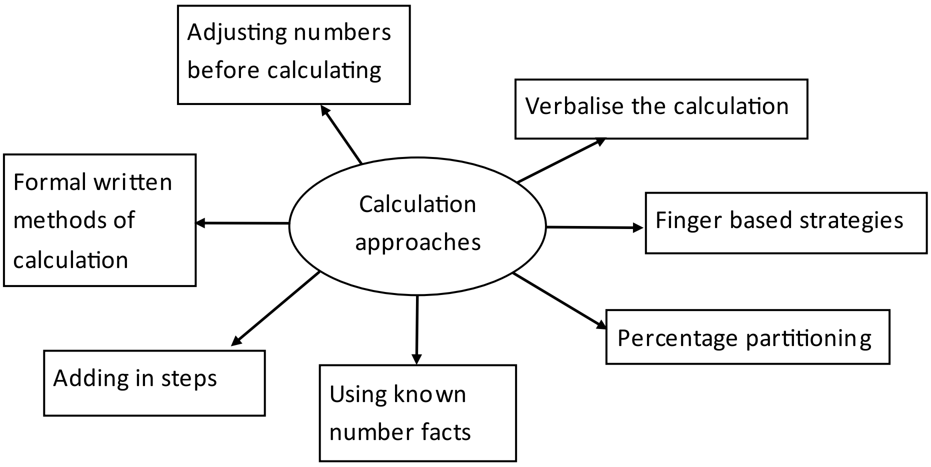
Chapter Four demonstrated how my analysis provided a great number of preliminary categories. Using the data analysis phase of my research I grouped these codes into broader overarching themes as the overlap between themes became apparent, as demonstrated in figure 5.1.




(Figure 5.1 The development of preliminary codes to a single overarching theme)

The thematic map figure 4.2 shows how the overarching themes are linked together. This diagram is a simplified version of figure 4.1 where we can see all the preliminary codes clustered around the overarching themes. Figure 5.2 is a section of the thematic map (figure

4.1), displaying the overarching theme *Calculation Approaches (mathematical)* detailing its preliminary codes (shown the in boxes).




(Figure 5.2 Calculation Approaches –mathematical, thematic map)

The assigning of preliminary codes to an overarching theme could be considered as subjective as I relied upon finding similarities between codes using my judgement. For example, the overarching theme *Checking Work Completed* contained five preliminary codes as can be seen in figure 5.3.

**Checking Work Completed**

- Agreeing on answers
- Querying approaches
- Querying answers
- Checking work completed so far
- Spotting mistakes

(Figure 5.3 Preliminary codes to overarching code model)

The five preliminary codes contained within *Checking Work Completed* were grouped together as they contained examples of the children reviewing work before continuing. The two examples below are coded utterances from *Checking Work Completed.* They are similar as they both contain comments related to 'answers' and therefore are grouped together to form a bigger all-encompassing group. It is also true that these codes are significantly different from the other codes in the other overarching themes and thus they are grouped together to form one theme.

Example One - Coded as *agreeing on answers*

| |
|---|
| Caroline: (In response to A) the answer to the first one is 21 |
| Ana: (looks over at B) yeah that's right. |

(9th June/Task One/$C_2$ /Ana and Caroline)

Example Two - Coded as *querying answers*

| |
|---|
| Julia: But how come you got the exact same answer? |
| Imogen: It's right |
| Julia: But I double checked it |
| *Julia starts to look at Imogen as she calculates, appears to double check her (Imogen's) work* |
| Julia: How come that one is exactly the same? |

(10th June/Task One/$C_1$/ Imogen and Julia)

In addition I compared which utterances received multiple preliminary codes using the NVivo software. The overlap between themes is demonstrated in figure 4.2 in the previous chapter.

## 5.2 Overarching Themes

In the following sections I will discussion the themes which I have developed from the observation data. The overarching themes developed from the preliminary themes are:

- Problem Solving Approaches (general)
- Calculation Approaches (mathematical)
- Problem Solving Approaches (mathematical)
- Checking Work Completed
- Prior Learning

Table 4.1 in the previous chapter listed the preliminary codes and demonstrated how these were grouped together to form the broad overarching themes.

### 5.2.1 Problem Solving Approaches (General)

The overarching theme *Problem Solving Approaches (general)* was linked to the procedural aspects of problem solving. The term *general* was attached to this theme to differentiate it from the other problem solving theme which was more mathematics centred. It was the generic unspecialised nature of the codes assigned to *Problem Solving Approaches (general)* which meant they could be used to label the same codes in other subjects in the primary school curriculum. For example the code *delegating* could be attached to the group work dynamic in another subject lesson as well as a mathematics lesson. Burton (1980) notes that the 'skills and procedures used in problem solving are not only appropriate to mathematics' (p. 56).

**Administration and Communication**

I previously mentioned the group work nature of the tasks. A number of the preliminary codes in this theme were related to group administrative tasks and incorporated communication skills. These codes were;

- *Delegating*
- *Sharing the workload*
- *Working collaboratively*
- *Working independently*

The nature of the tasks required the children to work together in a collaborative nature and some of the children in the group were very good at devising roles or assuming roles within the group. Reportedly collaborative work with peers has the benefit of allowing children 'the opportunity for developing reasoned arguments useful in mathematics' (Mercer & Sams, 2006, p. 510). Therefore, it might appear strange to include working independently in this selection of codes. This is related to one incident of a child, Ana who decided to work independently from her group. Ana was from $C_2$ the more able class; she started the first task with her group but wanted to work independently. After encouragement from Priya and Caroline, the other members of her group, she began working collaboratively. It has been suggested that group work 'brings an element of shared responsibility' (Boaler, 2008, p. 11). The team work nature of the task was very helpful, especially for those children who stated that they were puzzled. I defined the coded *puzzled* as any occurrence whereby children did not know what to do. The following collection of samples demonstrated some of the utterances coded as *puzzled.* In both the Examples One and Four the children mention that they are confused, a verbalisation of their puzzlement. Within Example Two it can be seen how the group dynamic is helpful in resolving the situation.

Example 1

| |
|---|
| Darshana: We are chatting because we are a bit confused |

(10th June/Task One/$C_2$ /Darshana)

Example 2

| |
|---|
| Charlotte: We struggled to first off with<br>Class Teacher: What did you struggle with?<br>Charlotte: It was ...we didn't get about the VAT bit<br>Class Teacher: What didn't you get about the VAT bit?<br>Charlotte: It was ...we were just going round in circles I don't know, it kept going wrong<br>Class Teacher: It kept going wrong, Anika, what bit did you do in your group?<br>Anika: Well we only just worked it out together, we check and checked and rechecked and we corrected each other |

(11th June/Task Two/$C_1$ /Charlotte and Anika)

Example 3

| |
|---|
| Darshana: Finding out the camouflage gear VAT that is quite hard, it keeps going reoccurring |

(16th June/$C_2$ /Darshana)

Example 4

| |
|---|
| Stacey: I'm so confused with this self-destructing paper |

(16th June/$C_1$ /Stacey)

The other members of the group aided these individuals. This became a form of collaborative learning experience which was an additional benefit of setting these tasks for the children to complete. Boaler (2009) suggested that the 'collaborative nature of mathematics is an interesting feature of this subject' (p. 27). It has additional benefits to the children as the act of explaining work to others deepens understanding, reinforcing knowledge (Boaler, 2009). Burton (1980) noted problem solving is aided by working in groups (p. 53).

Other elements of this theme were concerned with how the children communicated their ideas within their working groups. I have used the term communication skills previously; the following tasks are concerned with the children describing what they were doing, justifying their approaches and telling each other the answers to questions. Once again these codes are not specific to the learning of mathematics but they demonstrate some of the communication that occurred between the children during classroom discourse. There was some correlation with the overarching themes *Checking Work Completed* and *Mathematical Problem Solving Approaches*.

