WHAT DETERMINES OIL PRODUCTION?

A CASE STUDY OF NIGERIA AND THE UNITED KINGDOM

A thesis submitted for the degree of MPhil

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ABSTRACT

Nigeria and the United Kingdom are leading oil producers within their region. Both countries are linked by their exploration and production maturity within their regions and the fact that they produce similar oil grades. Their institutional similarities and their economic status as developed and developing economies provide the platform upon which this study basis its comparative investigation.

On account of the oil price phenomenon and oil supply concerns by way of reserves, this study investigates the effect of the Hotelling theory, the Hubbert theory and Engineering decline curve theory on actual production rates within Nigeria and the United Kingdom. It develops individual models for both countries, applying each theory to each country to analyse the individual effect and the effect in comparison to one another. The level of adherence of each country to these production theories is measured, following which a combination of all three theories is applied to both country cases to quantify the level of significance and relationship to actual production behaviour.

The results leave us with the understanding that the Hubbert theory does indeed capture the behaviour of production in both countries even where it was not expected in the case of Nigeria. The reserve constraint remains a key factor in future production plans for both countries. The results of the empirical analysis provide evidence of strong support in the United Kingdom for the Hotelling theory and weak support in the case of Nigeria. Oil price also remains a key factor in production modelling, though less so in the case of Nigeria. The engineering modelling approach on the other hand failed to explain Nigeria's production profile, while it captured that of the UK; indicating that production in Nigeria is yet to decline. This result is corroborated by the projected peak production date seen in the Hubbert forecast model for Nigeria, and the weaker support Nigeria shows for the Hubbert theory.

The study concludes by developing a combined model using all three theories to quantitatively analyse which of them best explains the country oil production profile. The results lead us to conclude that despite the fact that there is weak support for a production theory; an empirical analysis of the data does show that the relationship may not be insignificant. The augmentation along with the combination of the production models sheds more light on actual behaviour as it provides a more in-depth understanding on actual oil production behaviour.

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1 CHAPTER 1: INTRODUCTION AND GENERAL BACKGROUND

1.1 Introduction to the study

Oil & Gas Exploration and Production (E&P) involve an extensive and cohesive combination of investment and application of various technical disciplines. The main goal of all companies involved in the exploitation of a hydrocarbon reserves remains the same; that is to produce the reservoir in the most efficient and cost effective way thereby generating adequate returns on their investment. Jahn et al (2008) study notes that the prime objective of the oil company is to discover commercial hydrocarbons from which it can create profits subsequent to development. The study also explains that with this comes a range of uncertainties, risk factors and estimations, which in turn are fed into the planning, decision making and budget development of these projects. For major capital projects, it is common to perform risk analysis at several stages of the development planning, so that risk items can be identified early and actions planned accordingly. Optimal production is controlled by a myriad of extraneous geologic, engineering and economic factors which are modelled using different techniques by stakeholders in the bid to maximally exploit the mineral resource. Therefore, production volumes are modelled before and during extraction operations for project planning and revenue estimation purposes.

Oil price and reserves affect production, and they have contributed significantly to the debate and outlook on world oil supply. As such the oil market continues to be the subject of focus as we continue to see evidence of the effect of economic and political factors on oil production on a country scale and world scale.

Oil production is forecasted for revenue projection and planning in the case of the producers, and to shed light on time of depletion for dependent economies. Though all oil producers are different, production plans and models vary, their prime objective remains the same i.e. to discover commercial hydrocarbons from which to create profits subsequent to development (Jahn et al, 2008). A significant portion of the world hydrocarbon reserves are located in developing countries, and the need to study and analyse international exploration and production operations of these economies is underlined by the fact that over 75% of world's proven reserves currently lie with them.

Studies such as Toft and Duero (2011); Zhao et al (2012) and Hunter (2014) carry out research into the effect of economic policies on optimal production. The studies carry out a comparative investigation into the production volumes of selected countries, where they quantify and highlight the effect of the selected input factors. These factors vary from contract type and corruption measures, to oil price. Kemp (1990) examine the effect of national policies on UK North Sea production and highlight the impact of depletion policy on the UK and its producing counterparts in OPEC. Nakle (2007) explores UK oil tax regime and eventual revenue in the face of volatile oil prices, the results of which highlight the importance of government policy on the overall profitability of the business. Anshasy and Bradley (2012) investigate the role of oil price in determining fiscal policy in oil exporting countries. The results of all of this research highlight the importance of economic factors such as oil price volatility, which has brought about a shift in focus for producing countries as they prepare their economies for post-depletion. This bears importance for Nigeria and the UK which are both reliant on crude export revenues. Nigeria's produced petroleum typically accounts for 75% of government revenue and 95% of total export revenue (IMF, 2014).

The driving issue with hydrocarbon reserves is that it is a non-renewable resource which therefore means that it is possible to 'run out'. In addition, not all discoveries are commercially recoverable, and so oil price may affect production. Watkins (2002) carries a country comparative study in which the results highlight that reserves, and therefore production, can change for various reasons. The increase in UK reserves has been attributed to revised oil-in-place findings. However, it remains a possibility that this could well increase if recovery factors can be improved via technological advances. Nigeria's reserves on the other hand were predicted to reach depletion within 30 years (Odukwe and Enibe, 1988), a prediction which has clearly not materialized in the face of increasing reserve estimates due to new oil finds. This research shall investigate the factors which have made the most impact on production in the two countries over the last 25 years.

Though reserve discoveries may boost dwindling supply, studies such as Brandt (2007); Reynolds (1999) and Bardi (2005) argue that reserves are not the only factor which affects production. They expand on the prevailing Hubbert (1956) theory which proposes that oil production will increase, peak and then slowly decline to zero, following a bell shape curve. They propose that shape and symmetry of the bell curve are affected by technology and production policy. Ebrahimi and Ghasabani (2015) argue that the multiple-peaks Hubbert approach better describes oil production in some cases thereby accounting for factors which might boost production. This approach however highlights that when forecasting it is possible to end up with multiple depletion scenarios. Reynolds Kolodziej (2007); Reynolds and Pippenger (2009) on the other hand show that not all cases support the Hubbert theory. In order for Hubbert theory to apply to Nigeria and the UK, it is expected that their production profiles should follow a clear bell shape curve.

The effect of other factors such as oil price, OPEC production and cost on oil production has been well established and analyzed under the Hotelling's (1931) theory which proposes that oil production volume should move in response to oil price. In a bid to control production, Hotelling theorizes to manipulate optimal price of an exhaustible resource in relation to its scarcity (production) and growth of interest rate. The theory states that the price of a depleting resource like conventional oil should rise over time at the same rate as interest rate because its value should increase as the reserves are exhausted.

Via econometric analysis, various works such as Pindyck (1976); Lin (2008a); Lin et al (2008); and Lin and Wagner (2007) conclude that the real oil market may not always fit the Hotelling model, so there is a need for the inclusion of augmentations such OPEC and technological progress to the model. Lin (2008) and Reynolds & Baek (2012) are not in full support of the Hotelling theory, and they highlight even further additional factors such as capacity constraints and crude oil quality differentials, which need to be included for a more accurate real world model. The studies note that production in some countries and regions may not show support for the Hotelling theory.

Production modelling is also carried out using the reservoir production engineering technique which is based on Arps' (1945) theory known as decline curve analysis. Decline curve analysis models the boundary dominated flow within a reservoir to develop a decline profile for the producing area. Studies such as Kaufmann (1991); Sorrel et al (2010); Jakobsson et al (2012) carry out a rare related analysis in which they use both economic and engineering modelling approaches to develop production models. The studies highlight the shortcomings of both approaches and attempt to fill the information gap characteristic of each.

Bentley et al (2007) observes that though high oil prices encourage exploration, brings on economically marginal fields and permits more expensive recovery, these effects are limited in a country which is past its peak production; and even so, each country needs specific analysis.

Although many of these studies are in agreement with the theories they investigate, a literature review of available research on the subject of oil production volume modelling and its determinant factors shows that production modelling theories are often augmented in attempt to capture real world behaviour. Economic production theories and engineering production these theories are often used individually and separate from each other such that a model combining their relationship is uncommon.

It is expected that since Nigeria regulates it production as a member of OPEC, its production model should show little support for the both the Hubbert and Hotelling theories, while the UK would show stronger support. This proposed study will establish the level of support of Nigeria and the UK for these production models. Using these model relationships, it will measure the level of impact each has on actual production to therefore set the basis for further comparative analysis of selected factors on optimum production. This proposed research distinguishes itself by focussing on a country comparison rather than popular or group or regional comparison as seen the research discussed in the literature review.

Most studies use an either/or approach whereby either an economic or engineering model is used exclusively. Rarely is the combination approach used. This study however builds on the works by Kaufmann (1991); Sorrel et al (2010); Jakobsson et al (2012) to develop an approach that incorporates both economic and engineering models. It will investigate the factors which affect the extraction operations in the two prolific oil production nations, and it will explore the explanatory power of leading economic and engineering theories on oil production.

This study focuses on Nigeria the leading producer of oil in Africa in comparison to the United Kingdom, a leading European producer which produces the same crude oil grade as Nigeria does. The aim of this thesis is to understand factors affecting the oil production by using both economic and engineering models. The results are compared in an attempt to highlight possible differences and the reason for these, subsequently then setting the basis for further analysis of economic factors which affect production in both regions. The choice of comparators is based on the fact that both the UK and Nigeria are mature producers, have close economic ties, and produce a similar grade of crude oil at 38^{o}API (International benchmark) and 37^{o}API Nigeria respectively. And despite these similarities, their production profiles and petroleum policies are very different. Annual data is used to alleviate the data confidentiality and availability issues, and to circumvent documentation inefficiencies in Nigeria. The use of annual data is evident as normal practice from petroleum economic studies examples of which are evidenced in Bindemann, 1999; Abdo, 2014 and Azhgaliyeva, 2014.

Using a case study scenario over a period 25 years, annual data for oil production, operating expenditure, oil price, and interest rates on a country aggregate level are used to develop the models for both countries. The results are compared against the model expectation to estimate their level of support for the economic and engineering production models. Using the results, forecasting models based on the Engineering production and Hubbert production models are then built and extrapolated for both countries to predict depletion in both country cases. Following this an econometric model which combines the theories into a single model is developed. This model measures which of the production theories is strongest; i.e. which of these competing effects best describes the production behaviours in both countries, and how do they compare against the reserves constraint in describing production.

The ensuing results highlight the impact of the geological and economic constraints on oil supply in both countries. The results of the analysis support evidence that the Hotelling theory best describes the Nigeria production while the Hubbert model best describes the United Kingdom production model. The results shed light on the limitations of various production modelling theories. They show the Hubbert theory to be the most robust in explaining actual production profiles in both countries, while the Decline Curve theory (Engineering) has the weakest explanatory power. The results of this study indicate support that despite the varying explanatory power of the production model approaches in the two countries, when used in combination these theories can provide a more comprehensive perspective on production behaviour. These results support the findings highlighted in Sorrel et al (2010); Jakobsson et al (2012).

This work is organised into a study of three parts. Following an introduction to the country case study comparative method on which this study is based, the rest of the work is divided into three studies. One study develops a Hotelling model and tests the Hotelling theory and expectations in both countries; and another develops a Hubbert model and Engineering decline curve model and tests the two theories against the production profiles in both countries. Following this, the third study develops a model which combines the three models to measure which of these competing theories best quantifies and explains the production profiles in both Nigeria and United Kingdom. The results of all are then discussed at the end of each chapter with a final conclusion chapter which highlights further expansions for future works.

1.2 Methodology

This case study approach for this research is in accordance with the case selection method – *typical* (Seawright 2008); with the choice of Nigeria and the United Kingdom exemplifying a stable, cross-case relationship. Indeed, this study is about something larger than the chosen case itself – i.e. the need for focus on developing economies via the comparison method to each other and their developed counterparts. The study will respect the empirical data and organise a methodological stance to reflect that respect – as mandated by Blumer (1969); Crossley and Vulliamy (1979). The strength of case study as a research method lies in that it provides the ability to maximize the extent to which behaviour observed in one context is can be projected onto another; opening up for cross-case analysis (Crossley and Vulliamy, 1979; Seawright, 2008). Though a country might possess its own economic and social peculiarity, countries may be grouped regionally for comparison; or selected using characteristics such as maturity within the oil industry (new or mature), hydrocarbon producer type (oil or gas producer), etc., to allow for linkages in comparison.

A quantitative comparative analysis of two countries implies (1) a more comparable study than when the same company is involved, (2) a study that envelopes multiple companies operating and taking decisions based not only on internal policies but also on government, economic, social, and fiscal policies, and policies bordering on corporate governance issues; giving a more robust scope.

The choice of comparators is based on the fact that both countries are mature producers within their regions, producing similar crude oil grades, and both countries have close economic ties following post-colonial relations; yet their production profiles and petroleum policies are vastly different. In addition, they are both in different geographical locations and also of different economic classifications; as such the selection is in line with the three basic criteria for choosing right comparators in comparative method research – i.e. being historical, spatial and counter factual Buckley and Chapman (1996), in reference to Buckley et al, (1988).

This work will focus on the upstream sector of the petroleum industry i.e. pertaining to the exploration and production operations of field or reservoir up to the wellbore. The two countries are characterized by the following:

1. Both countries have close economic ties following post-colonial relations
2. Both countries produce similar crude oil grades. The UK produces the Brent Crude 38^{o}API (which is the international bench mark for oil grades) and Nigeria produces the Bonny Light Crude – 37^{o}API.
3. Both countries are mature producing countries. The UK's first oil beginning commercial production in 1965 and Nigeria's production beginning in 1956.
4. Despite this connection, both countries operate different petroleum tax structures and differing petroleum policies
5. The United Kingdom is classed as a developed economy, a mature producer with a stable environment, whilst Nigeria is a developing economy, and mature producer with a volatile environment.

1.3 Data Collection

This study will rely in the most part on secondary data. The dataset has been identified as annual crude oil production data and financial budget and annual reports for Nigeria and the United Kingdom for the period of 1989 to 2013 compiled and released by secondary sources such as Department of Energy & Climate Change UK (DECC), HM Revenue and Customs UK (HMRC), International Monetary Fund (IMF), US Energy Information Agency (USA EIA), Nigeria Federal Inland Revenue Service (FIRS) and Nigeria Department of Petroleum Resources (DPR). Annual data is used to alleviate the data confidentiality and availability issues, and to circumvent documentation inefficiencies in Nigeria. The use of secondary data sources will allow for easier access to data, and allow for a wider range of coverage.

2 CHAPTER 2: LITERATURE REVIEW

2.1 Existing Studies on Oil Production Modelling

2.1.1 Oil Price and Oil Production Interaction

Oil price modelling in relation to crude oil supply modelling has long been a popular subject of study which has appealed to economics scholars. Research has continued to strive to understand and quantify the factors which drive oil price, thereby making it a popular subject matter within the energy economics realm.

Hasab (1968) notes the oil market as being different from that of any other commodity, both in its structure and in its price mechanism. He states that the two reasons for this are (1) the fact that in the extraction and processing of crude oil and natural gas, monopoly capitalism has developed a special form of the division of labour with the oil and natural gas deposits being generally situated in developing countries, and the processing industries and trading companies concentrated in the advanced capitalist countries; (2) the oil industry with its vertical structure is one of the most highly monopolized industries in the world, citing the Organization of Petroleum Exporting Countries (OPEC) petroleum reserves and production in comparison to its consumption in comparison to the rest of the world. Such a description sets the stage for the research into the production of this particular energy resource and its relationship with the world market scene.

The interrelationship between oil price and oil production and the factors which affect them have been the subject of a long running debate in the academic and energy market arenas. Suarez (1990) conducts a study on the long term evolution of oil prices from 1860 – 1987, which concludes that oil price behaviour and stability will be maintained to the extent to which certain equilibrium is reached between importer, exporter and multinational companies. The study notes that oil price stability depends basically on the relationship of forces between importer developed countries and multinational oil corporations on the one hand, producer developing countries on the other, together with the rate of technological development both at the oil harnessing and production level and the utilization level.

Stevens (1995) approaches the subject or oil price and production relationship from a theoretical point of view. Stevens (1994) provides a theoretical framework to explain the determination of oil prices from 1945 – 1995 based on supply and demand curve interactions. The study notes a change in oil price behaviour from 1980 when the market became more competitive, transaction more transparent and information increased; and in 1986 when oil began to be traded as a commodity which introduces additional determinant factors on benchmark crudes e.g. North Sea maintenance programmes influence on Brent crude and pipeline capacity constraints effect West Texas Intermediate (WTI) crude production.

Samii (1987) investigates OPEC control and impact on oil price as a result of their producing power. The study shows the interrelationship between oil price and production concluding that production flexibility and oil price flexibility are required for a more stable market for both producers and importers. Other studies investigating oil price behaviour in relation to production include Marquez and Pauly (1984); and Cremer and Weitzman (1976).

More recent studies such as Beirne et al (2013) discuss the impact of a country's economic growth on oil price over a period. Beirne et al (2013) conducts a quantitative analysis which assessed the impact of China's growth on oil price at a country aggregate level. The study concludes that China's excessive growth has a significant impact on oil price, such that it adds a premium to the price of oil which increases over time. This impact is captured by quantitatively measuring the China-factor on oil price over the period which measures the significant relationship between the two.

Henriques and Sardorsky (2011) discuss volatility of the oil market and its effect on strategic investment on a company scale. The study highlights oil price volatility as an important topic because on the production side, oil is an essential input into the production of most goods and services. While most companies do not consume crude oil, they do consume petroleum products such as gasoline, jet fuel and heating oil, which are all derived from crude oil; and as such the prices of these petroleum products move in line with the price of oil. The results of the empirical study find that in periods of high volatility, there is increased investment delay, though the long term effect changes investment behaviour as the effect of not growing the company becomes more pronounced.

Devlin and Titman (2015) discuss the oil price risk in developing countries. The study highlights oil price fluctuation from month to month to be caused by temporary changes in global economic and political conditions that affect supply and demand for oil. For example, political problems causing temporary production disruptions in a country which causing a rise in oil price, and the subsequent resolution of the issue or production increase by another country causing it to fall again. The study discusses that the use of financial instruments in developing producer countries to regulate oil price behaviour is lacking, mainly as a result of poor government coordination, and status quo matters of capitalizing on high revenue and maintaining political support instead of implementing what may be best for the economy in the long run.