**Problem Solving Processes**

The following codes were more concerned with the actual process of solving the problems;

- *Conjecture*
- *Trying out – experimenting*
- *Non-mathematical reasoning*
- *Generalising*
- *Light bulb moment – realisation*
- *Illogical answers*

The codes *conjecture* and *trying out* - *experimenting* are strategies that do not necessarily require mathematical thought. I am proposing that the cognitive load for these tasks is not as the same as the mathematical themes (Calculation approaches – mathematical, Problem solving approaches – mathematical). The example below shows two children discussing their strategy for solving MI6 task three, calculating the amount of self-destructing paper. There was some mathematical understanding as they knew the answer was between 150 – 175 but it appears that they are trying numbers randomly.

| |
|---|
| Dev: So it's less than 175<br><br>Freddie: Try ...<br><br>Dev: So it's less than 175<br><br>Freddie: So its in-between 150 and 175, erm try ..<br><br>Dev: At least we know more than one<br><br>Freddie: Try 160 |

(13th June/Task Three/$C_2$ /Darshana, Dev and Freddie)

Conjecture was linked to some examples of non-mathematical reasoning, especially those focused on the spy sneakers (trainers) question in Section 4.4.3. I found the difficulties the children had with the sneaker question surprising. For example the image in the casebook of one trainer (figure 4.3) seemed to confuse them into considering that the sneakers were not sold in pairs as is usual for shoes. It emerged that the children were confused by the information supplied with the image of one shoe which gave the weight of one shoe but the price of the pair. However, it was not explicit that the price was per pair and weight per shoe. Wu and Adams (2006) note that solvers cannot solve a problem 'without a clear understanding of all the parameters related to the problem situation' (p. 98). It could be suggested that this doubt was due their lack of understanding of real-life situations but they did eventually reason that it would make sense to buy shoes in pairs by relating this to previous experiences.

The code of relating information to previous experience was one of the deductive codes I adopted from Polya (1957). Another example of lack of experience was evident when they were trying to calculate the amount of exploding paper the secret agent purchased. The children thought that 150 sheets of paper would be too much, whereas in fact it was the correct answer. I did consider whether my expectation of these children's real-life experiences was unreasonable. This was an example of a possible mismatch between the children's real-life experience and the demands of the problem. Within my literature review I mentioned that problems have to be closely matched to the solver, so that they perceive the situation as being problematic and they want to find a solution. There is the suggestion that children mature at 'different rates and their learning is strongly influenced by culture and experience' (Dunphy, *et al.*, p. 93). The issue of matching problems to solvers is a very complicated one.

These issues the children had could be due to lack of life experience. This led me to believe that these children would benefit from role play and the use of manipulatives to support the problem solving process. I am reminded of Lave's (1988) work on situated cognition. She suggested that when a shopping problem is given in class the participants ignore the story and treat it as a vehicle to disguise the mathematics contained within (Lave, 1988). Her research looked at the mathematics people employed in observed real-life situations. It truly was real-life and included going shopping and looking at diet plans. She then analysed the mathematics observed in these situations. A model for Lave's (1988) ideas in the primary environment would include setting up real-life events which allowed the children to employ the techniques and strategies previously learnt. This idea is once again echoed in Voutsina's (2012) research, involving the repetition of activities to improve performance in problem solving. As the data collection phase of my research continued I noted that children were becoming more confident. This was demonstrated by their use of estimating, Voutsina (2012) found that over time their participants developed better and quicker strategies.

My code *non-mathematical reasoning* was similar to Lave's (1988) real-life examples of the use of non-standard units of measurement. She discussed the instances of her participants adapting standard units by substituting them for containers which had a similar capacity. This method eliminated the use of standard measuring equipment. It is possible to suggest that this approach made the task easier for the participants, although this is just hearsay. Within my study I also included the notion of a common sense approach to tasks. *Non-mathematical thinking* was an interesting concept as it appeared that the children approached the problems with a different frame of reference, applying life experiences to the practice of problem solving. The casebook pages contained extra information and some children worked like detectives looking for any clues and not just the numbers. The casebook (appendix 2.0) showed that the secret agent was wearing shorts and a t-shirt this information was used by Julia and Imogen in Transcript 1.1 to conclude that these are summer apparel. The children then deduced that the secret agent had been to a hot country.

The problems in the MI6 scheme were essentially mathematical but to find the final solutions the children had to apply knowledge either learnt in other lessons or life skills and general knowledge. I was curious about how the children in the school, who received several extra private tutoring sessions and engaged in very little extra-curricular activities, would complete these tasks. I was not able to make any comment about this as my study was not designed to collect this data. However, it would be an interesting study for the future to investigate the impact of examination tuition on the reasoning skills of learners.

I have mentioned Putnam's (1987) discussion regarding pupil's transference of mathematical skills to problem solving in my review of literature, Chapter Two. His argument was regarding the 'application of mathematical concepts, skills, and strategies to various problem solving settings in which they should be useful' (Putnam, 1987, p. 687). It could be argued that the examples of *non-mathematical reasoning* illustrated the transference of skills from other areas of the curriculum. It also includes the use of common sense this was evident in the case of the children trying to work out how many spy sneakers were packed (Section 4.4.3, Task Three). The application of knowledge and relating ideas has some parallels with the code *estimating*. When deciding how much camouflage gear was purchased Freddie didn't work it out and asserted that it had to be one item. This example occurred during Task Three and showed Freddie's conviction in his answer, the certainty he had and confidence in his ability. I noticed that in general, the children became more confident as they worked through the tasks.

The analysis of the transcripts revealed examples of children solving the problems or part solving problems with flashes' of inspiration. I coded these as *light bulb moments* and they were close to eureka moments and the children appeared to get quite excited. This exhilaration is satisfying to witness and in some way echoes the sentiment of the positive emotions problem solving can evoke. Such emotive responses have been described as those of 'pleasure, involvement, tenacity and satisfaction' (Burton, 1980, p. 570). No negative attitudes or behaviours were observed towards the work. Previous studies have investigated attitudes in relation to Mathematics including the TIMSS 2011 survey (Sturman *et al.,* 2012). In the 2011 cycle of the TIMSS survey they asked Year Five (9 - 10 years old) and Year Nine (13 – 14 year olds) questions regarding their confidence and attitudes towards Mathematics. Their results for England indicated 'that on average, their participants;

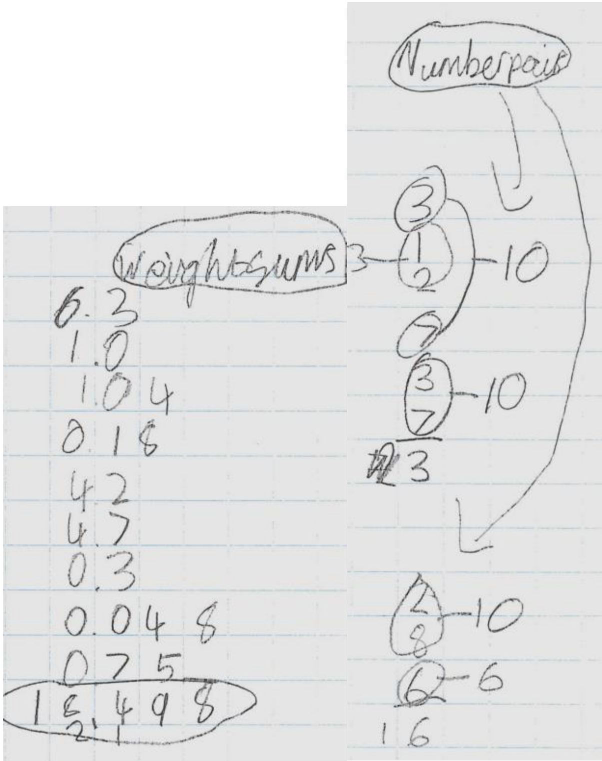
- *'Somewhat Like* learning mathematics and science
- *Somewhat Confident in* mathematics and science
- *Somewhat Engaged* in their mathematics and science lessons'.

(Sturman *et al.,* 2012, p. 55)

The final code, *illogical answers,* noted the utterances which were illogical or nonsensical. This term appears to critic the participants because they were young children and my coding could be potentially subjective. I have previously discussed subjectivity and generalisability regarding my findings and thematic analysis. Polya (1957) noted the fact that teachers often observe students achieving incredible (incorrect) results and that these children are not disturbed by their answers. The few instances I coded as *illogical answers* were examples of children accepting answers that I judged were not sensible. This included the School Trip Letter task whereby children thought that it acceptable to charge £2200 for entrance into the zoo. Another example occurred when one group attempted to solve the padlock MI6 Task Five (figure 4.5) this required a numeric answer but one group thought it was algebraic. These children did achieve correct answers eventually due to the intervention of other children and the class teacher. It might be considered unfair to call these answers illogical as one could argue that children might not know the price of entry to the zoo or have used a padlock with a code. These are abstract ideas but I was confident that these children had enough knowledge of the world to know that a trip to the zoo should not cost over £2000. The instructions for the padlock question did also ask for a numerical solution when solving the equations. Although this is similar to my earlier conversations regarding the preliminary code, *conjecture*. Burton (1980) suggested that 'the content (of the problem) and the skills of mathematics need to be accessible' (p. 50) and that the challenge lies in solving the problem. There are some parallels with De Lange's (1981) ideas on mathematising here but his study found that the children struggled to separate the mathematics in the problem ready to solve whereas my results showed a misunderstanding of context.

### 5.2.2 Calculation Approach (Mathematical)

Analyses of the data also revealed the overarching theme, *Calculation Approaches* this incorporated instances of the children attempting calculation procedures. As previously mentioned the term 'mathematical' was attached to reinforce the fact that the methods the children used were rooted in mathematical techniques. These included both standard and non-standard methods of calculation as detailed in the National Curriculum Appendix One (DFE, 2013b). Within the observation data there appeared to be a reliance on the use of formal methods of written calculations by the children. The children used column addition and both long and short multiplication but they shied away from using long division, preferring to use an adapted short method which used a two digit divisor. However, the children did adapt calculation strategies such as Freddie's addition method in work sample 4.4. Freddie was trying to make the very long column addition more manageable by using an additional step 'finding ten'.



(Work sample 4.4, Freddie, Class $C_2$)

Polya's (1957) earlier research proposed the four stage problem solving model. The first stage was described as understanding the problem which leads to selecting the correct strategy. The second is to make a plan of how to attempt the problem. The third stage is concerned with carrying out problems, incorporating the ability to prove that an answer is correct and the final is focused on reviewing answers. The first MI6 task included the selection of techniques to add several numbers. The first idea the children had when completing these addition sums was to construct column sums before they set about the mechanics of calculating the answer. I was surprised to observe these children using their fingers as a calculation aid as I perceived this as an immature strategy. The children appeared to only count on their fingers when solving the addition calculations in this first task. I can infer from these observations that the children were comfortable using their fingers as an immediate step between reliance on apparatus and the move to mental methods. In the data analysis chapter I noted how the children were only counting on one hand whilst continuing to write with the other. I considered this as an importance occurrence because it showed a maturity from counting-on using both hands as seen in younger or less experienced learners to a more sophisticated strategy using one hand. It was curious as to why this strategy only appeared during this first task. In previous research had been reported that 'children will use finger representations for counting and calculations during some stage in their development (Domahs, Krinzinger and Willmes, 2008, p. 359). They also suggested that this behaviour continues even when children have been discouraged from using them. As their teacher I knew that the participant children were not dissuaded from using their fingers whist I taught them. I concluded that this was possibly a regression to more comfortable and secure method whilst the children gained confidence with the tasks.

The incidence of mental methods was also incorporated into this theme. Reportedly, move from the concrete to the abstract and mental strategies has been encouraged in English schools due to the emphasis on mental calculation strategies (Threlfall, 2002). The children used several strategies to solve equations, which included *using known number facts* where they recalled information about numbers, such as number bonds, times tables and division facts. They then applied this knowledge to questions or calculations. The occurrences included the children 'making ten' when solving column addition and halving numbers to calculate percentages. Using number facts is in essence applying prior knowledge which may have been learnt previously through the rote learning of multiplication facts or number bonds. Mastery of number facts is embedded within the National Curriculum (DfE, 2013a). It stated in the National Numeracy document, Teaching Mental Maths Strategies (QCA, 1999) that, 'children should learn number facts by heart' (p. 3). It is apparent that these children have a good understanding of number facts and this also shows the benefit of this component of the National Curriculum.

The learning of number facts by heart is a contentious issue. It could be suggested that children need to know the process before applying it; Skemp (1989) describes this as the 'importance of learning with understanding' (p. 159). In Section 4.4.5, I described an interview with two children regarding the methods they used to solve the padlock equations. I was surprised by the comment from Yash. His justification for choosing a method was judged on how quickly he could solve the equation and that this was the way he had been taught by a private tutor. This could be considered a shallow view of mathematics where the learner just wants to find the answer at any cost. I understand that this could be considered a correct response to problem solving if we consider this definition;

> *problem solving in mathematics contains the solving of non-routine problems unfamiliar to the solver which require them to think creatively (Ernest, 1991).*

However, problem solving involves ‘cognitive tension’ which Burton (1980) describes as a ‘cycle of struggle and arrival which is the true impetus of learning’ (p. 50). I wondered whether the learner in this case really got the feeling of satisfaction problems solving is supposed to provide. His comment regarding the method of calculations and tuition reminded me of rote learning. Skemp (1989) suggests that it is not sensible to teach a method without the associated understanding as the result is success at a mechanical level. I discussed this in Chapter One in relation to my motivation for this research. Conversely Putnam (1987) suggested that the development of effective computational skills frees up the mind for higher level tasks. He suggests getting to a point of automaticity with the ‘skills learnt in varied settings that they are triggered effortlessly when needed’ (Putnam, 1987, p. 693). This does validate the tuition comment as Yash could trigger the required response needed.

The use of number facts was also used by the children as a shortcut, for example, as in partitioning numbers and adjusting numbers before attempting calculations. It was noticeable that these problems contained many questions where the children had to calculate percentages they used a method previously used in class which focused on finding 10% first. The children were quite deft at manipulating numbers to make calculations which initially appeared challenging to calculations which were more manageable. *Adjusting numbers* was used to describe instances where numbers were, for example, multiplied by ten to negate a decimal, 1.2 would become 12.

Although the children did use mental strategies they had a good repertoire of formal written methods of calculation. They also had an understanding of when to apply these strategies which is consistent with phase one of Polya’s (1957) four phase model. They knew which algorithms to use and this included the conversion between miles and kilometres. This question caused a lot of ‘perceived’ confusion in the groups but the children soon decided to use a division calculation. Interestingly, the children in this study would not use long division and preferred to complete short division with jottings of multiplications alongside. Their method shares some similarities with the ‘chunking process’ of finding near multiples of the dividend. I noted several examples in my data analysis of column addition, short division, long and short multiplication. Within the column addition questions the children constructed great long lists of numbers. Sometimes they broke the calculation down into steps. What interested me was how they persisted in adding the numbers in the order given in the case book. I wondered whether these children were conditioned to complete the calculation as given. They had not considered adding the numbers in a different order so as to use their knowledge of number such as, bonds to ten. Even when they were struggling some still continued with the multiple number additions whereas others did start to adapt. They used their initiative to break the sum up into chunks and add it in steps. Within this approach there is question of transference of skills and the passive learning approach. Boaler (2009) noted how children are taught to only follow rules and find it hard to apply knowledge. However, I have noted how the children did start to become more confident and take more risks by estimating answers especially after Task Two.

The *estimation* of calculations became apparent as the children progressed through the tasks from Task Three onwards. I suggest that the children were adapting their strategies by moving from the inflexible attitude they had to addition to then thinking about the answer before starting the calculation. They appeared to realise that there might be easier ways of solving these problems as they worked through MI6. There are some parallels with the Overarching theme of *Prior Learning*, as the children used known facts and previous knowledge to answer these tasks which is similar to Polya's (1957) phase two.

Task Five, the padlock codes differed from the other tasks as the children were presented with calculations to solve. It contained the majority of the examples of formal methods. I observed children using known number facts to narrow down their starting position when calculating the square root of 625. Tara knew 20 x 20 = 400 and therefore started with 24 x 24 before moving to 25 x 25. Yash looked at 625, noticed that it was a multiple of 5 and that the square root had to have a 5 in the unit's column. He used his knowledge of square numbers and multiples to help him find this answer.

There was very little mathematising needed in Task Five which was similar to the Sudoku activities, as they did not require decoding of the language surrounding the problem. The bracket type questions were familiar to the children as they had covered them in preparation for entrance examinations. The group work dynamic was different for these questions. They were more concerned with working independently and then checking the answers with their peers.