Oil price also remains important even from an importer country point of view. Studies such as Gokmenoglu et al (2015) investigate the oil price effect on a country scale where the study tests for a long run relationship between industrial production and oil price in Turkey. The results show there is a significant long run relationship between oil price and industrial production, and advocate for further studies.

Studies aiming to further understand the interrelationship between oil price and production include Horne (2004) which examines what would be OPEC's optimal crude oil price given its production capabilities and overriding intention to manage production in order to maintain stable and suitably high oil prices with high export revenue yield. Ratti and Vespignani (2015) carry out an econometric study of OPEC and non-OPEC production over quarterly time periods, in which it also builds a predictive economic model. Cologni and Manera (2014) investigate the economic determinants of oil production in selected small oil exporting countries using the method of theoretical and empirical analysis. The study notes that the output behaviours differ by country, with some showing oil price to have no significant effect on their production output volumes, while some show only show evidence of impact on production after a lagged period.

Bentley and Bentley (2015) discuss oil price with oil production and reserve constraints. The study investigates how oil supply and reserve quantities have affected oil price over time, and concludes that historically oil prices have been driven primarily by the management of the potential oversupply of oil. The study also highlights that oil reserves have a significant impact on oil price dependent on the conservatism of categorisation of reserves as either possible or probable. This categorization in addition to policy changes within the oil industry, government and government ministries are pertinent to understanding and managing the effect of oil production on oil price.

Although many other studies such as Adelman and Watkins (2005); Almeida and Silva (2009); Prat and Uctum (2011); Aleksandrov et al (2013); D'Ecclesia (2014); Wu and Zhang (2014); and Kilian and Lee (2014) continue to investigate oil price behaviour and the factors that influence it, there would seem to be a much more limited focus on the reverse effect oil price has had on production. This is where this thesis differs from other existing studies. The main focus of this proposed study is on oil production determinants i.e. economic and physical (reserves). It expands the oil price modelling study carried out by Bentley and Bentley (2015) by approaching it from a reversed perspective. It will instead investigate the effect oil price on oil production models with reserve constraints; further playing on the already established endogenous relationship between the two as highlighted by the studies above.

Production models are usually defined by production functions which represent the relationship between and output and combination of input factors. General and specialized production models can be described and estimated using a variety of functions and theories, such as the Cobb-Douglas production function, the Hotelling theory and Hubbert production theory; and these production models are constrained by various factors such as price, cost, reserves, labour and interest rate.

2.1.2 Hotelling Theory

The Harold Hotelling (1931) principle is used especially in the case of production of an exhaustible resource, which is crude oil in this instance. It proposes that production should move with oil price, thereby controlling price increase and decrease. The theory is often used in relation to the Hotelling Valuation Principle by Miller and Upton (1985a, b) which states that the Hotelling principle relies on the fact that the value of the reserve is primarily dependent on the price (Jamal and Crain, 1998). The whole theory therefore links total production, reserves and oil price in an effort to understand oil price behaviour. Studies such as Thompson (2001); Lin and Wagner (2007); and Arora and Tanner (2013) test the theory to see whether production has an effect on oil price and whether oil prices do indeed respond to interest rates. The studies conclude in support of the theory, though they note that the theory may not capture all factors such as technology and stock (reserves) and actual production practioner's preference. Further literature review of the expansion of this theory in relation to the context of this study is discussed in the following chapters.

The Hotelling theory is often applied to general production models without particular emphasis on a discipline or sector. Guo et al (2015) extends the Hotelling production model to include production technology and labour input. The study highlights the significant effect that production technology and labour (which is measured using minimum wage) have on production and therefore productivity within two firms with differing location factors.

Lin and Wagner (2007) expand the Hotelling principle of optimal production of a non-renewable resource in relation to stock price effect and technological progress. The study is carried out on 14 minerals over a period of 35 years, and it concludes that technology is a very significant factor in driving resource price down, which is in line with the Hotelling theory though with modifications to the basic theory.

Other studies like Young and Ryan (1996) incorporate a risk factor into the Hotelling production model. Production models are developed for non-hydrocarbon minerals and attempt to adjust for risk effect. The results of the study present measured success as the models fail to capture the risk variance over time at an industry level.

Crude oil production in relation to oil prices is examined in Harold Hotelling's (1931) research in which he proposes that oil production volume should move in response to oil price. In his study Hotelling theorizes to manipulate optimal price of an exhaustible resource in relation to its scarcity (production) and growth of interest rate. The theory states that the price of a depleting resource like conventional oil should rise over time at the same rate as interest rate because its value should increase as the reserves are exhausted. This rule, which forms the basis for economic theory of non-renewable resources, is based on the assumptions of an efficient market in which crude oil is regarded as exhaustible.

With respect to the use of the Hotelling production theory in the oil and gas industry, Reynolds (2013) investigates the uncertainty of natural resource economics using the Hotelling theory. The study discusses the shortfall of the Hotelling principle citing the inappropriate use to which it is being applied when it comes to oil production. It highlights the limits of the Hotelling theory in accounting for sunk costs regardless of future oil production; and also notes that though the Hotelling rule may be useful for production modelling of individual resource mines and regional assets such as water, it is not applicable to oil, and the way it is currently being produced. The study concludes by calling for collaboration between economists and engineers.

Despite the popularity of the Hotelling theory amongst economists, studies such as Pindyck (1976); Lin (2008a); Lin et al (2008); and Lin and Wagner (2007) have also felt the need to expand on the Hotelling theory to better capture real world oil market. Though their research tends towards agreement with the theory, they expand their model analysis to include factors such as technological progress and competitive market behaviour and thereby develop augmented models of the Hotelling theory. Other studies such as Reynolds and Baek (2012) are not so much in agreement however. They analyze the relationship between oil production and oil price using the Hotelling theory as a platform for investigation; and their study highlights the determinants of oil price to include marginal cost of extraction, the backstop price of the next best alternative, demand, reserves and interest rates. However, further expansion of the Hotelling theory highlights a limitation in the theory which proposes that when interest rates begin increasing faster than oil price, production should decrease, such that this in turn should drive oil price up. Reynolds and Baek (2012) identify one of the inherent problems with using the Hotelling theory where oil must be discovered before it can be extracted to be that – in the real world, no one knows ahead of time the volume of reserves there are, or where they are and therefore cannot plan when to find and use these reserves in the pricing model.

Further understanding of the relationship between crude oil production and oil price on a country aggregate level include studies such as Bandyopadhyay (2008); Murakami (1976); Watkins and Adelman (1995); Cologni & Manera (2014); Luther (1996) and Chamarran (2002). They highlight the need to account for even more extraneous factors such as OPEC membership, market behaviour, and political stability. From these studies one may conclude that the Hotelling cannot fully explain production behaviour without capturing all of the other elements highlighted above.

The scenarios discussed by the Devlin and Titman (2015) bring OPEC contribution to the world oil price market into the forefront. This is firstly, because all of its member countries are classed as developing economies by the UN, and secondly because they account for over 70 percent or oil that is traded in the world oil marketed, and the hold over 60 percent of the world oil reserves deposits and are therefore crucial to future world oil supply.

Hubbert production modelling on a country and regional or group scale such as OPEC in relation to oil price, production and reserves is investigated by numerous studies such as Bandyopadhyay (2008); Filis et al (2011); Lin et al (2015) and Hochman and Zilberman (2015). They investigate the past effect country and regional oil production have had on oil prices with particular focus on OPEC production and its behaviour with the global oil price market. The studies all agree that OPEC production has been instrumental in regulating and global oil market, and though its current level of control seems to be going through some changes; its collective influence on the global market remains significant.

2.1.3 Hubbert Production Theory

The Hubbert peak theory which was first proposed by Marion King Hubbert stipulates that the production curve is bell-shaped and symmetrical. In addition to understanding production behaviour, the production model is often used with the intent to forecast future production. Works which apply the Hubbert production model include Bartlett (2000); Gallagher (2011); Jackobsson (2012); Orbach (2012); Saraiva (2014); Reynolds (2014); Bardi (2015); Jaromir et al (2015); Jianzhou et al (2015); Ebrahimi et al (2015) and Guangfei (2015).

The Hubbert (1953) and (1956) peak oil theory predicts that the regional oil production and world oil production would follow a bell shaped curve. The theory predicts that the world oil production is presently in decline having reached and passed its peak, giving rise to various studies which aim to investigate and forecast production profiles and decline curves of regions and indeed the world. This economic theory which is constrained by geology being reservoir volume ties into Arps' (1945) engineering production profile analysis method which utilizes reserve volumes to capture production decline curves at field and regional levels. Both the Hubbert and the Engineering decline curve theories predict depletion rates.

Giraud (2012) explores the Hubbert theory as a forecasting technique in modelling production output. The study examines the use of the Hubbert theory to forecast production and warns that the linearized version of the Hubbert curve should not be applied to forecast the ultimate reserves until or unless the production peak has already occurred. Following this it can then be used to model depletion rates in relation to reserve constraints.

Brandt (2006) tests the asymmetry of the Hubbert production curve for the United States on a state, regional, national and international level by generating and measuring the quality of best-fit curves to production profiles. The results of the study indicate that asymmetry in regional production is common and should therefore be expected. The results advocate that sharp production peaks can occur, and they conclude that with regards to using the Hubbert theory as a forecasting method, other curve shapes will need to be allowed for. They also state that a more inclusive Hubbert method should be developed whereby it includes social, economic and environmental factors.

Brandt et al (2010) further expands the Hubbert oil production model theory to further examine the power of the Hubbert model to explain and predict production and depletion. The study concludes to highlight that the simple mathematical Hubbert model is impractical, as it does not capture political or economic factors which eventually affect production peak dates. It also calls for increased integration between the economic and physical factors affecting oil production, stating that more detailed mechanistic models exhibit greater fidelity in explaining and reproducing historical production, therefore making them more useful for near term predictions. The physical factors are the petroleum reservoir properties which describe rock properties and flow behaviour of the reservoir fluids between the reservoir layers.

Others studies like Gallagher (2011) stick to the mathematical curve presented by Hubbert, though with emphasis on the cumulative logistic curve and its fit to actual production profiles. The results of the study show that this method is able to explain oil production behaviour and even capture production swings which may occur for numerous reasons. Thereby capturing a more inclusive curve over the production lifetime

Szklo et al (2007) discuss forecasted oil production in Brazil using the Hubbert model. The study investigates why the Hubbert theory explains Brazil production better than the Hotelling theory. It investigates how the definition of oil reserves affect production modelling using different reserve values to predict peak and depletion using the Hubbert theory. The results highlight the importance of accurate assessment of oil resources, and the effect of pre-set recovery factor targets for certain regions on the overall shape of the bell curve and depletion forecast given the reserve constraints.

Studies like Brandt, 2007; Reynolds, 1999 and Bardi, 2005, also analyse the expected bell curve symmetry which the original Hubbert theory proposes, and discuss that there are various factors which might cause a shift in the shape of the expected curve, and in the case of Ebrahimi and Ghasabani (2015), multiple peaks might even be observed in production profiles. Though these studies are in agreement with the Hubbert theory in particular instances, they do expand on the theory to produce extended results. Reynolds Kolodziej, 2007; Reynolds and Pippenger 2009 show that there can be variations to the curve and that not all regions may show evidence of support for the theory.

Chavez-Rodriguez et al (2015) carries out a study on oil production in Peru, in which a Hubbert model is applied using reserve constraints which vary in how conservative they are. The results show how aggregate production can exhibit a multi-cycle curve, which is an expansion of the standard Hubbert single-peak curve.

Bartlett (2000) carries out a quantitative analysis of Hubbert style curves on world U.S. and world oil production using an Excel template. Just like Bartlett (2000), this proposed study will carry out a similar quantitative analysis, using Excel to describe and forecast a Gaussian best fit curve constrained by ultimate oil recover over the 25-year period for Nigeria and the United Kingdom.

All of these apply the Harold Hubbert's peak model to aggregate oil production models with the objective of understanding how bell curve theories apply in their regional areas of investigation. The studies focus on application of the Hubbert theory in selected countries such as Peru, Brazil, and groups such as OPEC; and they all conclude that in order for the theory to better explain oil production, additional factors such as total ultimate reserves, investment and technology will need to be accounted for.

2.1.4 *Engineering Decline Curve Theory*

Production decline curve analysis is a reservoir engineering empirical technique that extrapolates trends in the production data from oil and gas wells. The purpose of a decline analysis is to generate a forecast of future production rates and to determine the expected ultimate recoverable (EUR) reserves. The method measures reservoir boundary dominated production decline, and can only be used when the production profile begins to decline as a field reaches a boundary dominated production flow. In essence, production as a result of reservoir conditions must be evident for the application of this analysis for future production modelling. Typically, decline analysis is conducted on a plot of rate versus time or rate versus cumulative production for a single well or field, however this study will investigate on an aggregate level. Hook (2009) provides a descriptive analysis of reservoir engineering and modelling techniques which use the decline curve analysis for field application. The study shows the production principles applied in engineering production modelling from an engineering perspective. Decline curve analysis is a study which is routinely done on a well and field scale primarily within the oil industry disciplines of reservoir engineering, petroleum engineering and petroleum geoscience.

Khanamiri (2010) study describes decline curve analysis as the most widely used reserves and production estimation method by engineers and companies. It is a method which fits observed production rates of individual well, groups of well, reservoir or in the case of this study a country, to a mathematical function in order to predict performance of the future production by extrapolating the fitted decline function which is based on Arps (1945) mathematical equations. This engineering method of estimation has been built upon over the years by studies such as Fetkovich (1980); Carter (1985); Fetkovich et al (1987); Agarwal et al (1999) and Li and Horne (2001) and (2003).

Fetkovich (1973) carries out a decline curve analysis of oil production using type curves. The study expands on the standard engineering time-rate curve analysis extrapolations and simulations carried out within the industry, to incorporate boundary flow, reservoir drive mechanisms and reservoir types. The study successfully manages to define and a clear engineering tool with high diagnostic powers which better assist production modelling and reservoir development in the form of production decline curves.

Hook (2014) investigates depletion rate using decline rate analysis on a selection of fields. The study investigates depletion which is driven by production decline by tying into the Arps' (1945) theory which explains depletion rate behaviour in the decline phase of production, explained by decline curves. The study discusses the importance of capturing the effect of reservoir flow dynamics such as pressure, compressibility and reservoir flow rates dictated by porosity and permeability in the modelling of production output. The results show a strong correlation between decline rates and production rates, with high depletion rates correlating strongly with high decline rates on a field scale and regional scale.

Further building on the Arps and Fetkovich studies, Li and Horne (2006) carries out a study in which they attempt to predict and match production in reservoirs with different permeabilities. The results highlight the fact that exponential decline curve models tend to underestimate reserves and production rates while the harmonic model tends to over predict reservoir performance. The study takes into account reservoir characteristics such as fracturing, permeability, and measures the power of these decline models including an expanded Li and Horne model which includes even more phase flow properties. It concludes that this decline curve model which measures in between the exponential and hyperbolic, best describes production decline and volume predictions.

The decline curves created by Arps' (1945) are simply a plot of production versus time, in a semi-log or log-log plot. A review of the theory is conducted by the SPEE (2002) in which an overview of the decline curves is carried out and recommendations for utilization are made.

The studies discussed above are conducted on petroleum reservoirs and both oil and gas wells to further investigate reservoir boundary dominated flow periods which is what the decline curve theory is based on, to account for reservoir pressure behaviour such as parameters such as variable bottom hole pressure, increased pressure draw-down effect in the case of gas wells, fractured and water driven reservoirs; and the separation of transient and reservoir boundary dominated flow.

Although M.J Fetkovich's (1980) production type curve model study successfully expands the Arps (1945) traditional production model to include production throughout the full production to include transient flow and not just boundary dominated, this study will focus on the application of the Arps' production model which focuses on depletion, and ties into the Hubbert reserves constrained model for a more consistent oil production model comparison study.

2.1.5 Cobb-Douglas Production Theory

The Cobb-Douglas production function is the aggregate production function developed by Paul Douglas in 1927 and Charles Cobb in 1947 to estimate total production or real value of the output goods in various countries. The Cobb-Douglas production model utilizes the input factors of production namely labour, capital and productivity. This ability of the production model to estimate the function form as a linear relationship, allows for the input of other production factors thereby providing a solid starting point for this study proposes to develop a combination production model.

Kazmi (1972) outlines the assumptions of the Cobb-Douglas function to be that (a) there are constant returns to scale, (b) there prevails perfect competition in the economy and (c) the factors of production i.e. labour capital are substitutable in the process of production.

Studies such as Reder (1943); Herbert and Ferdinand (1963); Goldberger (1968); Wu (1975) and Charnes et al (1976) carry out studies in which they test and extend the production functions capabilities and limits. They result in varying levels of confidence in the Cobb-Douglas production function's ability to explain optimal production output. However, the function remains relevant to production modelling as it continues to be used in a variety of industries, though its use in the oil industry is uncommon as extensive literature review of existing works have shown.

Grubbstrom (1995) describes production modelling as the process of transforming one set of resources (inputs) into a second set. The basic Cobb-Douglas production function lends itself to including other factors such as technology, government policy, OPEC and infrastructure, which are described by their own input variables. Though studies such as Labini (1995) disagree with the model in its basic form, and call for the use of a dynamic relationship between the variables, unifying their exponents in order to account for technology, others such as Diamond et al (1980); Pendharkar et al (2008); Aiyar and Dalgaard (2009) and Reyes-Santias et al (2014) all apply the model in its basic form to a diverse number of industries, where they successfully carry out empirical analyses on production all defined using Cobb-Douglas production model.

Diamond et al (1980) examines optimal taxation, using a Cobb-Douglas example. The study incorporates labour uncertainties and consumption into the production function, highlighting these to be crucial to more structured and richer model.

Shahabuddin (1985) carries out a study into the application of the Cobb-Douglas function in agriculture. The study tests at a farm level whether the function should be used as indiscriminately as it is often used, and if technology input is well represented for a variety of agricultural outputs. The results show that the production function restrictions are indeed validated in some of the production outputs, while in some cases it is inconclusive; therefore, confirming that its indiscriminate use may not be appropriate.