These formal calculations methods often occurred in conjunction with *verbalising* the calculation. The act of *verbalising* appeared as another calculation strategy the children used. Here the children would state what they were doing, which appeared to help them in deciding the final answer. This could have also been a consequence of group work and the shared talk that occurs in these situations. It can be argued that they were engaged in reasoning as they were talking about their ideas and the strategy they employed to solve. There was some overlap with the theme of *problem solving approaches (mathematical).*

### 5.2.3 Problem Solving Approaches (Mathematical)

So far in this chapter I have discussed the overarching theme *Problem Solving Approaches (general),* these codes were more concerned with the procedural aspects of the process. I noted that these were linked with administration tasks and some surface communication but were not necessarily related to the mathematics curriculum. This theme, *Problem Solving Approaches* (mathematical) does contain many examples of communicating the mathematics the children were completing. Within this act of communicating they were engaged in reasoning. Pepperell *et al.*, (2009) suggest that to be successful in mathematical communication children need opportunities to;

- *'Talk about mathematics – listening, reflecting and responding*
- *Explain why methods work*
- *Pose their own mathematical questions'* (p. 2)

The children explained their mathematics to others at times this was instigated by my questioning as the class teacher. Explaining was also an important feature of the collaborative learning, as was the debate process the children entered. The process of debating included how the children justified the mathematical approaches and agreed on methods of calculation. *Justifying their approach* has some overlap with the theme of *checking their work* and this was due to the fact that the act of justification was in response to a query about their approach or answer. *Generalising* was also included as children drew together ideas from other examples they had solved and this was used to justify ideas and explain patterns.

There were incidences of the children restarting their work. These discussions about restarting were, at times, prompted by my questioning as I was curious about why the children needed to restart their work. The children were able to reflect on the work they had competed and their next steps. *Suggesting start positions* demonstrated the discussion in the groups as well as the logical thinking of some of the participants. Although a start point could be suggested it did not always lead to the children following it through. The code of *selecting a start point* continued from some of the suggested starting points. When looking at the transcripts the children appeared to need some direction in how to start tasks. I observed one group of girls, Ana, Caroline and Priya from class $C_{2,}$ who formulated a plan (see Transcript 2.1 in Chapter Four) to attempt to solve the first MI6 problem. Polya (1957) discusses the student teacher interaction and suggests that the best way a teacher can help is to put themselves in the children's shoes, making suggestions regarding what the problem is asking.

Looking at more detail at the mechanics of MI6 Task One there were several different ways of attempting it. Ultimately the task was to find out where the secret agent had been and the problem asked for the comparison between the distances already travelled and the possible trips she could have taken, the units used were crucial. However, it did not matter which units were used, just that they were the same. From the observations I noticed that the children were preoccupied with calculating the conversions between the miles and kilometres. This impacted on their ability to find a starting point to this problem. The distance the secret agent had travelled was in kilometres and the distances in the casebook were given in miles. The MI6 teaching notes suggest that the children convert from miles into kilometres by adding the trips together first. It is more logical to add all the totals and then consider converting to make the units the same, ready for comparison. If the children converted the measurements into kilometres first, before adding, then they would have to complete more than 15 conversions. Although it would have been simpler, to leave the totals in miles and convert the pedometer distance into miles before comparing them.

There are often several ways of solving a problem. It is possible that if a learner understands the problem and recodes what is known they can find the solution quickly. They can isolate what is needed by scrutinizing the problem and making a plan. Polya (1957) suggested that, 'when the solution obtained is long and involved, we naturally suspect that there is some clearer and less roundabout solution' (p. 61). It appears that this is human nature to tackle problems in this way. It is also possible that this is strategy that some children had assimilated as they progressed through the MI6 tasks. This could also be a reason why there were several examples of *non-mathematical thinking* with the children trying to find a quicker answer.

The strategies pertaining to the Sudoku tasks were concerned with reasoning and logical thinking. The data collected from the children completing Sudoku puzzles demonstrated some additional problem solving techniques only applicable to this problem type. These were concerned with the specific way the grid was completed such as using rows first and were strategic. Yet, there has been some suggestion that inclusion the of Sudoku in the mathematics curriculum ‘strengthens the mathematical skills that are required to solve such puzzles, which include trial and error, guess and check, logical reasoning, narrowing down of choices, looking for patterns, the process of elimination, and others’ (Tengah 2011, p. 53). In addition there were instances of these skills in the MI6 tasks.

### 5.2.4 Checking Work Completed

This code follows on from the overarching theme of *Problem Solving (mathematical)*. It is an extension of that theme but differs as it focused on the many processes of checking work. It is also phase four of Polya’s (1957) four phases of problem solving and was noted as strategy by Elia et al., (2009). The analysis of the data revealed many examples of the children checking their work. These were collected together in a separate theme as the reviewing of answers is an important aspect of problem solving. Thinking logically about how to conduct the problem solving process and producing an answer that is reasonable are important to the problem solving process. The codes *querying approaches* and *querying answers* are features of mathematical reasoning.

Within the first MI6 problem the children become quite confused and doubted their answers. The addition calculations resulted in three identical answers. They did not consider that they were correct and their initial thought was that they were wrong. If they were comparing these answers to previously solved problems then it is reasonable to assume that this reaction is valid because it is unusual to conclude with three answers the same. The children queried their answers, some even wanting to go back and recalculate their answers. I could suggest that these children were using passive methods but the fact that they were questioning their answers is positive. Reasoning is an important skill in mathematics, deciding that something makes sense is part of being mathematical (Boaler, 2009).

### 5.2.5 Prior Learning

The overarching theme *Prior Learning* draws parallels with the other overarching themes of

*Problem Solving Approaches (general), Calculation Approaches (mathematical)* and *Problem Solving Approaches (mathematical)*. Fig 4.2 showed the interconnectedness of the overarching themes and demonstrates how *prior learning* is linked to the other themes. There is an argument that *Prior Learning* should not be a discrete theme due to this interconnectedness, as it is present in so many problem solving strategies. However, because elements of it do feature in so many other overarching themes its importance needed to be emphasised.

It is possible that to be a successful problem solver there has to be some application of previous knowledge and prior learning. Polya (1957) suggested that relating ideas to previous knowledge is beneficial to the problem solving process. He remarks how a 'good idea is based on past experience and formally required knowledge' (Polya, 1957, p. 9). It is evident from the data collected that the children brought a lot of their prior learning and experiences to the tasks. For example, in describing their calculation approaches I mentioned *using known number facts* where the children recalled information about numbers, such as number bonds, times tables and division facts.

The MI6 padlock task, Task Five contained examples of formal methods. Within this task there was an example showing the children recalling how to solve equations involving B.I.D.M.A.S and written methods of both long and short multiplication and division (figure 4.5). This was a very clear illustration of the children using their knowledge of calculation. Another example was when Tara and Yash calculated the square root of 625. Tara knew the square root of 400 was 20 so she started with 24 x 24 before moving to 25 x 25. Yash used his understanding of multiples of 5, and surmised that the square root of 625 had to have a 5 in the unit's column. Pepperell *et, al.*, (2009) suggested when solving problems the children have to make decisions about the best way of completing problems. This must include the recollection of previous work. The examples of the children partitioning percentages were also directly related to the classwork these children had completed previously.

I have noted how I observed several examples of *prior learning* which are not necessarily tied to academic learning. These do include the children using their previous experiences from everyday events to help answer the problems. I have already discussed the spy sneakers problem in relation to *non-mathematical reasoning*. The difficulties were centred on the photo in the Agents Store (figure 4.3) which shows only one shoe but some other items sold in pairs are photographed in multiples. This was the argument one child used in trying to decide if the price £45.20 was per shoes. They were further confused because the weight was shown as per shoe not per pair. The children did eventually reason and relate their previous knowledge to this problem deciding that sneakers are sold in pairs. In this example, the children used their knowledge and understanding of the wider world to assist them in solving the problem.

Previous research (Cooper *et al.*, 2003) demonstrated that children 'import a variety of extra-school realistic considerations into their solutions' (p. 452). This is similar to Beswick's (2011) comment that the non-mathematical reasoning 'evoked by a problem are necessarily dependent upon the problem solver and specifically the experiences and knowledge they bring' (p383). This affirms the importance of children's experiences when developing problems. The transcript of the children trying to solve the trainer problem included discussion about shopping for shoes as they tried to reason and make sense of the problem. This illustrates the importance of prior learning and the application of this knowledge to the problem.

In the literature review I commented on how problem solving tasks are often introduced to children at the end of units of work. I noted the comments of Gravemijer and Doorman (1999) as they stated that the application of real-life problems occurred at the end of learning. At the time I was querying whether this was in fact the correct time to use problem solving. However, my research has shown that the children used their previous knowledge of concepts as a strategy. Therefore, there is a rationale for using problem solving in this way. Conversely this also prompted the idea that problem solving would be a useful opener to activities, my data has demonstrated the variety of strategies both mathematical and non-mathematical children employ. This is a very useful method of assessing knowledge and understanding before teaching.