Using this method as a vehicle, this study endeavours to quantify the effect of production theory independent variables on aggregate production output in selected country cases. In keeping with the function's assumptions outlined in Kazmi (1972), the Cobb-Douglas function allows us to successfully include (1) a reserve constraint which is implicit in both the Hubbert model and Engineering decline curve model theories, which describes a bell curve which peaks at maximum production capacity following which it begins an inevitable decline; and (2) the oil price constraint inherent in the Hotelling Principle which dictates the oil price movement dependent on production and interest rates. Thus the Cobb-Douglas model allows this study to capture multiple production models in a combination model that provides better understanding of oil production and the factors that drive it.

2.2 Conclusions

Though many oil price modelling studies abound, there would seem to be not as many which discuss the backward relationship between production and oil price i.e. how oil price affects oil production, and if there is indeed a significant relationship between these.

Comparative country case analyses of production volume and economic factors in petroleum economics, includes studies such as Blake et al, 2006; Kolstad and Soreide, 2009; Toft and Duero, 2011; Zhao et al, 2012 and Hunter, 2014. These studies focus on developed and developing economies on an individual country basis, and in comparison, with each other. The application of the comparative method sheds light on the significance of the unique characteristics of the comparators, providing additional dimension to the quantitative research. The following chapters follow the example laid by these works in pursuing its objective of a comparative quantitative analysis.

With regards to the subject of understanding, managing, and even predicting oil production at a country aggregate level, research studies have analyzed and applied various production theories and models to country production profiles with varying results. The literature review of oil production volumes and economic theories presents the much discussed Harold Hotelling's 1931 production model theory and Hubbert (1953) and (1956) peak oil production theory as a basis for understanding oil supply behaviour. A review of the available research highlights studies which analyze and discuss the level of adherence of various country production profiles to variations of the Hubbert and Hotelling models; though with measured and sometimes inconclusive results.

Though extensive review of available research has indicated that research which includes the combination of production and engineering theories is scarce, few studies such as Jakobsson et al (2012) and Sorrell et al (2012) do attempt to build models which incorporate both economic and engineering perspectives. This study however builds and expands on this further by carrying out a comparative analysis and quantitatively measuring the level of adherence to the proposed theories of the selected country comparators – United Kingdom and Nigeria; and the factors which seem to have the most effect on these results.

In keeping with the above mentioned research studies using annual data, the next few chapters will investigate adherence of annual oil production volumes in the United Kingdom and Nigeria to these economic production theories.

3 CHAPTER 3: Does the Hotelling Model explain change in Oil Production?

3.1 Introduction

Our investigation of oil production starts by looking at the effect of oil price on production. Oil prices are a global phenomenon and they are expected to affect oil production directly.

Nigeria is a developing economy which has the largest oil reserves and is the top oil producer in Africa. It continues to be of interest to the oil and gas exploration and production (E&P) industry, despite its socio-economic and political volatilities. It is one of the twelve members of the Organization of Petroleum Exporting Countries (OPEC) which accounts for 40% of world oil production and 72.6% percent of world oil reserves. In addition to this, its production is regulated by OPEC whose mandate is to ensure stabilization of the oil markets via the regulation of the production volumes if its member states.

The United Kingdom on the other hand is a developed economy, and is the largest oil producer in the European Union. Its crude, the Brent crude oil 38^{o}API, is the international benchmark for petroleum oil grades. Both of these countries are mature producers depend on oil for export revenue, so a sound understanding of the factors that affect their production volumes is pertinent to control and optimization

It is evident that oil prices affect demand and so supply at an aggregate level of the world (Biern et al, 2013). However, it is unclear about if an individual (country) production responds to a change in oil price since the price is an economic phenomenon of demand and supply at the world aggregate level. This motivates this chapter which will investigate the relationship and effect of crude oil price and production cost on production volumes in both countries over a 25-year period. It will investigate whether we can hold to the theory according the Harold Hotelling (1931) which relates crude oil production to oil price. A quantitative comparative study will be carried out to analyze whether changes in oil prices affected production volumes in these two countries in line with the Hotelling rule; and how significantly is this impact. Other extraneous factors such as operating cost, production quota systems and interest rates (bond yield rates) and their impact alongside oil price on output volumes will be investigated for a more robust study.

The probable availability of national oil and gas production data in public domains and held at public institutions, marries well with this Case Study approach as this provides access to a wide range of data information from multiple sources. Financial reports are also available to the public as they are required by national and international laws to be released. This will most likely reduce exposure to non-disclosure and company internal policy issues which may impact the objectives of this research. Each of the two countries will be treated as a whole with published reports and primary data providing enough empirical data for a thorough quantitative analysis. This comparative method of study is validated by (Eisenhardt, 1989 and Buckley et al, 1996).

The method of data analysis for this study will be in keeping with the statistical mode of analysis described by Hartwig and Dearing (1982) and Lewis-Beck (1995), which facilitates the analysis of the relationship between variables instead of a confirmatory mode of analysis which may be less open to a wider range of alternative explanations and randomness. This study will measure how oil price relates to country oil production volumes using a time a series data estimation approach. The proposed research aims to shed light on the magnitude and impact of oil price relative to interest rates on production and revenue generated. Statistical methods utilized in this research will be undertaken in an attempt to quantify this impact. Multivariate regression analysis has been applied, which has the advantage of bringing in more information to bear on a specific outcome. This allows one to take into account the continuing relationship among several variables, which is especially valuable in observational studies where total control is not possible. With this method, one is able to study the effects of several variables acting simultaneously instead of singly.

Bowley (1903) stipulates that in analysis of data, quantities shall be estimated in relation to corresponding estimates for previous years, thereby dealing with a series. i.e. statistical study should be considered in the light given by corresponding estimates for previous years. Additionally, time-series analysis will assist to possibly develop dynamic causal effects, develop forecast and/or regression models, should the scope of this study extend to this purpose. Bowley (1903); Johnson (1939); Fisher (1932) and Chen et al (1990) provide guidelines for series analysis, variance and covariance, small sample techniques and missing data treatment, providing ample support for this proposed research. These guidelines include that – the totals (in the case of this study – the dependable variable) shall be homogenous in that quality which concerns the argument. The study should endeavour to conceive the ideal measurement necessary to support each deduction, whilst rejecting estimates that do not necessarily give the same view as the ideal measurement. It is advised that with the development of small sample techniques, there is likely to be a return to the more accurate and individual studies of the past. And finally to avoid difficulties caused by missing data, focus should be shifted to time periods in which data is available for all the variables, despite the fact that this may shorten the data range

3.2 Existing Studies and Arguments

This chapter carries out a literature review into the existing body of academic research on oil production modelling, the impact of oil price, operating cost and OPEC on world oil production volumes. Oil price remains an ever topical subject as a result of the significant direct and trickle effect it has on the revenue streams of leading economies. This study concerns itself mainly with the production output volumes. What has been the effect of oil price on Nigeria and UK’s production over the last 25 years? Has this been in line with the Hotelling model? Does the empirical analysis of both countries show a relationship with operation costs, and persistent high or low oil prices? Or are there perhaps other factors influencing production in both countries, and to what degree?

Crude oil production in relation to oil prices has been examined in Harold Hotelling's (1931) research in which he proposes that oil production volume should move in response to oil price. In his study Hotelling theorizes to manipulate optimal price of an exhaustible resource in relation to its scarcity (production) and growth of interest rate. The theory states that the price of a depleting resource like conventional oil should rise over time at the same rate as interest rate because its value should increase as the reserves are exhausted. This rule, which forms the basis for economic theory of non-renewable resources, is based on the assumptions of an efficient market in which crude oil is regarded as exhaustible. It also assumes that the five main determinants of oil price are – marginal cost of extraction, the backstop price of the next best alternative, demand, reserves and interest rates (Reynolds and Baek, 2012). The study discusses that when interest rates begin increasing faster than oil price, production should decrease, such that this in turn drives oil price up. In other words, when oil prices begin to fall we expect production output to be controlled. See figures 2 showing other factors which may decrease production such as political disruption in the case of Nigeria and declining investment in the United Kingdom. Smith (2013) conducts a review of the research methods and models used in the study of extractive resource taxation. He notes in his theoretical study that Hotelling's analysis is predicated on the assumption that each owner will strive to maximize the net present value of his resource. As such, he concludes that the theory is a statement of the aggregate effects of those underlying optimization efforts than a recipe for optimal development of a particular resource deposit.

Further studies to expand this theory such that it better mirrors the real oil world market include Pindyck (1976); Lin (2008a); Lin et al (2008); and Lin and Wagner (2007). They research the impact of OPEC, technological progress and competitive market behaviours on the Hotelling model, concluding that the real oil market may not always fit the Hotelling model, so there is a need for the inclusion of these augmentations in the model.

Though the Hotelling rule which builds on Cournot's (1838) competitive markets research is a popular one amongst economists, more recent studies such as Lin (2008) and Reynolds & Baek (2012) are not in full agreement with the theory. The study by Cynthia Lin (2008) analyses further Pindyck's (1978) the expatiation on Hotelling's theory; which was expanded to include unlimited reserves. Cynthia Lin (2008) builds on this by using a more recent and updated dataset, in short intervals and sub-periods. The study concludes that Hotelling model will yield more realistic results if further augmentations such as capacity constraints, OPEC and non-OPEC models, crude oil quality differentials, are taken into account.

Reynolds and Baek (2012), identify one of the inherent problems with using the Hotelling theory where oil must be discovered before it can be extracted to be that – in the real world, no one knows ahead of time how much reserves there are or where they are and therefore cannot plan when to find and use these reserves in the pricing model. So in essence the Hotelling rule may not necessarily translate to country and company exploration and production activities. This conclusion is buttressed by studies such as Watkins (1992); Watkins and Adelman (1995). Lin (2009) notes that few empirical studies of the Hotelling model have been done in the context of world oil industry.

The assumption of an efficient market with perfect oligopoly as Cournot (1838) proposes may not also be realistic to the oil industry, given the existence of OPEC, Non-OPEC, National Oil Company and International Oil Company producers all with different market stakes and agendas. The fact that there is no fixed number of producing companies, that exploration and production operations are have been integrated, and that there is the existence of varying stakeholder proprietary ownership within each of these corporations, means that the Hotelling model becomes less clear cut in its explanation of the real world of oil and gas exploration and production business.

It remains therefore that there are other extraneous factors which affect which affect oil price. An example of some of these factors both on at world scale and country scale is depicted in the figures below. Watkins (1992) addresses this issue in his study of oil price and production, concluding that even with the addition of more realistic features, it is unclear if Hotelling's model explains the real world, therefore setting the premise for a more realistic investigation which includes set time frames.

See the figures 1 & 2 for a historical view of the world geo-political impact on crude oil price and volume trends Nigeria, and the effect of recent events on world oil price.

Figure 1 World Oil price trend and geopolitical impact

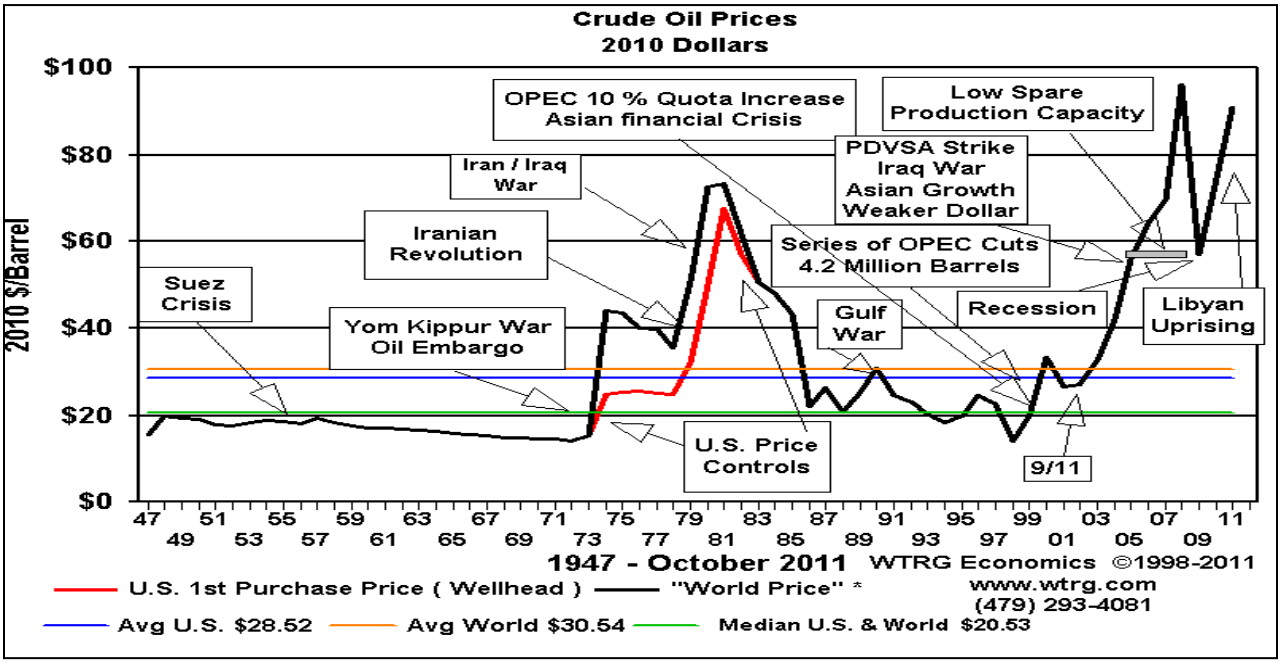
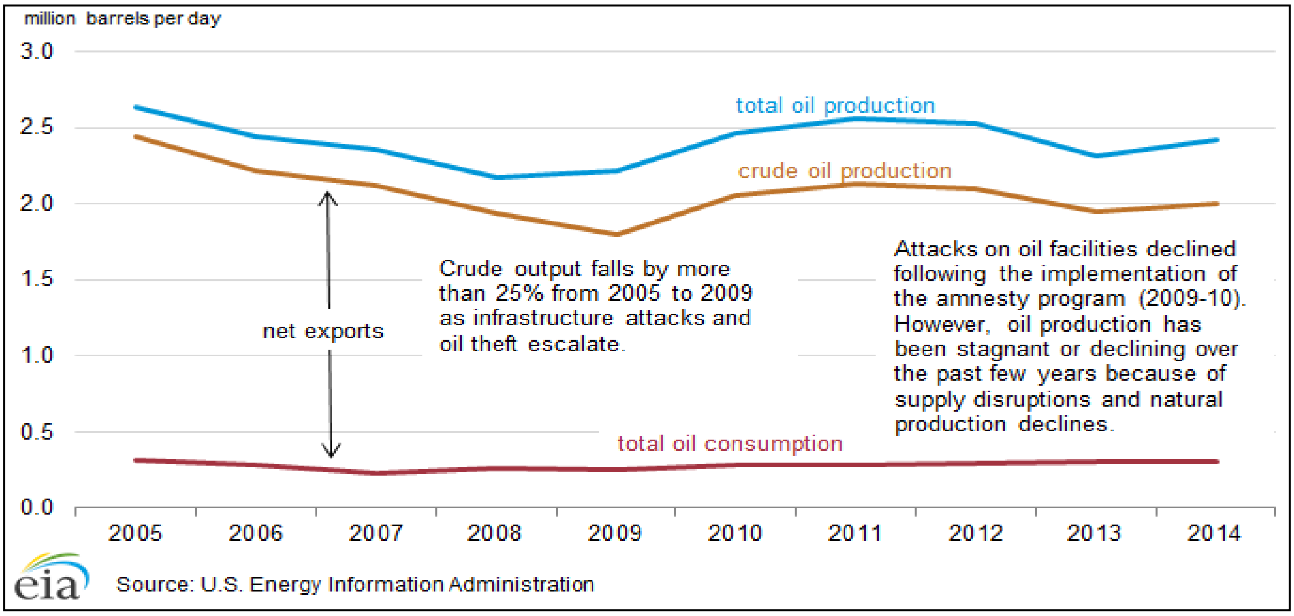


Figure 2 Nigeria's Production disruptions



Source: US EIA

Bandyopadhyay (2008) discusses the relationship between oil price and production volume output with regards to OPEC's price making influence. The study highlights the balancing act required to maintain rent seeking intentions and high oil prices for these export countries whilst controlling output volumes and spare capacity. He discusses the effect of these factors, highlights the opposing needs of importing developed countries and exporting underdeveloped countries, and concludes that oil price moves to the beat of a myriad factors, one of such being control or influence OPEC's factor. This however is not a control mechanism that will all ways work as various factions struggle to agree a regulated price band for optimal revenue.

Murakami (1976) simulates country crude oil production with focus on some OPEC countries. In it he includes some factors such economic policy and economic variables such as, GNP, demand and supply gap, oil price, industrial capacity, etc. He concludes that due to the effect of even more extraneous influences e.g. varying economic structure, it is best to model individual countries with their unique variables in order to view the effect of these on their production.

Devlin and Titman (2008) conduct a study of the managing oil price risk (due to price volatility) in developing exporting countries. They highlight the need for the savings funds to be set up by governments to deal with windfalls and revenue depletions due to production and price fluctuations, concluding that this can only be achieved with sound a fiscal framework, accountability and transparency. We note that Nigeria has a Petroleum Revenue Fund which was set up to for precisely this function. Over the years the country has tried with minimal success to use the fund to stabilize the economy following reductions in volume outputs and fluctuating oil prices as a result of socioeconomic volatilities and infrastructure weakness.

Finally, Cologni & Manera(2014); Luther (1996); Chamarran (2002) conclude that crude oil production is not necessarily related to Oil price and Operating cost and demand. They propose that further empirical research is required to investigate the effect of political and economic institutions on production output. Proposed variables include including variables such as political stability, foreign investment policies.

Prior to such an investigation, the next chapters analyse the impact oil price, operating cost and OPEC have on Nigeria and UK production. They will test the relationship between these variables against the basic Hotelling model within a competitive market. The comparative empirical investigation is carried out using a case study approach

This proposed work differs from others in that it will study the variable relationships in the reverse. The Hotelling theory and its further derivations aim to control oil price by varying production. However, this study aims to look at the effects of these variables *on* production in Nigeria and the United Kingdom over the last 25 years.

Again, the choice of these two representative countries is enforced by the similarities of their crude oil types which also attract similar prices in the oil market, in addition to other technical commonalities of field reservoir type, and reservoir pressure systems.