## 5.3 Final Remarks

It was the aim of this chapter to discuss the data, in relation to my research question. The participants, Year Six children used a variety of strategies and techniques to solve the problem solving tasks. There was a substantial amount of rich data to analysis and the result was a large selection of codes, these were condensed into the five overarching themes detailed in this chapter. The data demonstrated that the children were mainly employing general problem solving approaches. The real-life nature of the problem solving activities was a significant aspect of this research as were the context. The strategies also showed some parity with Polya's (1957) four phases as noted in this chapter.

The discussion has also highlighted the importance in matching the problem solving activities to the children's own experiences. I am suggesting more than differentiating on the grounds of mathematical ability but we need to be mindful of the children's experiences and what these mean to their concepts of situations. I had explored context in relation to real-life situations during the writing of the literature review. The context and how learners mathematise is an important consideration for the developers of problem solving activities.

# Chapter Six - Conclusions

## 6.1 Introduction

This chapter aims to conclude my research by summarising the main findings and the conclusions I have drawn from the data. I begin by discussing the results before moving on to the limitations of my research and the issues of validity and generalisability. I conclude with a discussion regarding the implications of this research for practice and future research.

## 6.2 Conclusions and Generalisations

I presented a detailed case study of a series of lessons, containing problem solving activities in a focus institution using observation data collected by participant observation. It aimed to answer the following broad question;

> How do Year 6 children engage in mathematical problem solving activities?

This included an examination of the mathematics a sample of Year Six children (aged 10 -11 years) were engaged in whilst attempting problem solving tasks. As I had decided to utilise thematic analysis there was an expectation that the data would provide further questions. These formed the additional questions of my research which were:

> What strategies do the children employ to solve mathematical problems?
> What mathematics do the children use to solve mathematical problems?

My research questions were influenced by Polya's (1957) four phases of problem solving;

> *First - Understand the problem*
> *Second – See how the various items are connected …to make a plan*
> *Third – Carry out the plan*
> *Fourth – Look back at the completed solution, review and discuss it* (Polya, 1957, p.5*)*

The review of the literature expanded Polya's (1957) phases and suggested some problem solving strategies. These strategies included;

- Relating ideas to previous knowledge
- Speculation
- Solving part of the problem
- Graphical representations
- Recall previously solved problems
- Conjecture
- Generalising

I employed these to aid the deductive coding process. Although I have stated that I am a constructivist, using my own interpretations, I found that using Polya's (1957) four phases influenced and helped me to focus my initial coding. This led to a more interpretivist paradigm as I began interpreting the data in light of these four phases. However, I was using a Grounded Theory approach and I revisited the data corpus on many occasions. Therefore, these deductive codes led to the formation of inductive coding. It consequently meant that the interpretivist approach led into the eventual constructivist approach that continued throughout the remainder of the study. This analysis resulted in many preliminary codes which condensed down to the resultant five overarching themes:

- Problem Solving Approaches (general)
- Calculation Approaches (mathematical)
- Problem Solving Approaches (mathematical)
- Checking Work Completed
- Prior Learning

It was clear from results of this study that the children were actively engaged in problem solving. The transcripts and copies of the children's work verify their attempts at problem solving. In answering the main research question the preliminary codes demonstrated the 'how do' which are the approaches taken during mathematical problem solving activities query from the research question. The largest overarching theme was *Problem Solving Approaches (general)* which encompassed the non-mathematical procedural aspects of problem solving (see table 4.1). These procedures could be considered as almost 'house-keeping' tasks such as *delegating tasks.* This theme also included problem solving behaviours and actions which were not exclusive to mathematical problem solving.

The subsequent question regarding the strategies and the mathematics the children used can be answered by looking at the overarching themes. These broad themes can be further partitioned into those focused on either strategies or mathematics as can be seen in table 6.1.

| **Additional research questions** | **What mathematics do the children use to solve mathematical problems?** | **What strategies do the children employ to solve mathematical problems?** |
|---|---|---|
| **Overarching themes** | • Calculation Approaches (mathematical)<br>• Problem Solving Approaches (mathematical) | • Problem Solving Approaches (general)<br>• Checking Work Completed |
| | • Prior Learning | |

(Table 6.1 Additional questions links to overarching themes)

Table 6.1, also shows how the overarching theme *Prior Learning* impacts on both of the additional research questions. This overarching theme was rather important. When examining the preliminary codes, many could be assigned as *Prior Learning*. For example, the mathematical strategies the children used was from previous experiences such as *Using known number facts*. Prior Learning was listed as a discrete code as it contained the codes; *relating ideas to previous knowledge*, *recall previously solved problems* and *applying knowledge from previous problems*. These preliminary codes did not have any commonalities with the other preliminary themes but they all included an element of using past knowledge so were grouped together into an overarching theme.

The results also showed that group work was an integral feature of these problem solving activities. The codes representing the group dynamic have appeared in the overarching theme *Problem Solving Approaches (general) Problem Solving Approaches (mathematical)* and *Checking Work Completed* (table 6.2).

| | MAIN OVERARCHING THEME | | |
|---|---|---|---|
| | **Problem solving approaches (general)** | **Problem solving approaches (mathematical)** | **Checking work completed** |
| **Preliminary codes** | ♦ Sharing the workload<br>♦ Delegating<br>♦ Describing what they are doing<br>♦ Telling each other answers<br>♦ Working collaboratively<br>♦ Working independently | ♦ Explaining their work to others<br>♦ Justifying mathematical approaches<br>♦ Agreeing on calculation methods<br>♦ Justifying restarting work<br>♦ Suggesting starting points<br>♦ Selecting starting points | ♦ Agreeing on answers<br>♦ Querying approaches<br>♦ Querying answers |

(Table 6.2 Group work codes)

I noticed how, at times, the children's lack of knowledge of real life situation hampered them. When I selected the problem solving activities for this study I believed that these tasks would offer an academic challenge in a context that would inspire the children. Therefore, it was surprising to note how the context impacted on the problem solving process. Although, as the children progressed through the tasks, they become more confident and flexible in their thinking. For example, the instances of *estimation* occurred after task three.

The data collected demonstrated that when children attempt problem solving activities, the resultant actions and behaviours are many and complex. Within the *Implications Section* I have discussed some of the suggestions this research has raised for mathematics education in general.

## 6.3 Limitations

There are obvious limitations to this research due to the fact that it is a case study in one establishment by one researcher and therefore it has limited generalisability. Yin (2014) suggested that the researcher should 'consider their case as the opportunity to shed light about some theoretical concepts or principles' (p. 40). My research has attempted to investigate the principles of problem solving in the classroom as this is explicitly referred to in the National Curriculum as one of its three main aims with an expectations that children 'solve problems by applying their mathematics to a variety of routine and non-routine problems' (DFE 2013b, p. 3). I have used the methodology of case study using participant observation in an effort to gain a pseudo 'fly on the wall' perspective during the observed problem solving lessons, to gain an insider's view. Case study is a proven method that 'allows the researcher to focus on a case and retain a holistic and real world perspective' (Yin, 2014, p. 4).

When referring to case study generalisations I was intrigued by the term 'fuzzy generalisations' proposed by Bassey (1999). He asserts that case study research can make the claim that findings may be 'possible, likely or unlikely' to be found in another similar study (p. 12). This was in reference to the replication of the study within other similar situations. There are several famous single case studies as noted by Yin (2014) who suggests that their findings have been used to 'develop broader theoretical perspectives' (p. 43). I have acknowledged the limitations of my study yet, when I conducted my review of literature I was unable to find a similar study. The relevance of this study is embedded in the fact that there is very little current research on problem solving and in particular, practitioner research. The small sample used in the study may be viewed as a limitation but it facilitated in-depth observation and the generation of rich data. Another limitation was the challenge facing researchers who conduct studies in their own institutions. Although I was aware of this and made efforts to reduce the effect of this, it is possible that the data may have been influenced by my dual researcher/teacher role. In an attempt to limit this I did revisit my codes on several occasions and re-read the transcripts and re-coded excerpts after reflection. I also discussed my findings with colleagues in the focus institution.

## 6.4 Validity

It has been suggested that it is difficult to demonstrate validity within case studies (Cohen *et al.*, 2001). This is because validity is concerned with truth and this states that the results demonstrate a truthful answer to the research question. The question of truth is easier to prove in positivistic studies. I have clarified by theoretical perspective which enhances the validity of my research.