3.3 Data and Analysis

3.3.1 The Data

The table below consists of data over the period of 25 years used to define variables used in the regression analysis. It consists of mostly financial and oil production data associated with petroleum exploration and production in the United Kingdom and Nigeria. The data has been collected from public sources namely – *Department of Energy & Climate Change UK (DECC), HM Revenue and Customs UK (HMRC), International Monetary Fund (IMF), US Energy Information Agency (USA EIA), US Federal Reserve Bank and Nigeria National Petroleum Corporation (NNPC).* The study is framed within a 25-year period 1989 – 2013 which allows for final submissions and publication of data from public sources, and reduces data sensitivity issues.

Figure 3: Data: United Kingdom and Nigeria

Period: 1989 - 2013

Country Data and Units	Source
Exchange rate (GBP/USD)	Bank of England
Oil & Gas E&P Capital and Operating Expenditure (US dollar)	Department of Energy & Climate Change UK (DECC) and Nigeria National Petroleum Corporation (NNPC)
Bond yield Rate (percentage)	Federal Reserve Bank of St Louis
Oil production, barrels/day	US EIA
World oil price (US dollar/barrel)	US EIA, IMF
Oil reserves (barrels)	US EIA

Figure 3 Data: United Kingdom and Nigeria

3.3.2 Initial observation and overview of E&P operations

From the data it is evident that the UK's crude oil production is in steep decline, peaking at 980 million barrels. Over the last 13 years, production volumes are seen to fall by 65% with a 45% fall from 2006 to 2012. This is despite the overall steady rise in oil price from $18 to $111 per barrel over the 25-year period.

Production peaks in year 2000, despite low oil prices at under $30 per barrel. After which from 2002, it begins to decline steadily. Though crude prices almost double from 2009 to 2013, UK production volumes continue to fall. See figure 5 showing UK's production profile to be in steady decline. On observation this production volume decline is despite rising oil prices. UK operating cost continues to rise despite this declining production. High operating costs have led to postponement of activity and the stalling of projects – thus adding to the reduced production and ultimately reducing the attractiveness for future projects. Rising operating cost in the North Sea over the past decade is a known phenomenon. See appendix.

According to Mckinsey & Company, DECC, and Oil and Gas UK, rising operating cost of UK oil production is cause by the following reasons:

1. Maturity of the UK Continental Shelf basin. The basin having produced for over 50 years has reduced oil reserves and with increasing complexities within the reservoir as pressure continues to decline as a result of production
2. The geological and geographical locations of the reservoir assets require more complex and expensive reservoir management.
3. High cost of labour in the UK oil industry. The increasingly high cost of labour in the industry has no direct bearing on production level decline.
4. High cost of facilities maintenance and management

5. High level difficulty of engineering operations due to harsh terrain, again without recourse to production decline.
6. Increased requirement of specialist equipment and machinery
7. Increase in safety and environmental regulation with its attendant cost increase and without consideration for production levels.

Ref: McKinsey & Company, Oil & Gas Practice, June 2014; UK Department of Energy and Climate Change (DECC)

We also observe that UK gross revenue has not seen the same sort of sharp decline as it has in production volumes. The revenue is buoyed by very high oil prices which compensate for dwindling output. For this study, revenue is simply calculated as the product of oil price and production.

On initial observation of Nigeria's data, production volumes are regulated by OPEC, suggesting that Cournot-Nash's (1838) theory of competition within the oligopoly of the oil market does not look to be affirmed during this 25-year time frame.

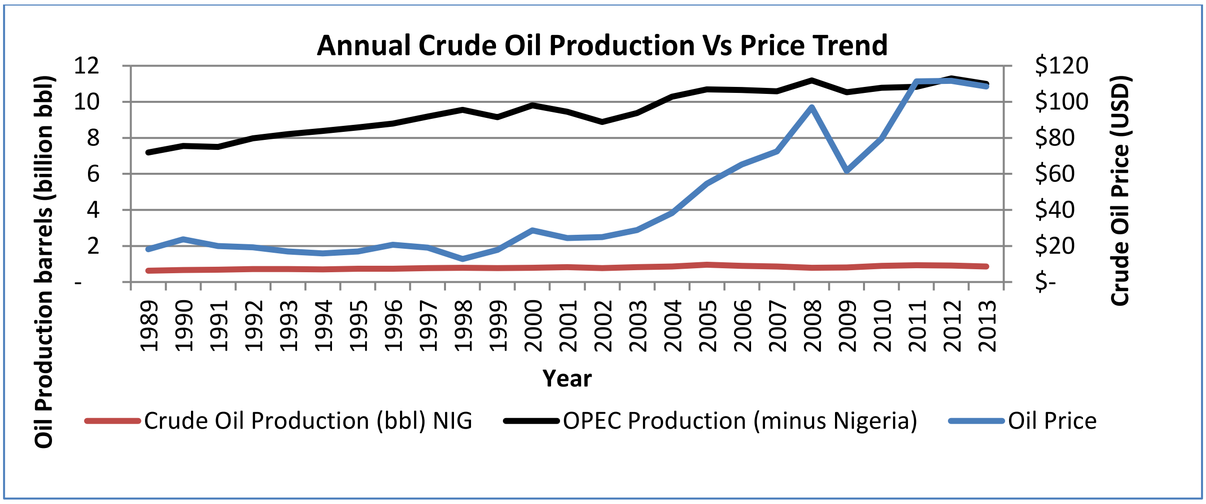


Figure 4: Nigeria and OPEC oil production versus oil price

Nigeria's production has increased over the last 25 years despite obvious sharp changes. It peaks in 2005 at 959 million barrels that year. The sharp changes in production trend are as a direct result of domestic political, social and economic factors within the country.

Despite this, Nigeria has managed to maintain production volumes at a high of above 700 million barrels per annum. Even with the production declines following political unrest and socioeconomic disturbances, Nigeria is still able to meet its production quota. (US EIA and BP Statistics Report 2013). We also see corresponding dips in production trend as oil price falls. Further quantitative analysis should buttress a collaborative production output relationship between Nigeria and OPEC.

The data also suggests that Nigeria's operating expenditure is not clearly driven by oil price. Available country OPEX data for the period of 2002 – 2012 shows no obvious relationship. From the available data, we see that Nigeria's operating expenditure is much lower that the UK. Operating expenditure has remained stable and low despite increased production levels and especially in comparison with the high levels in the UK.

It is important to note for the current and future studies that UK Production decline is attributed to the following reasons. (i) Reserves Depletion – whereby the recoverable resources have become exhausted (ii) Reduced production flow due to the physical limitations of the reservoir e.g. permeability and reservoir geology (iii) Reduced production due to reservoir pressure decline. As production continues pressure falls, and the UK continental shelf is a mature basin.
Source: McKinsey & Company, Oil & Gas Practice, June 2014; UK Department of Energy and Climate Change (DECC)

Therefore, the only way to alleviate this aggregate production decline is by expanding recoverable reserves which are still limited by physical extent of formation (Hook, 2009). This is fast becoming the overriding issue with the UK coupled with the increasing Operating Costs within the region.

3.4 Empirical study

In related research Mohn and Osmundsen (2006) conduct an econometric study of the oil and gas exploration economics of the Norwegian Continental Shelf using the above described analytical tools. In their study they build a multivariate model to test for long-term structural relations between drilling efforts and explanatory variables, and an error-correction model to capture short-term dynamics and data sluggishness. This two model approach allowed the study to separate between temporary and persistent (structural) effects.

Azhgaliyeva (2014) conducts an extensive econometric study of the effect of fiscal policy on oil revenue fund in Kazakhstan. This work presents a strong example of the vein in which this proposed study intends to proceed. Following co-integration and unit root tests (Engle and Granger, 1987), the study concludes with evidence that oil production and oil price have long-run relationship, but there is no statistically significant relationship between production and oil price in the short-run. The long term relationship is captured by the error correction term, and that in the short-run effect is captured by the coefficients of other explanatory variables. The research is in line with econometrics methods discussed in Johnson (1939). Other similar studies which are related to this proposed work include Michael and Hunka (1960); Powell (1999) Azhgaliyeva (2014); Iwayemi and Fowowe (2011); Kretzschmar and Kirchner (2009); Smith (2013)

It is important to establish the characteristics of the data in this time series, as this is often non-stationary. It is also important to know whether or not the variables have a unit root (Davidson and MacKinnon, 2004). In the case of this study however, due to the length of the time series, the risk of misleading results due to persistent time series is minimized, thus allowing for the use of ordinary least square regression.

In order to quantify the impact that the explanatory variables have had on production, an econometric model is specified. A revenue model and a production model are specified for each individual country. A linear regression model is set up with the production (Q) and revenue (Q^R) being the dependent variables and crude oil price (P), operating expenditure (X), and OPEC production (Q^E) as the explanatory variables with exogenous assumption.

According to Hotelling (1931) model that production output of oil is determined by the return of investment in oil relative to investment return in alternative assets, we write Hotelling theory in a specific form as follows:

$$Q_t = \alpha + \beta P_t + \theta X_t + \lambda Q_t^E \ldots + \varepsilon_t \quad (1)$$

or

$$Q_t^R = \alpha + \beta P_t + \theta X_t + \lambda Q_t^E \ldots + \varepsilon_i \quad (2)$$

Where Q_t is oil output of a country at year *t*, P_t is world oil price that affects the investment return in oil and X_t is an investment return in financial asset, such as corporate bond. The model in (1) and (2) also takes into account the constraint of OPEC production in each country denoted by Q_t^E in (1) and (2). Following the premise for country models mentioned in the above sections, a model for each country is developed in order to identify country specific differences. Annual data is used for the purposes of this study. The models set oil production as a function of oil price, operating cost, bond yield rates and OPEC production. The same is done for the revenue model; however, OPEC variable does not exist for the UK model. The above equations describe the simple model, where Q is the production volume and Q^R is the gross revenue derived by simply multiplying production volume and oil price. P_t and X_t, explanatory variables which need to be defined are the oil price and expenditure of the oil industry at a country aggregate level, and ε is the error term. The term Q^E refers to OPEC production (minus Nigeria's production) which allows us to capture the relationship between the two variables and measure their collaborative relationship if any, and r is the corporate bond yield rate. This linear model captures the spread of the data (figure 5).

Figure 5: UK versus Nigeria oil production and revenue trends

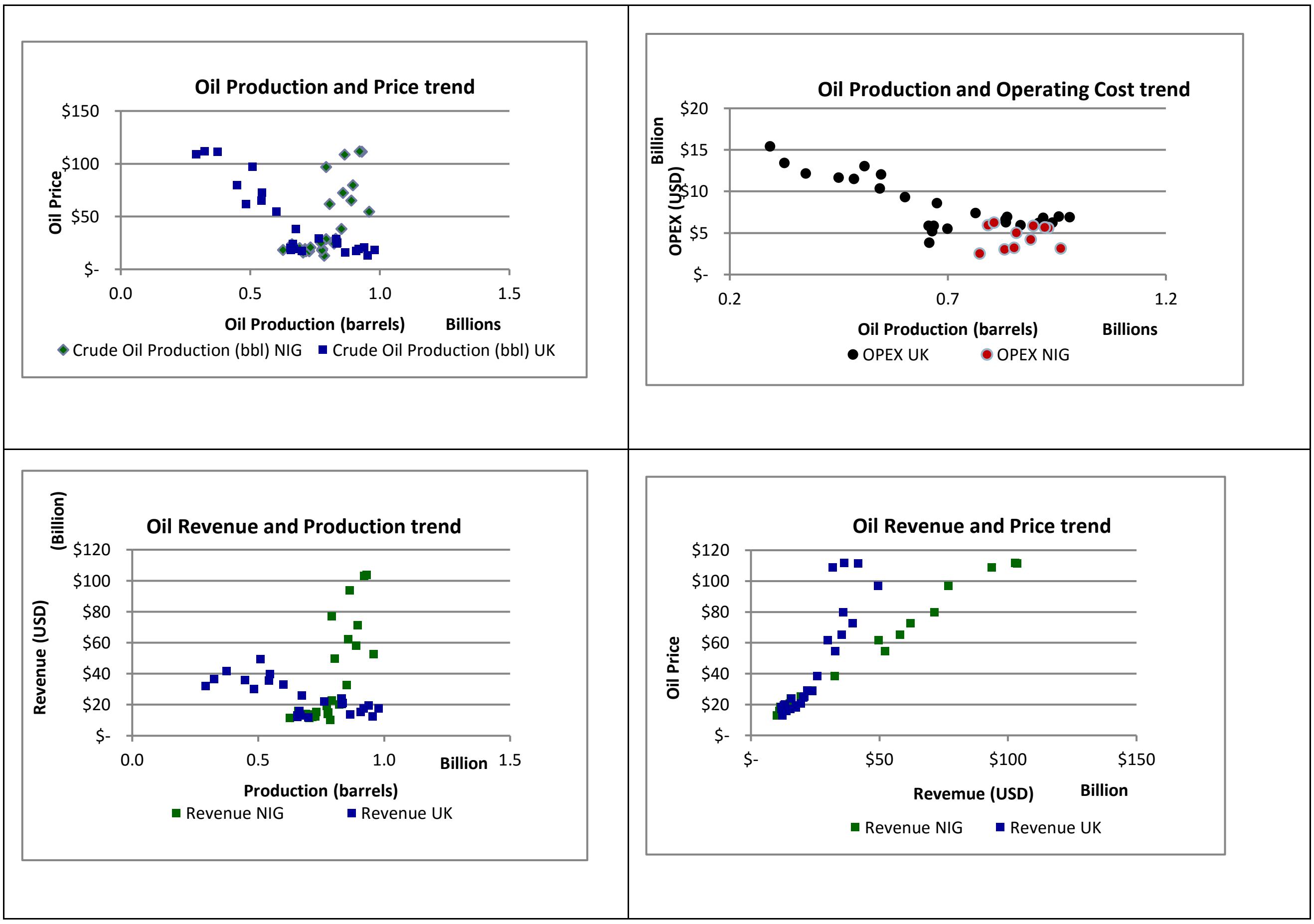


Table of Variables and Results of ADF Unit Root test

Explanatory Variables
Oil Price$_t$
OPEC (excl. Nigeria)$_t$
Country Production
Corporate bond yield rate$_t$
Δ Corporate bond yield rate$_t$/ΔOil price$_t$
Operating Cost

Variables	ADF (t)	p value	Stationary	Level
LnP	-4.68	0.0013	yes	1%
Lni	-5.17	0.0004	yes	1%
LnQnig	-5.70	0.0002	yes	1%
LnQuk	-3.14	0.0373	yes	5%
CapexNig	-4.20	0.0135	yes	5%
CapexUK	-3.63	0.0132	yes	5%
OpexNig	-2.96	0.0805	yes	5%
OpexUK	-2.89	0.0621	yes	5%
Cost per bbl UK	0.11	0.96	*no*	
cost per bbl NIG	-3.59	0.04	yes	5%

In order to quantify oil price effect, lagged variables are used to incorporate feedback of oil price on production over time. The appropriate number of lags is determined by the data to be a maximum of two. Annual data of 25 years is used in this analysis in accordance with Oil industry practice for both engineering and economic analysis. The spurious correlation is tested for and confirmed as non-spurious by the stationarity tests results shown above.

3.5 Results and Discussion

In accordance with the Hotelling theory, we expect that when interest rate goes up production should fall in order to drive price up. The test results (Tables 1 and 2) show that the Nigeria's oil production and revenue models support the Hotelling theory. We measure a significant negative relationship in the two models which corroborates the Hotelling theory. In the revenue measurement, there is a statistically significant negative relationship with bond yield rate and change in price. The corporate bond yield rate captures the oil market interest rates, which correspond with real world oil price. An increase in the bond yield rate shows a corresponding decrease in revenue and this is in line with Hotelling's rule. There is also a corresponding decrease in production which is required to maintain high oil prices and this also is in support of the Hotelling theory stipulations. The results of the Nigeria production model also demonstrate evidence of collaboration with OPEC production, showing a significant positive relationship between the two variables which is in tandem with the OPEC regulatory quota system. The significant negative coefficient of OPEC production when oil price is lagged further demonstrates this expectation that OPEC will reduce its production and therefore Nigeria's in the event of persistently low oil prices.

The UK on the other hand shows a weak support of the Hotelling theory (Tables 3 and 4). There is evidence of support of this from the production volume measurements. The revenue model however shows a positive relationship with interest rates and production, and therefore does not support the Hotelling theory. Ageing reservoirs and depletion (discussed in Chapter 4) have affected the UK's oil production over the years, resulting in production declines and widespread outages as a result of these technical problems. This is further supported by the results of the analysis (Table 3) which shows a negative oil price relationship with production despite increase in oil price. It is important to note that increases in tax rates in the oil and gas sector coupled with the technical issues have contributed to the sharp declines in UK oil production (US EIA, 2015). Despite large declines in production over the last few years, the UK remains one of the largest petroleum producers and exporters in Europe. Even though the UK is now a net importer of all oil and gas, it is still one of the top oil exporters in Europe, with the largest importers of its crude export being Netherlands and Germany (at over 55%). Though the UK government does not hold ownership interest in oil production, the oil and gas sector accounts for about 25% of its corporate tax receipts. So as a result of high oil prices, revenue has remained buoyed, and high tax rates are the mechanism through which the government maximizes its net revenue despite the dwindling oil output.

These above findings show evidence of support of Ratti and Vespignani (2015) who investigate OPEC and non-OPEC oil production and real oil price; in which they find that the negative effect on real oil price on non-OPEC oil production is larger in absolute value than the positive effect on OPEC oil production growth. Therefore, OPEC production contributes to real oil price while non-OPEC production does not. The results also corroborate evidence of collusion discussed by Smith (2009) within OPEC. We see evidence of a strong statistical positive relationship between OPEC production and Nigeria's production. This so is despite conflicting interests amongst member countries, which Smith (2009) also highlights. In the attempt to test the Hotelling theory, we have inadvertently generated support of evidence of collusion.

Table 1: Empirical results of Oil Production – Nigeria – 1989 – 2013

Dependant Variable: *Country Oil Prod.$_t$*	(1)	(2)	(3)	(4)	(5)	(6)
Oil Price$_t$	-0.000	-0.000				
	(-0.46)	(-0.14)				
OPEC(excl. Nigeria)$_t$	0.791***	0.756*	0.432**	0.379**	0.716***	0.706**
	(8.36)	(2.10)	(2.15)	(2.25)	(7.67)	(2.37)
Oil Price$_{t-1}$		0.026		0.058		
		(0.47)		(1.15)		
Oil Price$_{t-2}$		0.008		0.002		
		(0.17)		(0.05)		
OPEC(excl. Nigeria)$_{t-1}$		0.026		0.072		0.125
		(0.07)		(0.20)		(0.34)
OPEC(excl. Nigeria)$_{t-2}$		-0.053		-0.027		-0.132
		(-0.17)		(-0.09)		(-0.43)
Corporate bond yield rate$_t$			-3.035*	-3.811*		
			(-1.95)	(-2.01)		
Δ Corporate bond yield rate$_t$ /ΔOil price$_t$					-0.000	-0.000
					(-0.73)	(-0.79)
Intercept	2.322	3.755	10.765**	14.299**	4.038*	4.445*
	(1.08)	(0.71)	(2.28)	(2.20)	(1.89)	(1.82)
Adjusted R^2	0.833	0.666	0.857	0.739	0.798	0.740
Number of Obs.	25	22	25	22	24	23

Note: Data is for 1989–2013. Robust standard errors are in parentheses. ***,**,* denotes significance at the 1%, 5% and 10% level, respectively.

Table 2: Empirical Results of Oil Revenue: Nigeria – 1989 – 2013

Dependant Variable: *Revenue$_t$*	(1)	(2)	(3)	(4)	(5)	(6)
Oil Price$_t$	0.018***	0.016***				
	(8.76)	(3.29)				
Country Oil Prod$_t$		1.963**	-0.454	-0.327	1.128	1.732
		(2.93)	(-0.30)	(-0.27)	(0.70)	(1.03)
OPEC(excl. Nigeria)$_t$	1.517***	0.029	2.832*	2.819	4.480***	2.496
	(3.23)	(0.02)	(1.87)	(1.04)	(3.24)	(0.87)
Oil Price$_{t-1}$		0.002		0.005		
		(0.41)		(0.97)		
Oil Price$_{t-2}$		-0.002		-0.004		
		(-0.48)		(-0.01)		
OPEC(excl. Nigeria)$_{t-1}$		-0.468		-1.533		0.515
		(-0.41)		(-0.66)		(0.23)
OPEC(excl. Nigeria)$_{t-2}$		0.949		2.383		1.593
		(1.15)		(1.16)		(1.08)
Corporate bond yield rate$_t$			-25.033**	-30.213**		
			(-2.51)	(-2.89)		
Δ Corporate bond yield rate$_t$ /ΔOil price$_t$					-0.007**	-0.006**
					(-2.54)	(-2.28)
Intercept	-11.627	-28.67*	-30.077	-51.668	-102.016***	-117.221***
	(-1.09)	(-1.94)	(-1.09)	(-1.52)	(-5.14)	(-6.02)
Adjusted R^2	0.954	0.955	0.738	0.833	0.733	0.764
Number of Obs.	25	23	25	22	24	23

Note: Data is for 1989–2013. Robust standard errors are in parentheses. ***,**,* denotes significance at the 1%, 5% and 10% level, respectively.

Table 3: Empirical results of Oil Production – UK – 1989 – 2013

Dependant Variable: *Country Oil Prod*$_t$	(1)	(2)	(3)	(4)	(5)
Oil Price$_t$	-0.009***	-0.005**			
	(-10.09)	(-2.64)			
Oil Price$_{t-1}$		-0.004**		-0.008***	
		(-2.51)		(-3.84)	
Oil Price$_{t-2}$		-0.002**		-0.003**	
		(-2.29)		(-2.12)	
Corporate bond yield rate$_t$			0.286	0.282	
			-0.01	-0.06	
ΔCorporate bond yield rate$_t$ */ΔOil price*$_t$					0.012**
					-2.39
Intercept	20.693***	20.748***	22.597***	20.714***	20.322***
	-419.23	-463.35	-42.01	-63.54	-307.23
Adjusted R^2	0.834	0.895	0.87	0.865	0.17
Number of Obs.	25	23	23	23	24

Note: Data is for 1989–2013. Robust standard errors are in parentheses. ***,**,* denotes significance at the 1%, 5% and 10% level, respectively.

Table 4: Empirical Results of Oil Revenue – UK – 1989 – 2013

Dependant Variable: *Revenue*$_t$	(1)	(2)	(3)	(4)	(5)
Oil Price$_t$	0.012***	0.021***			
	-7.69	-5.84			
Country Oil Prod.$_{t-1}$		0.383	0.352	0.352	0.391
		-0.47	-0.86	-0.73	-0.64
Oil Price$_{t-1}$		0		0.007	
		-0.06		-0.2	
Oil Price$_{t-2}$		-0.001		-0.003	
		(-0.28)		(-0.80)	
Corporate bond yield rate$_t$			-27.178***	-19.116***	
			(-4.95)	(-3.15)	
Δ Corporate bond yield rate$_t$ */ΔOil price*$_{t\ t}$					-58.311***
					(-9.13)
Intercept	23.308***	2.983	16.506*	16.506	23.126***
	-321.24	-0.51	-1.78	-1.51	-8.17
Adjusted *R^2*	0.756	0.818	0.734	0.734	0.856
Number of Obs.	25	23	23	23	25

Note: Data is for 1989–2013. Robust standard errors are in parentheses. ***,**,* denotes significance at the 1%, 5% and 10% level, respectively.

3.6 Conclusions

Taking into account studies such as Cournot (1838); Watkings (1992) and Lin (2009), there is the expectation that Nigeria's production would not adhere to the Hotelling theory as a result of the OPEC factor. This however seems not to be the case. Overall, Nigeria shows strong evidence in support of the Hotelling theory, while the UK shows a weaker support. It still remains that there are external factors which have an effect on Nigeria's production; however, from the results we see the Hotelling theory of optimum balance between production and oil price to be evident. The test results imply that the Hotelling Theory may lose its predictive power when there is a mixture of import and export in oil production. This highlights a limit of the theory to explain change in oil production. A further study of the effect of will need to be pursued to enhance the investigation of this finite mineral resource production. Reserve estimates provide the finite quality required for production and depletion modelling on either a field or country scale.

4 CHAPTER 4: Hubbert Production Model Vs Engineering Production Model – How do they compare?

4.1 Introduction

Hubbert 1953 and 1956 peak oil model states that oil production is mainly determined by change in oil reserves in the long run. It predicts that the regional oil production and world oil production would follow a bell shaped curve. The theory predicts that the world oil production is presently in decline having reached and passed its peak. This theory has fuelled the bipartisan views held on the state of world oil reserves over the years. The two opposing schools of thought consist of those that believe the world is running out of oil, and those that believe we are not quite there and will not be for some time yet.

The Hubbert theory has been used to develop forecasting models for world production, country production and regional production, whereby the peak production and depletion rate are simulated to predict date of depletion and when we shall eventually run out. Studies such as Brandt, 2007; Reynolds, 1999 and Bardi, 2005 build forecasting models in which they expand on the simple Hubbert approach to discuss symmetry and factors that affect the expected bell curve shape and behaviour. These studies though in support of the Hubbert model, conclude that factors such as technology and reserve additions affect the shape and estimation of the curve, shifting and skewing it. In some studies, like Ebrahimi and Ghasabani (2015), multiple peaks are observed which show support of the additional factors highlighted by Brandt, 2007; Reynolds, 1999; and Bardi, 2005 whereby increases in production and multiple peaks due to economical, technical, or political reasons should be accounted for. Other studies such as Reynolds Kolodziej, 2007; Reynolds and Pippenger 2009 show that there can be variations to the curve and that not all regions may show evidence of support for the theory.

The engineering decline curve analysis is another production modelling technique used by oil and gas companies to simulate production profiles. The technique is based on J.J. Arps (1945) theory measures the reservoir boundary dominated production decline, and is employed by engineering disciplines in production modelling and history matching operations. It is the industry norm to use this technical evaluation in tandem with field and reservoir parameters. Few studies such as Jakobsson et al (2012) and Sorrell et al (2012) attempt to build models that incorporate both economic and engineering models in order to bridge the gap between their deviations and limitations. The approach of these studies is related to the proposed objective of the following sections which is to develop and quantify the relationship between the models.

In this study, the Hubbert theory and reservoir engineering decline curve analysis (Arps, 1945) are used as a vehicle for comparison to test the level of support Nigeria and the UK show for the Hubbert curve model and the engineering model. The study highlights other possible factors that may affect production in addition to the physical depletion of supply which the Hubbert theory propagates. It compares the economic and engineering forecasting models to investigate the differences if any in the depletion predictions.

From the results, we see that despite expectations of lack of support of the Hubbert theory in Nigeria given the extraneous effect OPEC has on its production, a Hubbert curve can still be detected. This weak support is backed by the engineering model which does not detect a production decline. The Hubbert and decline curve extrapolation for both countries show very different scenarios. The UK production Hubbert curve estimates a much more conservative depletion than the engineering model which fits a hyperbolic curve to estimate a slower decline curve. Nigeria in comparison to the United Kingdom shows evidence of a much longer time till depletion than the UK case.

The following sections discuss production, the modelling approaches used in this study, and discuss the result further building the platform for the combination model.

4.2 Existing Studies and Arguments

4.2.1 Oil Production Modelling

Recent observations in the oil market have however raised doubts about actual declines in world production solely due to reserves decline. Are we really experiencing a fall in production because of reserves depletion or are there other factors affecting production?

In oil companies, geoscience and engineering models form the crux of extraction plans, while petroleum economic models are designed to estimate profit returns on the project. All production operations within the petroleum industry are based on the results of production models and simulations. Fields and reservoir assets are appraised based on models, following which various production scenarios are modelled and an optimum plan is chosen based on various factors. These include company policy, margin of operation, political environment in which the asset is situated, contract type and duration, etc. Modelling continues throughout the lifetime of the field to compensate and bridge the gap between real life and model projections.

Though these studies are usually carried out on a field scale by petroleum companies, this work will compare and quantify the relationship between economic and production models at a country aggregate level.

Chavez-Rodriguez (2015) uses the Hubbert approach to analyse Peru's oil production. The study evidences that some countries may deviate from the single-curve Hubbert pattern, resulting in the need for a multi-cycle analysis. This multi-cycle is attributed to improvements on oil production infrastructure such as pipelines, and institutional and regulation changes to oil policy.

Bentley (2002) though also in support of the Hubbert peak oil model, identifies key estimation issues such as the danger of using reserve to production ratio as a measure for forecasting. The study highlights that since additional reserves will certainly be found, this pushes the risk of supply difficulties further away. The study also points out the matter of dubious reserves reporting, highlighting it to be a factor worth noting in projection analysis. This is discussed in more detail in further sections of this paper.

Ebrahimi and Ghasabani (2015) use a modified Hubbert curve for production forecasting and policy advisory and reform. The study carries out a production forecasting estimation using a variant multicyclic model on a regional scale, using Excel. It uses various production peaks visible over long periods of time to model multiple cycles of the Hubbert peak within regions, which in turn give multiple forecasts for each region.

Ratti and Vespignani (2015) carry out an econometric study of OPEC and non-OPEC production over quarterly time periods, in which it also builds a predictive production model. The study evidences some success with the use of progressive econometric prediction techniques in modelling OPEC production growth.

Kaufmann (1991) attempts to bridge the gap of mutual exclusivity, studies the relationship between the curve model and the econometric models. The study combines both models to highlight evidence that this method may account for deviations and variation in production volumes.

In a related study, Sorrel et al (2010) discusses oil depletion and forecasting methods. The study highlights the constraints of various forecasting methods. It notes the strengths and weakness of each to be that; (i) the curve-fitting models (for example – Hubbert), though straightforward and widely used, they lack an adequate theoretical basis, are sensitive to the choice of functional form, and neglect key variables and so perform poorly as a result (ii) though econometric models provide a better match to historical data, this may not translate to accurate future production forecasts (iii) models using field or project data (engineering) provide the most reliable basis for near-time forecasts, however, further modelling is hampered by proprietary datasets, lack of transparency and requirement for multiple assumptions. The study also mentions that hybrid in which two of these models are combined still are accompanied with disadvantages such as a lack of empirical validation and lack of sufficient data for parameterisation.

Jakobsson et al (2012) also carries out a regional field study in which it combines reservoir parameters and economics to build a production profile. The model derives different production scenarios all of which lead to the same qualitative results of the Hubbert curve in which production peaks at the point when a fraction of the recoverable resource remains in the ground. The study which includes reservoir engineering inputs is mainly only possible on a field scale in which the reservoir parameters can be measured. Such parameters do not translate to a country scale, therefore the engineering production modelling method of decline curve analysis which relates production and time would be more suited to the purposes of this study.

Sorrell et al (2012) conducts an analysis of depletion rates, decline curve analysis, and reserve growth of crude oil which highlights the various principles and mechanisms employed in attaining a better understanding of crude oil production at present and in the future. Though the study does touch on each of these models, it does not combine them. In support of normal industry approach, each is used separately as an evaluation tool.

The above studies which apply the economic and engineering production modelling techniques on various producing regions show that the theories may not always apply and though the results may give insight they may not always be definitive. A cross-model approach may yet provide more information as this study intends to investigate. This study will use the Hubbert theory and reservoir engineering decline curve analysis (Arps 1945*)* as a vehicle for comparison, and it will test the level of support both countries show for the Hubbert curve model and the engineering model; highlighting other possible factors that may affect production in addition to the physical depletion of supply which the Hubbert theory propagates. The study will also compare economic and engineering forecasting models and investigate the differences if any in the depletion predictions. After which an empirical analysis is carried out on the effect of reserves and selected factors on production in these two countries.

4.2.2 The Hubbert relationship between oil production and oil reserves

There has been a lot of discussion concerning the matter of modelling production peaks whether on a country, regional or world scale. The Hubbert model has received a lot of attention especially in the current ongoing discussion of crude oil supply and world economic dependence on this non-renewable resource. Though the proposed research sets out to test the Hubbert simple oil production model, we note that it is based on a mathematical formula which does not include geologic or reservoir physics considerations, a view which is also supported by Cavallo (2004)

Hubbert (1953, 1956) proposes that oil production or extraction for an individual oil producing region follows a bell shaped curve. This theory gave rise to the peak oil school of thought that oil production for any region and the world will peak, and then begin a steady decline till it runs out. The theory quantifies production over time given that reserves are known and do not change.

The original Hubbert theory says that for any given oil producing geographical area, the rate of production will follow a bell shaped curve. It is based on the assumption that (1) reserves are known (2) production starts at zero, (3) production rises to a peak that can never be surpassed, and (4) after the peak once production declines it does so until the resource is depleted.

Hubbert's curve theory is shown by below.

$$Q(t) = \frac{Q_{\max}}{1 + ae^{-bt}} \qquad (3)$$

Where Q_{max} is the total resource available (ultimate recovery of crude oil), Q(t) the cumulative production, and *a* and *b* are constants.

Studies such as Brandt (2007); Smith and Paddock (1984); Pesaran and Samiei (1995) develop models for the world and selected regions. Their results show support for the Hubbert theory, showing that these regions do follow the Hubbert curve model.

Brandt (2007); Reynolds (1999) and Bardi (2005) further extrapolate and expatiate in support of the Hubbert model. They conclude that there can be multiple peaks (multi-cycle) models as a result of new discoveries and technological advancements. The results of their study show that oil production and supply curves may not follow a symmetrical bell shape at a field, country, or world aggregate level. Brandt (2007) tests three assumptions of the Hubbert theory; (i) bellshape, (ii) symmetry and (iii) model application to larger region. The analysis which is done on four models defined by four regional sizes concludes that the Hubbert curve does not describe all production models in various regions. It notes that an asymmetric model in one direction better describes actual production history, and that the probability of a multitude of production curve shapes must be acknowledged in any future analysis using the Hubbert theory.

Bardi (2005) studies factors that affect the symmetry of the Hubbert curve. The study programs a simulation to discover that inputs such as technological improvements and production strategy can affect rate of decline of production, the peak position and ultimately the shape of the resulting curve. Brandt (2007) also carries out a regional study to test the assumption the oil production follows a bell shaped curve. The test also tests shape and symmetry on regional models, concluding that size of a region affects the symmetry of the curve. The study also advocates the expansion of the Hubbert model to include other factors that may affect production decisions.

Furthermore, Reynolds Kolodziej (2007); Reynolds and Pippenger (2009) carry out studies to show that not all regions follow the Hubbert curve as a result of factors such as technology. Reynolds and Baek (2012) study also highlight that regions can delay their production potential, resulting in non-symmetric or skewed Hubbert curve for the region. Nigeria which is an OPEC member falls within this category where evidence of collusion is supported in earlier sections of this study. Therefore, for this proposed study, we can expect the Hubbert curve on a gentle incline towards a peak, or quite possibly and extended peak.

Pickering (2008) conducts a study to examine the relationship between oil extraction rates and reserves. The study defines straightforward linear relationships between production and remaining reserves whilst allowing for costs and market structure. It finds that for non-OPEC countries there is a highly significant and positive relationship between extraction rates and remaining reserves, and therefore supports Hubbert's (1956) school of theory. The study however notes that this is not the case for OPEC countries which have the largest reserve endowments and yet show production declines. The study highlights the probable explanations for this – such as deliberate production management to avoid vulnerability to oil price volatility, heterogenous discount rates, and simple mismeasurement of reserves, which is highlighted in following sections of this study. So in essence, not all regions or countries may fit the Hubbert model. Pickering (2008) results support the intent of this study which is to compare two countries and their support of the Hubbert model curve.

Though mostly carried out in caution, other studies take things further even to predict the moment of peak oil. Sorrell et al (2010) carries out a comprehensive analysis of peak productions in various producer countries till date. The study establishes ultimately recoverable reserves which incorporates production, new technology and recent reserve discoveries. Using the Hubbert model, it then attempts to ultimately forecasts world reserve estimates, and eventually concludes that the world conventional oil production will reach its peak production before 2020. Reynolds (2015) investigates the ability of the Hubbert model to forecast world production and peak from past production. The study results in differing forecasts for various regions, concluding that a broader view of the Hubbert model to include other factors such as price allow for a more real forecast.

4.2.3 Engineering Production Profile Analysis

Other methods of oil production analysis include the decline curve analysis which is a reservoir engineering empirical technique that extrapolates trends in the production data from oil and gas wells. The purpose of a Decline analysis is to generate a forecast of future production rates and to determine the expected ultimate recoverable (EUR) reserves. The most commonly used trending equations are those first documented by J.J. Arps (1945).