I noted previously in Chapter Four my concerns regarding losing sight of the data if it was coded and sorted into categories without contextualising it. I was anxious to retain the context the data was situated in and therefore it was analysed and discussed in the order it was collected, one problem solving task at a time. Computer assisted qualitative analysis was used as a method of storing and sorting my data. The data was in the form of transcripts and copies of the children's work from the problem solving tasks. Bazeley (2007) suggested that there is an issue of reliability regarding the use of software for data analysis. He continues his explanation to suggest that the results are dependent 'on the skill of the user in both executing the method and using software' (Bazeley, 2007, p.7). This is cited as a disadvantage by Robson (2002) and that this 'proficiency takes time and effort' (p. 462). Therefore, the researcher has to be competent in the use of the software. I initially used NVivo as a coding tool I then used it to collate the overlapping codes which were then used to construct the thematic maps figure 4.1 and 4.2. I found it invaluable in storing and organising my data.

I used thematic analysis even though it has been the focus of some criticism in the literature (Gomm, 2004). Some of these criticisms are due to its perceived flexibility, to remedy this Clarke and Braun (2006) suggested researchers using this method 'need to be clear and explicit about what they are doing, and what you say you are doing needs to match up with what you actually do' (p. 96). They was also some suggestion that it lacks the kudos of other methods and as seen as an easy method for those new to qualitative methods (Clarke and Braun, 2006).

My rationale for the use of thematic analysis was influenced by several factors. Primarily I wanted to provide a detailed account of the themes I found within my data and thematic analysis is a recognised method for investigating patterns and similarities within this form of qualitative data. However, there are critics of this method who suggest that the resultant themes from thematic analysis 'tells us more about what was in the mind of the analyst than about what was in the mind of the interviewee' (Gomm, 2004, p.10). I acknowledge that the nature of thematic analysis is subjective but I have been transparent in my decisions throughout. Thematic analysis's rigour lies in the development of a clear and systematic method of analysis by the researcher.

## 6.5 Contribution To Knowledge

In carrying out this research I aimed to provide insights into mathematics education with special reference to problem solving processes children engage in. My study goes some way to explaining how children attempt to complete non-routine problem solving activities which were not *word problems* but closely linked to investigations. My findings demonstrated themes which are both mathematical and non-mathematically based. Other studies (Polya, 1957; Elia et al., 2009) have described the strategies learners employ. However, this study has demonstrated that in addition to Polya's heuristics other strategies were adopted by the children. It also demonstrated that these children did not use two of Polya's (1957) strategies of *graphical representation* and *partially completing a task*. The children's strategies did follow the four phases of problem solving Polya (1957) proposed. However, this study differed from Polya by exploring the overarching themes of the observed strategies and categorising these themes.

It can be seen from Polya's (1957) reference that this was nearly sixty years ago. I have noted in chapter two that in the intervening time the curriculum has changed even though I only focused on the mathematics curriculum since 1988. There is a noticeable shortage of published research in aspects of problem solving in the UK, although the concept of problem solving has been included in all the official policies and documents dating back to the Cockcroft Report (1982). Therefore, it is hoped that this study provides a new insight into children's strategies and behaviours relating to problem solving. This includes the continuation of an appreciation for appropriate and considered contexts in problem solving activities.

From my results I discovered that the children in the study were reliant on formal calculation methods. As the tasks progressed they did start to adapt and build in flexibility in their thinking. There was evidence of the children employing mental techniques when solving the calculation questions. For example they added in increments using knowledge of tables and changing repeated addition to multiplication. They demonstrated dexterity of number, moving between methods of computation and they could justify the approaches they employed. The group work element of the study was more important than I had expected. The school did not actively use group work in mathematics and relied on textbook tasks and skills learning. The group work element of the tasks appeared to promote learning and was very evident in the preliminary codes. I feel that this component of the study warrants further detailed exploration.

The study also provides some analysis of the types of published problem solving activities and what they entail which contributes to taking a fresh look at how problem solving is interpreted in published materials. The research reported here also provides a fertile ground for other researchers to select topics for further exploration.

## 6.6 Implications For Practice

The implications for others from my study for practitioners and policy makers are significant. My observations have shown a new perspective to the problem solving debate, for example my 'fly on the wall' approach has shown what mathematics the children employed over the series of observation lessons. It has demonstrated the discussion and decision making processes the children were engaged in and the value problem solving has by incorporating non-mathematical reasoning. I was also interested to see the impact of non-mathematical reasoning on this process.

The problem solving activities made the mathematics more real and relevant to these children. It would be interesting to investigate the longevity of learning that occurred during these tasks and whether learning at a deeper level is achieved by using more real-life situations. Any areas of learning I highlighted as wanting have been reported back to the senior management and curriculum leaders of the school. Considering the case study reported in this thesis and the different aspects, this should offer opportunities for practicing teachers to debate issues and perhaps replicate some of the activities and compare findings.

Within initial teacher training and continued professional development there is opportunity to strengthen the understanding of problem solving. I have mentioned how problem solving is one of the aims of the mathematics curriculum and yet it has been seen as a bolt on to units of work. From the results of this study it can be seen that problem solving is a complex, multifaceted skill in the acquisition of mathematical knowledge and understanding. When considering Ernest's (1991) comments regarding the promotion of problem solving pedagogies as he argues that we need to look at perceptions and personal philosophies of mathematics and not just assume all teachers will interpret the term problem solving as the same activity. At the onset of this research study I did not have clear definition of what problem solving was. From Ernest's (1991) comment I suggest that teachers need to be educated on what a problem solving activity is. From my review of the literature there are many suggestions borne out of this reading. Therefore, I would suggest a definition of problem solving activities should include the use non-routine problems appropriately matched to the individuals' understanding. The comment regarding understanding is important, at its most basic a problem is an obstacle to achieving a goal and therefore the solution involves some form of creative thinking. Importantly the activity needs to be intellectually challenging so that the learner perceives that their goal is blocked and also engaging so that they are motivated to find an answer. The learner needs to feel the need to solve the problem and the problem should provide an element of cognitive struggle. The problem has to be relevant to the life experiences of the solver. This study has shown that context is extremely important in enabling and conversely hindering problem solving. It has also highlighted the importance of group work in the classroom for these problem solving tasks.

## 6.7 Personal Learning

I approached this research with the curiosity of a teacher of mathematics and as a researcher. As a teacher my professional concern was concentrated on gaining a deeper understanding of what was happening during the attempts children made to solve mathematical problems. As a researcher I wanted to share with other professionals in the wider community my findings and further research projects they could lead to. I have previously mentioned that I came from a more scientific background and was more familiar with more positivist views of reality. Whilst completing this research I have developed as a researcher and have an appreciation for the many theoretical assumptions researchers can follow in the pursuit of answers.

As a class teacher I was concerned with the lack of mathematics for enjoyment that I perceived to be occurring and the impact of examinations on the Year Six children. I previously mentioned a perception that the curriculum in the final year of primary school is restricted by what some term ‘high stakes’ testing (Galton *et al*., 2009, p. 119). This impacted on the curriculum taught and focuses it towards test success. The possible influence of tutoring became apparent in Task Five as a child stated that they had completed the task previously with their private tutor. This child had been coached to select the quickest method. This might be a valid approach but I was concerned that the mathematics completed by this individual was not in the spirit of the task, just done as chore. To recap a previous comment from Boaler (2009) used in Chapter One, she noted that mathematics is more than facts to be memorised. The impact of tutoring is an area of study that should be explored further.

Lave’s (1988) work on situated cognition demonstrated how adults use mathematics in real life and the PISA 2012 report (2014) described the importance of problem solving for future economic gains. It does appear that in the current education climate, where gaining a selective school place or achieving a high place in the league tables is important, there is possibility that the subject is suffering due to the focus on academic skills and not the how to apply mathematics to real-life everyday situations.

In completing this research I have developed my own practice in the classroom and have included more problem solving activities. I have a better appreciation for the learning that results from these activities and the way it can inspire learners. I have also seen how other areas of children’s lives can be developed through problem solving such as life-skills including using money and persistence in the face of adversity. The group work aspect of the tasks allowed for the less confident children to ‘shine’ and become more confident in their ability. It also demonstrated to the children who thought maths was just about numbers and tests that this is a rich, interesting and enjoyable subject. Bearing this in mind I am reminded of a quote by Burton (1980) I mentioned in the introductory chapter of this work which summarised my view of problem solving as ‘an area of study which is thoughtful, provocative, beautiful and curious (Burton, 1980, p. 570).