It measures reservoir boundary dominated production decline, and can only be used when the production profile begins to decline as a field reaches a boundary dominated production flow. In essence, production as a result of reservoir conditions must be evident for the application of this analysis for future production modelling. Typically, decline analysis is conducted on a plot of rate versus time or rate versus cumulative production. Hook (2009) provides a descriptive analysis of reservoir engineering and modelling techniques which use the decline curve analysis for field application. The study shows the production principles applied in engineering production modelling from an engineering perspective.

This study will apply the decline curve analysis in the total annual production on a country scale. The decline rates of both Nigeria and the UK will be analysed and extrapolated to predict total depletion based on the production profiles of both regions.

The theory of all decline curve analysis begins with the concept of the nominal (instantaneous) decline rate (a), which is defined as the fractional change in production rate per unit time:

$$a = -\frac{(\Delta q/q)}{\Delta t} = -\frac{1}{q}\frac{\Delta q}{\Delta t} \qquad (4)$$

Another way of representing the decline rate is based on production rate (q) and the decline exponent constant b. The below equation describes the relationship where production decline rate ‘a’ is described as a function of production ‘q’ in relation to constant ‘b’ which measures the decline behavior.

$$a = k\, q^{b} \qquad (5)$$

When production is plotted as flow rate versus time, the nominal decline rate is equal to the slope at a point in time divided by the rate at that point.

The behaviour of the production data can be characterized based on the way in which the nominal decline rate varies with rate, based on the value of the decline exponent constant b. A decline curve can be described as exponential, hyperbolic or harmonic, when they fall between 0 and 1.

- Exponential: b = 0
- Hyperbolic: b is a value other than 0 or 1
- Harmonic: b = 1

However, before we proceed develop the decline curve model for Nigeria and UK, to it is worth discussing the reserves estimates upon which these all important models and simulations are based, as they are a crucial constraint to production output behaviour.

4.3 Quality of oil reserves reports

Petroleum reserves are a measure of the potential resource contained within a reservoir, and it is measured in barrels (bbls) or standard cubic feet (scf) for oil and gas respectively. The definition of reserves takes into account many factors such as social and economic, technical engineering, statistical probabilities and recovery. These in turn drive the quantifications and definitions outlined in the index.

Harold Hubbert (1956) theory holds that production of conventional oil, within any world region in the world, will increase and peak at a certain mid-point over the life time of the known supply of the resource, after which it will begin to decline. The assumption is that reserve volumes are known, therefore unchanging.

Reports show a steady growth in proven world oil reserves *(US EIA, BP, 2014).* It is worth noting that according to Mitchell (2004) and Wolf (2009), the truth is that there is no standardized or uniform global approach to estimation and certification of oil and gas reserves. Indeed, different global stock exchanges employ different reserve estimation rules Arnott (2004). There are various systems in place, some of which are more widely used than others. As such there remains always a risk of the estimate changing, should the reservoir asset be subject to an external audit in both national oil companies and international oil companies alike.

The bottom line is that reserves need to be both technically and economically recoverable to be viable. These are two separate factors which affect recovery (exploration and production operations), and therefore reserves separately. They can be utilized separately or combined in the assessment of profitability of a project. Technically recoverable reserves may not be economically recoverable. In the case of the offshore Arctic region, which contains a significant amount of oil, development and exploitation requires infrastructure development and technological progress before it becomes economically possible for extraction at current oil prices (IEA 2013b). This highlights the importance of oil price to production volumes.

4.3.1 OPEC Factor and quality of reserves reports

In discussion of quality or reserves we must mention the effect OPEC reports have on world supply. OPEC remains the most important commercial group relevant to Oil & Gas operations worldwide (See Table 5). It is an individual entity which has been single-handedly responsible for changing oil prices since 1973 (Rose, 2004). The group is both important to current crude oil production and world reserves estimates. OPEC forms 12 out of the top 20 countries with the highest crude oil reserves. They also account for about 40% of global crude oil production and 60% of international petroleum trade (U.S. Energy Information Administration (EIA). We note that all 12 OPEC countries are classed as developing economies.

By regulation of the production volumes of the member states, OPEC works to execute their mission, which is to ensure stabilization of oil markets (OPEC.org). The potential and dominant significance of this group to world supply requires further attention because, OPEC also publishes its own reserves estimates and production volumes which are sometimes looked upon with suspicion. Studies show that recently published estimate figures by OPEC countries may have been revised with a bias, which links export quotas to reserves estimates (IEA 2013b). Whilst this may be true, reserves are often revised annually, for producing countries worldwide; and published estimates are what future projections and plans are predicated upon. One main reason as to why OPEC members tend towards inflating their reserve estimates case is the issue of production quota limits i.e. production targets. The production quota placed on each member country is a percentage of the country's declared estimated reserves. This value ties directly to revenue to the country, so the smaller the reported reserves, the smaller their production quota and the smaller the quantity the country is allowed to produce and therefore export, resulting in a lower income generated to the government.

Still, this does not discredit OPEC countries from world reserve analysis, they accounted for 42.1% of total world oil production in 2013, and they still account for 36 of the 52 *'super-giant'* oil fields (i.e. fields with reserves of more than 5,000 million barrels of oil). OPEC member countries currently produce 40% of the world's crude oil, and their combined oil exports represent about 60% of the total petroleum traded internationally (BP statistical Review, 2014; US EIA, 2014). Proved reserves remain concentrated in OPEC as it controls 72.6% of the global total. (BP REPORT, n.d.).

Table 5: OPEC Countries World Reserves ranking (2013)

Country	**World Rank**	**Country**	**World Rank**	**Country**	**World Rank**
Algeria	16	Iraq	5	Qatar	12
Angola	17	Kuwait	6	Saudi Arabia	2
Ecuador	19	Libya	9	United Arab Emirates	7
Iran	4	Nigeria	10	Venezuela	1

***All of the above are classed as developing economies. **No 3-Canada & No 8-Russia are Non-OPEC countries.*

Owen et al (2010) carries out an in-depth theoretical study on the status of world oil reserve estimates and advises a revision downwards for true world estimates. The study highlights that the lack of adherence to reporting standards could explain the polarised views on the status of conventional oil reserves. Though the study warns that to determine whether or not and when oil supply and demand curves may diverge, or the direction in which they might move, access to a number of contentious and inherently uncertain data sets is required. Furthermore, to question reported reserve estimates could be politically sensitive and diplomatically offensive.

So since the revenue reality and potential is why investment in oil and gas operations continues, published reserve volumes are all we have, and so it will have to suffice. It is worth mentioning that reserves are reassessed and quantified periodically through the life of the field or project. At the start of projects, reserve estimates are audited by technical professional bodies e.g. Society of

Petroleum Engineers (SPE), World Petroleum Council (WPC), American Association of Petroleum Geologists (AAPG), Society of Petroleum Evaluation Engineers (SPEE) and joint venture members i.e. operating companies themselves, regardless of published quantities

4.4 Method and Data

4.4.1 Hubbert model

The standard bell shape curve equation is defined as:

$$f(x) = a \exp\left(-\frac{(x-b)^2}{2c^2}\right) \tag{6}$$

Or it can also be expressed for the purposes of this study below.

$$f(x) = \; Mexp^{-\left(\frac{C-x}{W}\right)^2} \tag{7}$$

Where M is the maximum value of curve (which occurs at C), x is value of oil production at the year (x), C is centre year – peak year, w = spread or width of curve. Hubbert (1956) bell shape curve theory says

$$Q(t) = \frac{Q_{\max}}{1 + ae^{-bt}} \tag{8}$$

Where Q_{max} is the total resource available (ultimate recovery of crude oil), *Q(t)* the cumulative production, and *a* and *b* are constants. The year of maximum annual production (peak) can be defined as:

$$t_{\max} = \frac{1}{b} \ln(a) \, . \tag{9}$$

4.4.2 Engineering model

Using Arp's (1945) production relationships;

Type	Exponential Decline	Hyperbolic Decline	Harmonic Decline
	Decline is constant b=0	Decline is proportional to a fractional power (b) of the production rate 0 < b< 1	Decline is proportional to production rate b = 1
Rate – Time	$q = q_i\, e^{(-a\, \Delta t)}$	$q = \dfrac{q_i}{(1 + b\, a_i\, \Delta t)^{(\frac{1}{b})}}$	$q = \dfrac{q_i}{(1 + a_i\, \Delta t)}$

Figure 6: Decline curve equations

Where *Di* is initial decline rate; Di = *(Qi – Q)/Qi*, usually for a particular time period (real data) (effective in this case); *Qi* is the instantaneous production at time 0; and Q is the production at time *t.*

4.5 The data

The table below consists of data over the period of 25 years used to define variables used in the regression analysis. It consists of mostly financial and oil production data associated with petroleum exploration and production in the United Kingdom and Nigeria. The data has been collected from public sources namely – Department of Energy & Climate Change UK (DECC), HM Revenue and Customs UK (HMRC), International Monetary Fund (IMF), US Energy Information Agency (USA EIA), US Federal Reserve Bank and Nigeria National Petroleum Corporation (NNPC). The study is framed within a 25-year period 1989 – 2013 which allows for final submissions and publication of data from public sources, and reduces data sensitivity issues. (Figure 7)

Period: 1989 - 2013

Figure 7: United Kingdom and Nigeria

Country Data and Units	Source
Exchange rate (GBP/USD)	Bank of England
Oil & Gas E&P Operating Expenditure (US dollar)	Department of Energy & Climate Change UK (DECC); Nigeria National Petroleum Corporation (NNPC)
Bond yield Rate (percentage)	Federal Reserve Bank of St Louis
Oil production, barrels/day	US EIA
World oil price (US dollar/barrel)	US EIA, IMF
Oil reserves (barrels)	US EIA
Annual GDP growth	IMF

4.6 Analysis and Results

The method of data analysis for this study will be in keeping with the statistical mode of analysis described by Hartwig and Dearing (1982) and Lewis-Beck (1995), which facilitates the analysis of the relationship between variables instead of a confirmatory mode of analysis which may be less open to a wider range of alternative explanations and randomness.

In order to quantify the impact that the explanatory variables have on production an economics model (Hubbert), engineering model (Arps) and an empirical model are specified for each country. A regression model is set up with the reserves (Q) being the dependent variable and crude oil price (P), reserves (R), operating expenditure (X), and OPEC production (Q^E) as the explanatory variables with exogenous assumption. The below model estimation is used using linear and quadratic Hubbert relationships in keeping with Pickering (2008); Beirne (2013); Reynolds and Kolodziej, (2008)

$$Q_t = \alpha + \beta OilPrice_t + \lambda Reserves_t + \theta Opec\ production + \mu\ \%GDPgrowth_t + Operating\ Expenditure\ldots + \varepsilon_t \quad (10)$$

$$Q_t = \alpha + \beta OilPrice_t + \lambda Reserves_t + \lambda Reserves^2_t + \theta Opec\ production + \mu\ \%GDPgrowth_t + Operating\ Expenditure\ldots + \varepsilon_t \quad (11)$$

4.7 Initial data observation

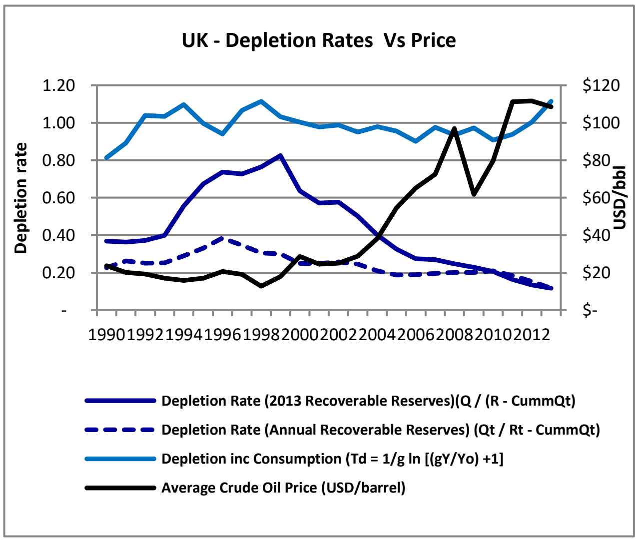
Initial visual observation of the depletion rates of the reserves for both countries shows a strong contrast. The UK is well into the depletion of its reserves whereas Nigeria is has barely produced 7% of its reserves. UK model on visual observation appears to follow some type of rough curve similar to Hubbert curve, albeit less symmetrical in nature. Depletion rate based on 2013 reserve volumes follows the curve as production, however on observation of depletion trends using annual reserve trends we see a much more stable depletion rate. The figures show efficient production of UK reserves whereby the depletion rate has remained steady and consumption has had minimal effect on production. The figures give an initial indication of the effect reserves have on production modelling.

From the data, Nigeria barely into the depletion of its reserves hence we are unable to see the expected depletion curve. Instead a high reserve to production ration is observed. This might be explained by the fact that we are still at the initial incline of the curve and we are approaching at a very gentle slope, however this needs to be tested. The depletion rate which incorporates consumption is calculated using equation below, where T^D is the depletion time taking into account consumption rate. g is the growth rate of consumption Y is the amount of reserves at present and Y_0 is the initial amount of the resource consumed.

$$T^D = \frac{1}{g}\, ln\left[\left(\frac{g^Y}{Yo}\right) + 1\right] \qquad (12)$$

The figures 8 and 9 below show the reserve and production trends which incorporate consumption and oil price trends.

Figure 8 - UK Production Depletion trends



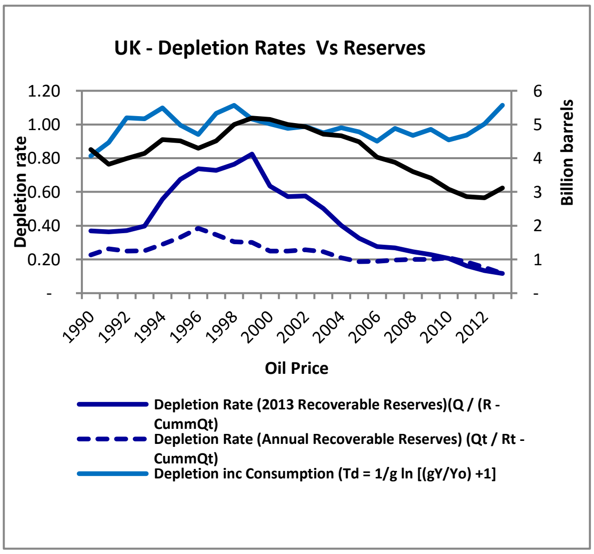
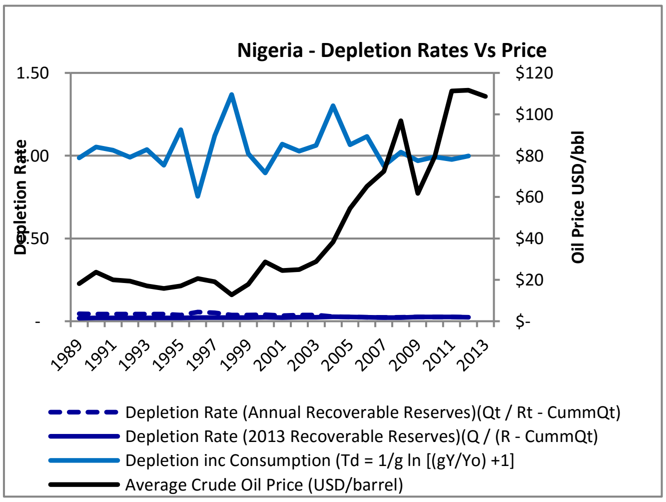
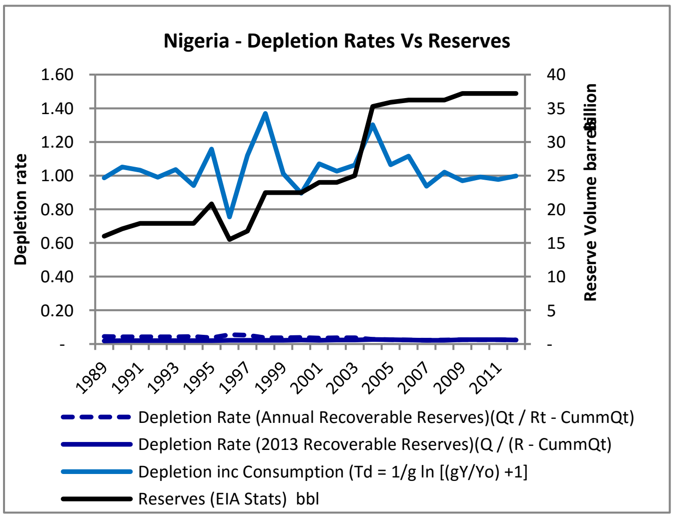


Figure 9: Nigeria Production Depletion trends





4.8 Hubbert Model analysis and results

The Hubbert model requires the peak production values, the year and the spread. Fitting a standard bell curve to the data using the below table and the following curve is estimated. From looking at the data we see that peak production is reached in 2005 at 959 million barrels per year for Nigeria and 1999 at 980million barrels per year for the UK. Using the solver application in Excel, which targets and sets a level of deviation from the curve using the sum of the error squared, the curve is then fitted to the data. The results show that production decline in both cases of Nigeria and the UK fit the Hubbert curve (Figure 10). The UK shows a steep production decline, while Nigeria shows a gradual slope though a curve is still evident. The model is then extrapolated and a depletion period is simulated to predict a total depletion date (Figure 11). From the results we can predict an earlier depletion date in the UK compared to Nigeria. Hubbert production for Nigeria and the UK is solved using the below table and equation 7.

The data is then extrapolated till the curve approaches zero, giving us an indication of the number of years to depletion for the models. Again noting Cavallo (2004), this modelling approach is a based on a mathematical formula which does not include geologic or reservoir physics considerations. Both country cases show support for the Hubbert curve despite expectations that only the UK production would show evidence of support. The best fit curves which are fit to the production peaks evident in the data show very different depletion rates. UK production headed toward steeper decline in comparison to Nigeria.

Table 6: Hubbert curve inputs

Nigeria	Peak year 'C' =	2005
	Max Production 'M' =	959mmbbl
	Width 'W' =	24
UK	Peak year 'C' =	1999
	Max Production 'M' =	980mmbbl
	Width 'W' =	24

Table 7: Hubbert curve results

	NIG	UK	NIG	UK
Year	Q (Million bbl) NIG	Q (Million bbl) UK	Q Hubbert Model (NIG)	Q Hubbert Model (UK)
1989	626	658	643	598
1990	661	664	659	653
1991	691	656	675	706
1992	711	668	691	754
1993	715	699	706	798
1994	705	867	721	835
1995	727	909	736	864
1996	732	940	750	885
1997	778	919	764	896
1998	786	955	777	899
1999	777	980	790	891
2000	792	833	802	874
2001	823	833	813	848
2002	773	836	824	815
2003	830	764	834	774
2004	852	675	843	727
2005	959	602	851	676
2006	891	544	859	622
2007	858	547	866	566
2008	793	509	872	510
2009	806	485	878	454
2010	896	450	882	400
2011	931	375	886	349
2012	922	325	888	301
2013	864	292	890	257

Figure 10: Hubbert Production curve

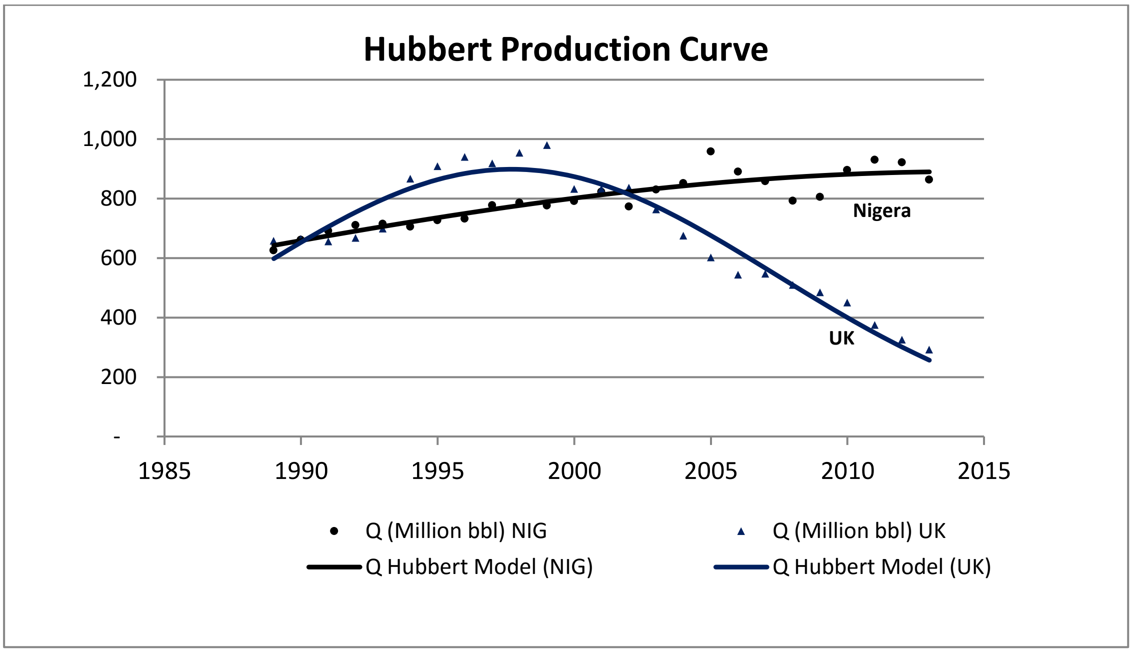
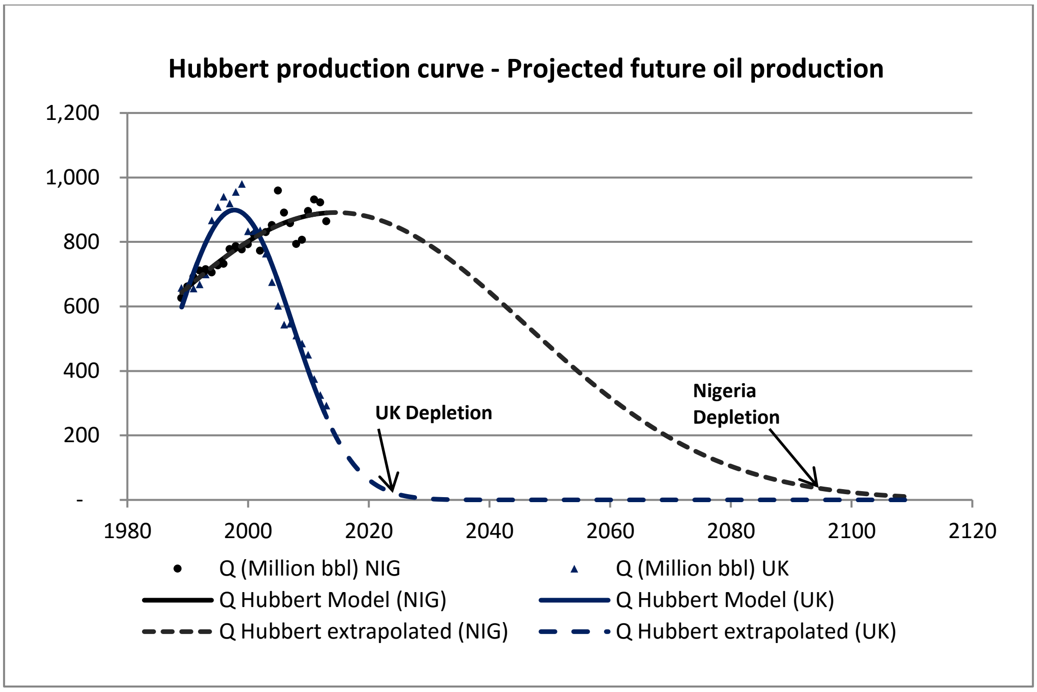


Figure 11: Hubbert forecast model



4.9 Engineering model analysis and results

The decline curve theory takes into account the physical laws i.e. the reservoir properties that affect hydrocarbon production. Though it is normally applied on a well-scale or field-scale the below analysis applies it on a country scale. The decline curve modelling approach is applicable here because it is virtually independent of the size and shape of the producing area therefore avoiding the need for detailed reservoir or production data. The only data required for the decline curve analysis and extrapolation is the production data. (Hook, 2009)

Results of the decline curve analysis show the UK production to peak and begin to decline from 1999 (figure 12). Using this as the point at which production decline begins, the decline curve is modelled using the table below. The model detects a decline in the case of the UK, and the results show that UK production fits a hyperbolic decline rate. This is so despite the fact that the exponential curve better matches actual production data (Hook, 2009). In the case of Nigeria, a decline is not detected, and no curve can be fitted (Figure 14). This may be an indication of Nigeria's regulated production maintenance of its quotas. The UK's production decline curve is extrapolated to estimate depletion.

(Figure 13) The results show that UK production depletion to less than 10 million barrels per year is estimated for 2072. Following this the results show a prolonged total depletion period of over 100 years. The lack of evidence of a decline production in the Nigeria may also be attributed to the fact that Nigeria may likely still be approaching its peak or is maintaining a peak, and this would be in line with its OPEC membership agenda.

Table 8: Production Decline curve results (Engineering Model) – Nigeria and UK

	NIG			UK			
Time **year**	**Exponential Decline Rate** **mmbbl/year**	**Harmonic Decline Rate** **mmbbl/year**	**Hyperbolic Decline Rate** **mmbbl/year**	**Exponential Decline Rate** **mmbbl/year**	**Harmonic Decline Rate** **mmbbl/year**	**Hyperbolic Decline Rate** **mmbbl/year**	**Projected Decline Rate** **mmbbl/year**
10	327	379	357	193	373	298	298
11	306	365	340	164	351	273	273
12	287	352	324	139	332	251	251
13	269	339	309	118	315	232	232
14	252	328	296	101	299	214	214
15	236	317	283	86	285	199	199
16	221	307	271	73	272	185	185
17	207	297	260	62	260	173	173
18	194	289	249	53	250	161	161
19	182	280	239	45	240	151	151
20	171	272	230	38	230	142	142
21	160	265	221	32	222	134	134
22	150	258	213	27	214	126	126
23	140	251	205	23	207	119	119
24	132	245	198	20	200	113	113

Figure 12: Production curves – UK – Nigeria

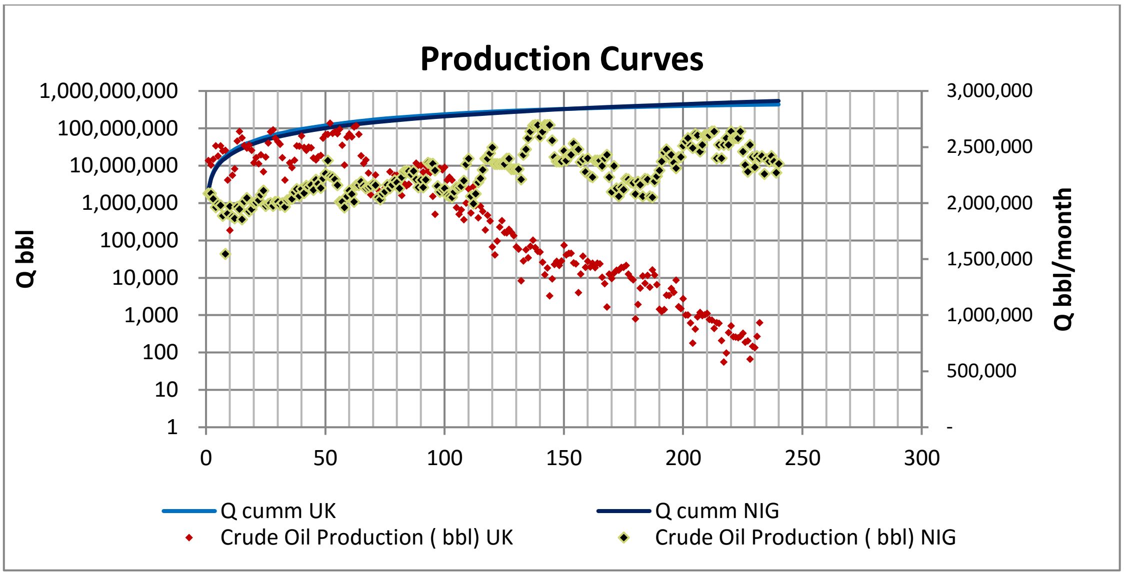


Figure 13: UK Decline curve (Engineering model)

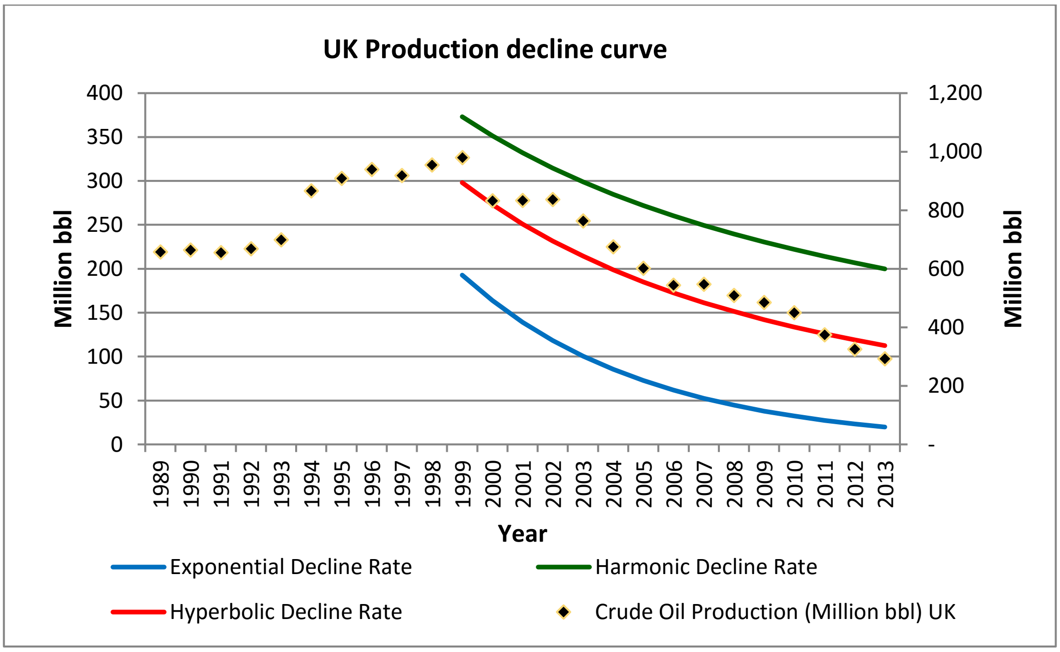


Figure 14: UK Projected Production Decline (Engineering Model)

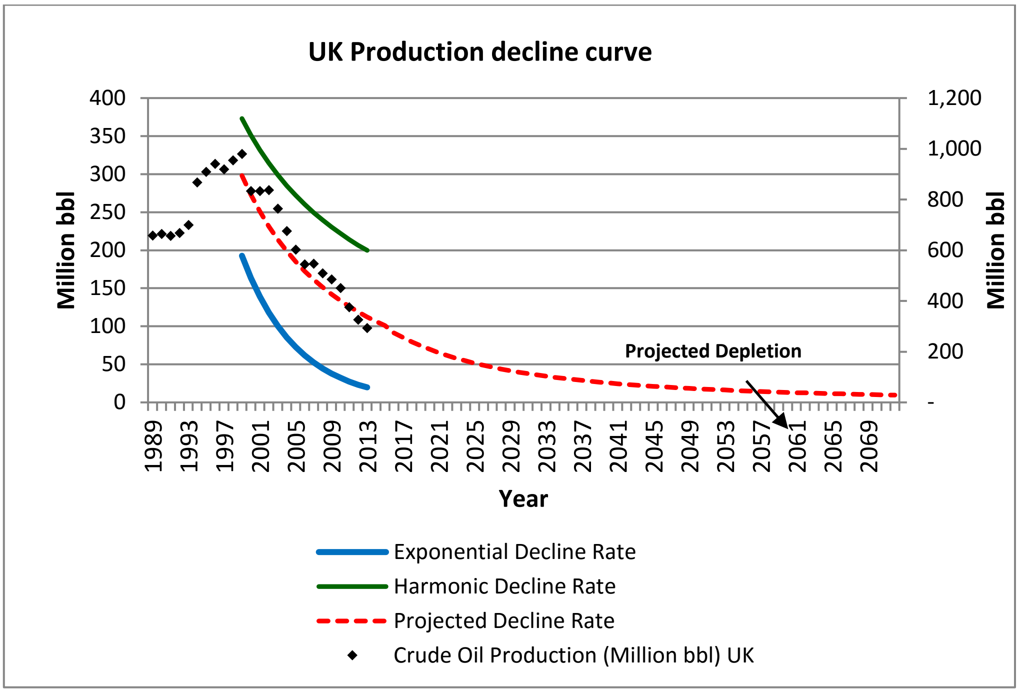
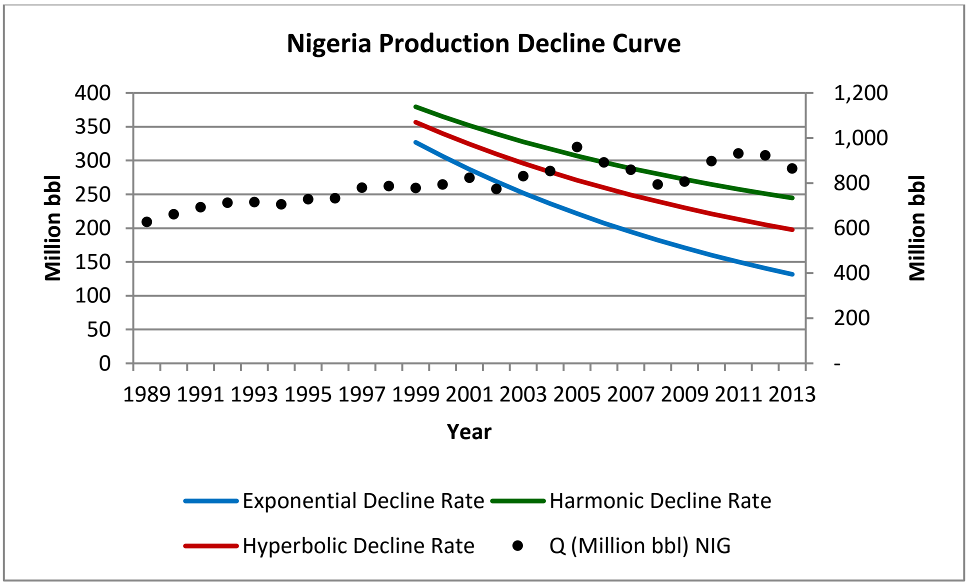


Figure 15: Nigeria Decline curve (Engineering model)



4.10 Conclusion

Results from this chapter show the significance of the use of various model approaches on the design of oil production activities over the life of a field or producing region. The measure of this impact affects matters such as production forecasting, where in the UK we see a conservative prediction of depletion from the Hubbert model in comparison to the engineering model. Hubbert theory projects a total depletion of volume by 2030 for the UK, whereas the engineering model forecasts a much less rapid depletion date with a sustained rate of 10 million barrels after 2072. Where the Hubbert model shows a significant relationship with the actual production, the difference in how conservative the Hubbert forecasts are, highlights how important production forecast models can become with respect to investment and revenue generation. The significance of the production theory relationship with actual production volumes lends gravitas to the level of impact a forecasting or production modelling approach can have on a country's production profile and therefore future planning. Subsequently, it may be concluded that the level of import of the Hubbert model to the UK future production is not insignificant.

Results from this study indicate that the engineering decline curve theory fails to explain Nigeria's production profile. Though Nigeria's production volumes also adhere to the Hubbert model just like the United Kingdom, the resulting bell curve shows that Nigeria's production is still yet to peak and depletion is much less steep than in the case of the UK.

The predicted production volumes are compared to actual production volumes in the next chapter using a combined approach. The varying explanatory powers of these theories and production models are further investigated in the following chapter which investigates the effect and level of significance of the combined variables on actual production volumes of both country cases.

5 CHAPTER 5: Production Modelling Methods: A Combined Approach

5.1 Introduction

A previous chapter has investigated the price effect on oil production at a country aggregate level from the aspect of empirical estimation of the Hotelling theory. Then we further investigated oil reserves and its effect on oil production using Hubbert theory and Engineering plan approach. This chapter will move further to carry out an empirical analysis of the relationship between oil production and oil reserves using the combined production function of a country. It will measure the level of impact of the combined variables on actual production volumes for the case of Nigeria and the UK, and it is expected that the results should provide more understanding of how oil production is determined.