## 6.8 Suggestion For Future Research

The observations of the lessons highlighted a possible need to develop problem solving techniques. There is some suggestion (see for example Voutsina, 2012) that as the children progress through problem solving tasks and complete more they will become more skilled in strategies and application of techniques. I made some comments in both Chapters Four and Five regarding the completion of the fifth task and how this demonstrated using known number facts to solve questions. If we treat problem solving as a skill then improvements in application of techniques will occur if the children are given the opportunity to practice. On starting this research I wanted to observe what the children did when they complete problem solving activities. I do appreciate that problem solving needs to be embedded in the curriculum and skills need to be taught for success to happen. The current National Curriculum and Programme of Study for Mathematics (DfE, 2013a) has problem solving as one of its three main aims it is therefore important that curriculum planners focus on this element of the curriculum.

Teachers' understanding of what problem solving means and how they address teaching it would be a worthwhile investigation. If an opportunity arises, I would replicate my research in a range of schools and compare the results.

This research is complete but my interest in problem solving and my constructions of how to enhance children's problem solving skills continue.

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# Appendices

## Appendix One – Ethics

### 1.0 Permission Letter To the Head Teacher

Brunel UNIVERSITY LONDON

[redacted]

9th May 2014

Dear Mr Roberts

Ref: Permission to conduct Educational Research

I am writing to ask your permission to conduct research at [redacted] as part of my Doctoral Dissertation. The focus of my thesis is Mathematical problem solving based on real life situations.

Participation in the study involves the children completing problem solving activities in class. These activities aim to continue to develop the children's mathematics skills in preparation for their secondary education. I will be videotaping observation lessons; this will not interrupt the normal class teaching. A sample of the children might be interviewed (audiotaped) and/or asked to complete a questionnaire about these maths activities. All recordings will be destroyed after the study has concluded and are just for accuracy.

All the information regarding the school and the participants will be kept confidential and pseudonyms will be used in the writing of this study. I will gain consent from the parents and children, they will be reminded that there participation in this study is voluntary and they are allowed to withdraw at any point.

If you require any further information regarding this study do not hesitate in contacting me. Thank you for help and support in this matter.

Yours Faithfully

R. Walden

## 1.1 Permission Letter To The Parents

Brunel UNIVERSITY LONDON

Brunel University, Uxbridge,
Middlesex, UB8 3PH, UK
Telephone +44 (0)1895 274000
Web www.brunel.ac.uk

Ref: Year 6 Mathematics Study

Dear Parents

I am currently conducting some research into Mathematics education and problem solving as part of a doctoral study. I am inviting your child, with your permission, to participate in this study. I have sought and gained permission to conduct this research from the Headmaster.

Participation in the study involves the children completing problem solving activities in their maths class. These activities aim to continue to develop the children's mathematics skills in preparation for their secondary education and to make the last term at [redacted] fun and enjoyable as possible. If you and your child decide to participate then I am asking permission to include them in videotaped observation lessons, this will not interrupt the normal class teaching. A sample of the children might be interviewed (audiotaped) and/or asked to complete a questionnaire about these maths activities.

You have the right to refuse permission for your child to participate in this study and the children have the right to withdraw from the study at any point in the process.

Both the School's and the Children's identities will remain anonymous and pseudonyms will be used in the final research report. The video recording will consist of group shots and will be destroyed after the study has concluded. Copies of the findings from my study will be made available. Please feel free to contact me for any further information or queries you might have.

Thanking you in advance for your help and cooperation in this matter, it is greatly appreciated.

Yours Sincerely

Miss R Walden

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INVESTIGATION IN PROBLEM SOLVING IN MATHEMATICS

I give permission for ______________________________(child's name) to participate in the Problem Solving in Mathematics study.

- I understand that we can withdraw from the study at any point.

Signed _________________________parent/guardian

Please print your name _________________________

## 1.3 Participants Leaflet

# PARTICIPANT INFORMATION SHEET

**Study title**
Investigation into what happens when Year 6 children attempt Problem Solving Activities.

**Purpose**
I am currently a student at Brunel University studying for a Doctoral degree. My research is focusing on Mathematics education. As part of my dissertation I aim to investigation the learning that occurs when children solve word problems. I am interested in how these activities promote learning in Mathematics in a primary school context.

**Why have you been ask to participate?**
You have been asked to participate in this study as it is aimed at children, like you, who are in Year 6. If you decide to participate in this study then, as part of your regular maths lessons you will attempt problem solving activities. Some of these will be group work and are all centred on the theme of 'Spy Maths'. You will be asked to attempt tasks which you should find challenging and hopefully enjoyable.

**Participation**
I have gained permission from the Headmaster to complete this project at school. Permission slips have been sent to your parents; however you have the right to withdraw from this study at any point if you do not want to be part of it.

**What happens if I take part?**
If you decide to take part then you will attempt the problem solving activities in class. My role is to observe how you work through these activities, I might videotape the lesson, and these will be group shots not close ups. I might also ask you to chat to me afterwards about the lesson or even ask you to complete a questionnaire.

You will not miss out on any other activities due to these maths activities or any subsequent chats, questionnaires. When I write about your lessons, I will not use your name, all conversations will be kept confidential and the video will only be viewed by my university tutors and me, it will be destroyed soon after the study has been completed. You also have the right to view the video if you want to.

**The results of the study**
The results will form part of my dissertation. I will not use any of your names or the school's name in my results and subsequent writing. Within my results I might paraphrase discussions using pseudonyms, your participation will be kept strictly confidential. I will produce a leaflet detailing the results of the study which you and your parents can view once I have successfully submitted the final dissertation.

If you have any further concerns regarding the study, do not hesitate in contacting me for more information. Thank you in advance for your help in this matter.

## Appendix Two – Maths Investigator 6 Tasks

## 2.0 Copy of Text Book Pages 6 – 7

## 2.1 Task One

Which trip did the secret agent take?

How far did she travel?

### Whole-class teaching

*"Hello! M here, head of MI. I'm pleased to welcome you to the team, MI6 – I already have a problem that needs your help. One of our top agents, Jane Blond 007½, has recently returned from a mission, but unfortunately her suitcase got lost along the way. It contains some important MI equipment and documents. It would be a disaster if it fell into the wrong hands – we must find it! I'm sorry to say, though, that Jane returned with a bug and is ill in bed. She is quite delirious and we can't find out much about where she went. Fortunately, we do have the kilometre counter that Jane was wearing on her jacket as she travelled – is that any use?"*

### Whole-class teaching

*"Hi MI6! I've found something else that may help. Before she left, Jane made a list of places she needed to visit and the approximate distances from place to place. She logged them with the MI Travel Department and they've kept a record on their system. Can you investigate the information and see if it gives us a clearer picture of where she might have gone? Thanks."*

### Whole-class teaching

*"Thanks for narrowing down the possible trips, MI6! I've just been to visit Jane in the MI Medical Wing, but she's still not making any sense. I've managed to get a copy of her medical notes – do they shed any light on where she might have been?"*

(Clissold and Pink. 2008c)

## 2.2 Copy of Text Book Pages 8 -9


(Clissold and Pink, 2008b)

## 2.3 Task Two

Which suitcase did she use?

## Whole-class teaching

"*Fantastic work, MI6! So, we know which locations Jane visited – but the suitcase could be in any one of them! It should have been transferred with her each time she changed planes, but something must have gone wrong. Before I start contacting the airports, we need to know what the suitcase looks like. I've found a receipt for a suitcase in her office. I think that we can assume it's the one she took with her. Can you identify it from the Agent Supplies Store, so we have a description to pass on?*"

(Clissold and Pink, 2008b)

## 2. 4 Task Three

What did the secret agent pack in her luggage?

Including the quantities

## Whole-class teaching

"*Hi MI6! Brilliant work. Agent Costa Lotta has contacted all the airports and it seems that each one has a suitcase identical to the one that you identified – clearly a popular choice! To try and narrow them down, it would be useful to know what Jane packed for her trip. Could you look into that? I've got to dash to a meeting. Thanks so much.*"

## Pupil activities

### Pair work

Children work in pairs to find the quantities of each item bought, using the catalogue pages (clue 4) and the receipt (clue 5); they leave the invisible ink to the last as it is the most challenging. Encourage them to consider, at each step, the best and most efficient methods to find the answers, i.e. mental, written or calculator methods. Tell them that they will have to explain their choices of methods during the review.

(Clissold and Pink, 2008b)

## 2.5 Task Four

What was the weight of the luggage?

## Pupil activities

### Pair work

Using the information in the displayed email and the information on the catalogue pages in the *Casebook*, children work out the total weight of items in Jane's suitcase. They then add the weight of the suitcase.

(Clissold and Pink, 2008b)

## 2.6 Task Five

What was the code on the padlock?

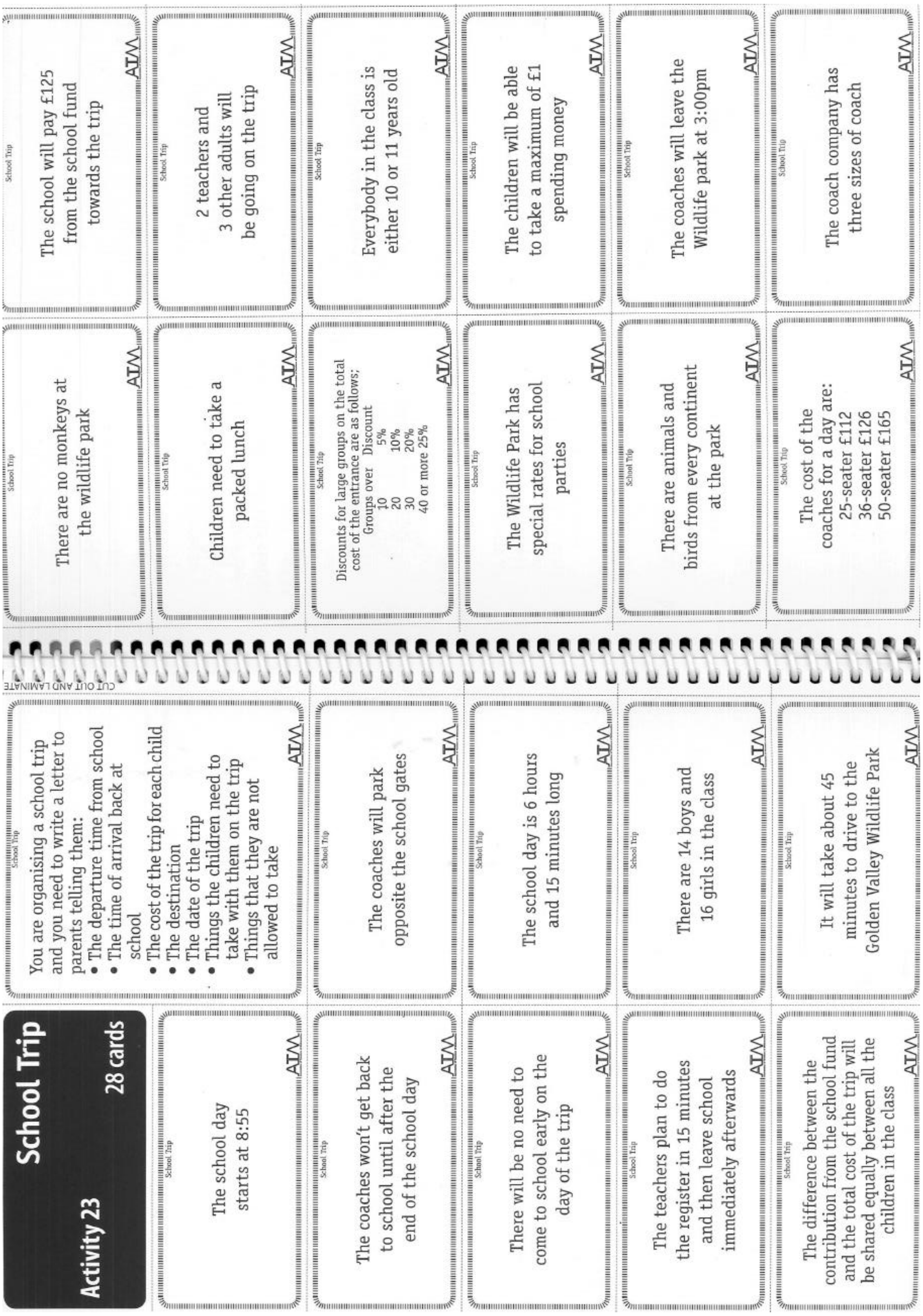
## Whole-class teaching

"*Hi MI6 – fantastic work! We're getting closer. Now the suitcases have been weighed, there are only two that could possibly be Jane's. Unfortunately, they're both locked with complicated padlocks. I've found some notes she left behind in her office – could these help us unlock one of them?*"

(Clissold and Pink, 2008b)

# Appendix Three – Zoo Letter Task

## 3.0 Information Cards



**Activity 23 — School Trip — 28 cards**

You are organising a school trip and you need to write a letter to parents telling them:
- The departure time from school
- The time of arrival back at school
- The cost of the trip for each child
- The destination
- The date of the trip
- Things the children need to take with them on the trip
- Things that they are not allowed to take

There are no monkeys at the wildlife park

The school will pay £125 from the school fund towards the trip

The school day starts at 8:55

Children need to take a packed lunch

2 teachers and 3 other adults will be going on the trip

The coaches won't get back to school until after the end of the school day

The coaches will park opposite the school gates

Discounts for large groups on the total cost of the entrance are as follows;

| Groups over | Discount |
|---|---|
| 10 | 5% |
| 20 | 10% |
| 30 | 20% |
| 40 or more | 25% |

Everybody in the class is either 10 or 11 years old

There will be no need to come to school early on the day of the trip

The school day is 6 hours and 15 minutes long

The Wildlife Park has special rates for school parties

The children will be able to take a maximum of £1 spending money

The teachers plan to do the register in 15 minutes and then leave school immediately afterwards

There are 14 boys and 16 girls in the class

There are animals and birds from every continent at the park

The coaches will leave the Wildlife park at 3:00pm

The difference between the contribution from the school fund and the total cost of the trip will be shared equally between all the children in the class

It will take about 45 minutes to drive to the the Golden Valley Wildlife Park

The cost of the coaches for a day are:
25-seater £112
36-seater £126
50-seater £165

The coach company has three sizes of coach

CUT OUT AND LAMINATE

### 3.1 Work Sample – Completed Letter

Dear Parent/Gardian

We would like to inform you that we would like to take your child on a school trip to the wildlife park on the 17th of June 2014. If you would like to come it is £2.70 per child.

Your child will need to bring a pack lunch in if they have lunches with the school they can prepare a lunch for you. The children can bring in a pound for the gift shop.

We depart at 9:10 am but come to school at normal time. We get back at 3:45pm. Come to school round about 3:3 0 pm.

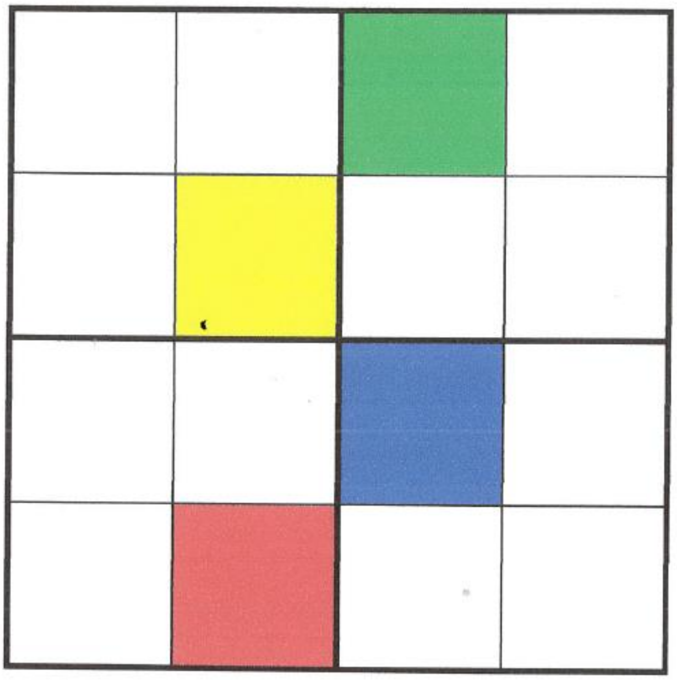
Kind Regards

## Appendix Four - Sudoku

### 4.0 Sudoku Example

# Colour Sudoku 1

Every row, column and mini-grid must contain the colours red, yellow, green, and blue.
Use counters or Cubes and logical thinking to help you.



**4.1 Sudoku Worked Example**

# Colour Sudoku 1

Every row, column and mini-grid must contain the colours red, yellow, green, and blue.
Use counters or Cubes and logical thinking to help you.