This investigation builds on the related production modelling studies Jakobsson et al (2012) and Sorrell et al (2012), and other related econometric studies conducted on a country case such as *Michael and Hunka (1960); Powell (1999)* Azhgaliyeva (2014); Iwayemi and Fowowe (2011); Kretzschmar and Kirchner (2009); and Smith (2013). Following from existing studies, the new development of this study is to take a comparative approach to identify which effect is stronger on oil production, i.e. oil reserves or engineering model planning. The study highlights the limits of conventional production function in explaining oil outputs such as Cobb-Douglas production function. The study extends the Cobb-Douglas function by taking into account the constraint of oil reserves or engineering planning on oil production. We apply the Hubbert theory to define the effect of oil reserves on production, and an engineering model to predict oil production according to changes in oil reserves of a field or a region.

The empirical analysis in this chapter will be in keeping with the statistical mode of analysis described by Hartwig and Dearing (1982) and Lewis-Beck (1995), which facilitates the analysis of the relationship between variables instead of a confirmatory mode of analysis which may be less open to a wider range of alternative explanations and randomness. This study will measure how the above models relate to actual country oil production volumes using time series data estimation approach. The proposed research aims to shed light on deviations each model may have from the real production volumes. Multivariate regression analysis has been applied, which has the advantage of bringing in more information to bear on a specific outcome. It allows one to take into account the continuing relationship among several variables, which is especially valuable in observational studies where total control is not possible. With this method, we are able to study the effects of several variables acting simultaneously instead of singly.

Following the premise for country models described in the earlier chapters of this study, a model for each country is developed in which the economic and engineering models are combined to build a relationship. Annual data is used for the purposes of this study. The equations below set oil production as a dependent function of economic oil reserves that are predicted by engineering production forecast models. The extended production function includes oil price, reserves, operating cost and expenditure and interest rate variables on a country aggregate level. Before developing the extended model of oil production, there is the need to establish the characteristics of the data in this time series, as this is often non-stationary. It is also important to know whether or not the variables have a unit root (Davidson and MacKinnon, 2004). In the case of this study however, due to the length of the time series, the risk of misleading results due to persistent time series is minimized, thus allowing for the use of ordinary least square regression.

5.2 Econometric Model Estimation

Using the models described in the previous chapters Q_h is defined as f(P, i, C), Q_H is defined as f(t) and Q_E is defined as f(Q, t). Where Q_h is the oil output predicted by the Hotelling model, Q_H is oil output predicted by the Hubbert model according to oil reserves, Q_E is the oil output of Engineering planning, i interest rate, P is oil price and C is the cost according to asset value changes, Q is real actual production and t is time. According to the (Cobb-Douglas) production function we define output of oil as $Q_c = f(K, L)$; or more specifically as $Q_c = A K^{\beta} L^{\alpha}$.

These two production are combined to develop the below econometric model. Cobb-Douglas model is employed as a starting point to derive an extended Cobb-Douglas function by taking into account oil reserve effect on production of oil. The new model is derived as follows.

Suppose that oil output (Qt) is determined by constrained Cobb-Douglas production, and the constraint is oil reserves effect predicted by Hubbert oil peak theory:

$$Qt = (Q^*/Q_H)^{\lambda} Q_H \quad (13)$$

Where Q* is the output of oil from Cobb-Douglas and Q_H is Hubbert output of oil predicted according to change in oil reserves available for exploration and λ is the constrained parameter.

When $\lambda = 1$ then $Qt = Q^*$; actual output equals to Cobb-Douglas prediction;

when $\lambda = 0$ then $Qt = Q_H$; actual output equals to Hubbert reserve-based prediction;

when $0 < \lambda < 1$, then $Qt = Q^{\lambda} (Q_H)^{1-\lambda}$; actual output lies between both Cobb-Douglas and Hubbert predictions.

So, $Q_t = (A K^{\beta} L^{\alpha})^{\lambda} (Q_H)^{1-\lambda}$ since $Q^* = A K^{\beta} L^{\alpha}$, Where A is oil production technology, K is the quantity of fixed assets employed for production and L is labour force hired for oil production. By taking into account the Hotelling effect on production, we further apply the adjustment function below:

$q_t = (Q_t/Q_h)^{\theta} Q_h$; where $Q_h = f(P, i, C)$, Q_h is the output predicted by the Hotelling theory.

$$q_t = Q_t^{\theta} (Q_h)^{1-\theta} \quad (14)$$

So, the actual output of oil (q_t) shall be:

$$q_t = \{(A K^{\beta} L^{\alpha})^{\lambda} (Q_h)^{1-\lambda})^{\theta}\} (Q_h)^{1-\theta}$$

$$= (A K^{\beta} L^{\alpha})^{\lambda\theta} (Q_h)^{\theta(1-\lambda)} f(P, i, C)^{1-\theta} \quad (15)$$

Following from the above forward adjustment, which involves incorporating the Hotelling theory as an input, reserves and their effect need to be taken into account with respect to the second output equation. To achieve this, we introduce the second stage of adjustment of output from the combined effects of Cobbs-Douglas model and the Hotelling model. We use the Hubbert output i.e. the output predicted by the Hubbert theory as an instrument to introduce the Hubbert effect into the model.

Therefore;

$$q_t = (A K^{\beta} L^{\alpha})^{\lambda\theta} (Q_H)^{\theta(1-\lambda)} (P^{\omega}, i^{\varphi}, c^{\rho})^{1-\theta} \quad (16)$$

Or we can replace the Hubbert constraint $Q_{H,}$ with the engineering planning constraint Q_E

$$q_t = (A K^{\beta} L^{\alpha})^{\lambda\theta} Q_E^{\theta(1-\lambda)} (P^{\omega}, i^{\varphi}, c^{\rho})^{1-\theta} \quad (17)$$

The study will test these two competing models – the Hubbert constraint versus the Engineering constraint providing insight to which is stronger. Taking the translog, we can estimate that;

$$lnqt = \mu\lambda\theta lnA + \mu\beta\lambda\theta lnK + \mu\alpha\lambda\theta lnL + \mu\theta(1-\lambda)lnQ_H + \mu\omega(1-\theta)lnP + \mu\varphi(1-\theta)lni + \mu\rho(1-\theta)lnC + (1-\mu)LnQ_E \quad (18)$$

When $\lambda' = \mu\lambda\theta$; $\beta' = \mu\beta\lambda\theta$; $\alpha' = \mu\alpha\lambda\theta$; $\theta' = \mu\theta(1-\lambda)$; $\omega' = \mu\omega(1-\theta)$; $\varphi' = \mu\varphi(1-\theta)$; $\rho' = \mu\rho(1-\theta)$; and $\mu' = (1-\mu)$

The model is therefore estimated as:

$$lnQ_t = \lambda' \, lnA + \beta' \, lnK + \alpha' \, lnL + \theta' \, lnQ_H + \omega' \, lnP + \varphi' lni + \rho' \, lnC \quad (19)$$

Versus

$$lnQ_t = \lambda' \, lnA + \beta' \, lnK + \alpha' \, lnL + \theta' \, lnQ_E + \omega' \, lnP + \varphi' lni + \rho' \, lnC \quad (20)$$

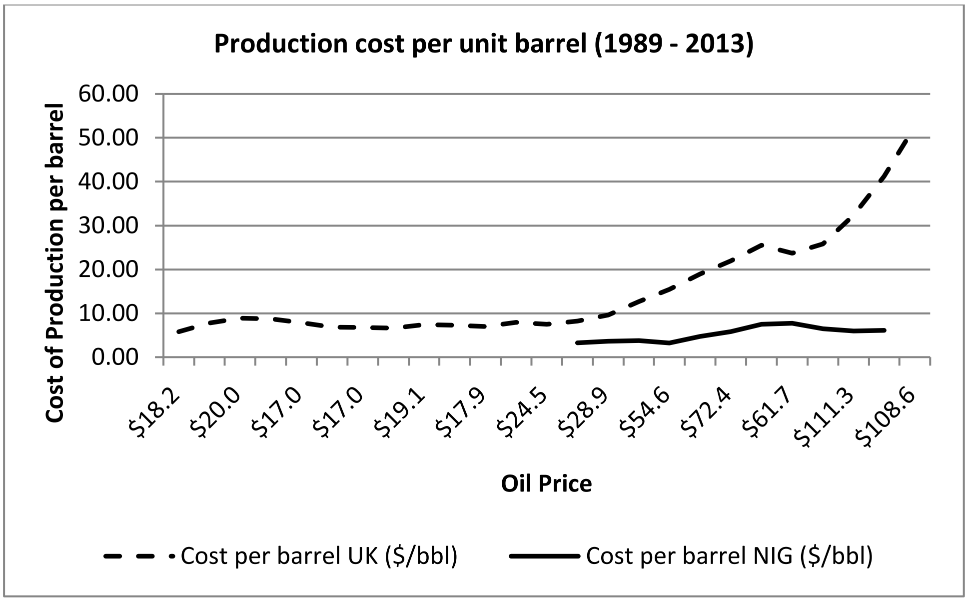
Cobb-Douglas labour is dropped as we assume a constant scale of production. The data which is time series is often non-stationary. It is essential to know whether or not the variables have a unit root (Davidson and MacKinnon, 2004). The time series element of the data is tested using an Augmented Dickey-Fuller (ADF) unit root test check for stationarity. Based on the p-values in Table 9 below, it can be seen that the ADF test statistic for the variables are significant. This rejects the null hypothesis of unit root, which can conclude that the variables are stationary. To avoid spurious results, stationarity and unit-root tests are carried out to check that the variables are non-trending and to establish a null hypothesis.

Table 9: Results of ADF unit root test

Variables	ADF (t)	p value	Stationary	Level
LnP	-4.68	0.0013	yes	1%
Lni	-5.17	0.0004	yes	1%
LnQnig	-5.70	0.0002	yes	1%
LnQuk	-3.14	0.0373	yes	5%
CapexNig	-4.20	0.0135	yes	5%
CapexUK	-3.63	0.0132	yes	5%
OpexNig	-2.96	0.0805	yes	5%
OpexUK	-2.89	0.0621	yes	5%
Cost per bbl UK	0.11	0.96	*no*	
cost per bbl NIG	-3.59	0.04	yes	5%

For a more robust analysis, additional factors such as capital expenditure, operating expenditure are included in the model. The capital expenditure variable described here is the capital expenditure of exploration and development operations which includes investments, gross operating surplus, exploration and the cost of appraisal wells drilled prior to developmental approval. The operating expenditure variable includes costs attributed to production and operation of oil fields e.g. exploration and production facilities, services personnel etc. For further analysis, cost of production per barrel is also included for additional observation of the effect of cost on production on production volumes. This variable is measured in accordance with (Mckinsey & Company, 2014) whereby the cost of production per unit barrel of oil is measured as a ratio of total operating expenditure of production to total production of oil in barrels. The figure below shows how the UK cost of production relates to data available for cost of oil production in Nigeria. Cost of production within the UK is in steep incline, the reasons for which are discussed in previous sections to include increased facilities maintenance expense, personnel expense, declining production due to regional maturity and increase activity and supply chain costs and inefficiencies. A main reason why Nigeria's cost of oil production does not follow the same increasing trend as the UK includes the fact that its reserves are not in decline, therefore enhanced production techniques such as those used in the UK North Sea are used sparingly. Other factors such as the much reduced use of specialized facilities and the maintenance of such, reduced environmental and weather effects on facilities and equipment, coupled with weak exploration and production operations environmental regulations, further contribute to the low cost of production observed in the data provided by the national oil company of Nigeria.

Figure 16: Oil production cost per unit barrel

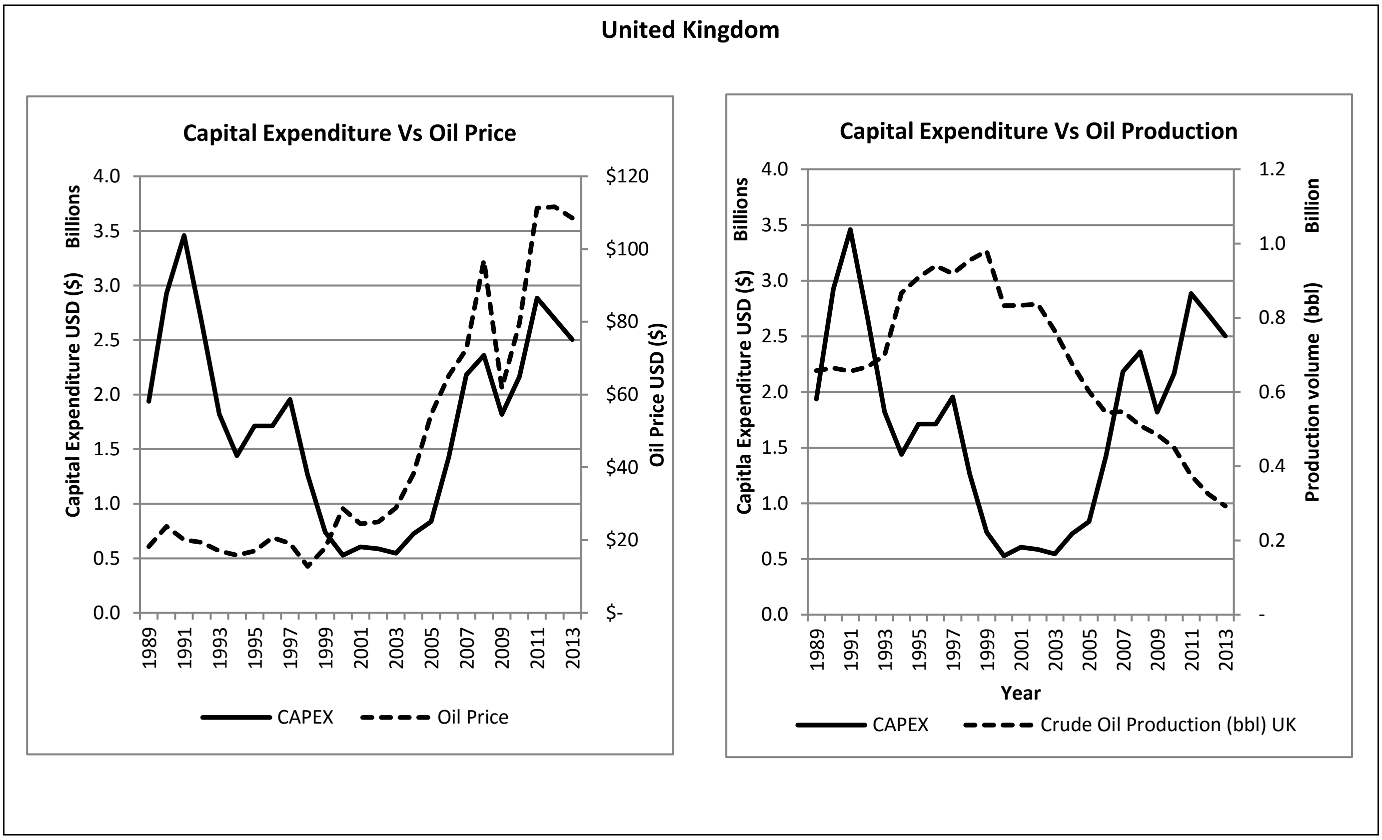


5.3 Results and Discussion

The empirical analysis in this chapter is designed to measure the impact of the Hubbert model, Engineering model and the Hotelling model on the real oil production in country model, thereby providing an empirical observation on whether these two countries being compared do indeed adhere to these theories. From the results shown in figure 10 and 11 below we see that in the case of the United Kingdom, the Hubbert model is shown to have a statistically significant positive impact on oil production. This is in line with the results of the Hubbert model test for the UK which shows strong support for the Hubbert theory. The models are corrected for collinearity by using cumulative values and increment ratios for some of the independent variables, following which the results show that the engineering approach has a statistically significant relationship with production. This observation is in also line with earlier results seen in the decline curve analysis carried out in previous sections, in which the UK production profile shows support for then engineering model, therefore showing support for the theory. Following from earlier chapters of this study, it is also expected that the UK production would show some support for the Hotelling theory. The results of this empirical analysis which combines the economic model by way of the Hotelling theory shows evidence of support of this expectation. A negative relationship between oil production and interest rate is observed from the analysis, therefore corroborating results from analysis of the UK model production model which was conducted in earlier chapters of this study. So it may be concluded that the observed negative impact is in line with the Hotelling theory.

Analysis of the impact of capital expenditure on UK production on the other hand is not as definitive. The data shows that investment trends can be observed to follow oil price as figure 17 below shows. We also see however that where capital expenditure is high and oil price was low, oil production volumes are high. So this trend may therefore explain the results seen in the empirical analysis. The cost of production, which captures production operating expenditure, shows the expected significant negative relationship with production. Cost of oil production in the UK has continued to increase despite dwindling oil production and this is supported by the results of earlier observations.

Figure 17: UK Capital expenditure trend



Results in the case of Nigeria also show a corroborating evidence of support of the Hotelling theory seen in earlier chapters of this study. A statistically significant negative relationship between bond yield rate and production can be observed in support of the Hotelling theory. The corporate bond yield rate captures the change in value of financial assets. We note that although observations may shrink to minimum required for empirical analysis, this number of data points is still in line with annual research data modelling and analysis standards carried out within the oil and gas industry. The results also show statistical significance in support of the Hubbert theory despite the observation of a weak Hubbert curve seen in earlier analysis, thereby providing an additional effect on production profiling. And in spite of the effect of OPEC factors and production quotas, Nigeria's production profile shows evidence of some support of Hubbert theory.

The results of the engineering model relationship with Nigeria's production on the other hand do not provide evidence of a significant relationship. The analysis from the earlier chapters shows that the decline curve falls outside of Nigeria's actual production profile; therefore, this approach does not capture a production decline. This inability to explain Nigeria's production may be due to the nature of the approach which is normally applied on a well or field scale and not on a country aggregate level. This being said, because this approach is the industry standard for production profiling, the method was applied to evaluate the significance of the results it would yield. The test of the engineering plan worked for the United Kingdom but not for the case of Nigeria. It may be that, the ability to maintain production capacity as a result of increasing reserves in combination with a slow depletion rate are reasons that make a decline rate not readily captured. Nevertheless, it is possible that following renewed production regulation and annual production quotas, a stronger Hubbert curve might be observed with future production, possibly a multi-cyclical curve as Nigeria regulates its production. The below tables show the results of the empirical estimate of models 13.1 and 13.2 for both countries.

Table 10: Estimation of Effect of Hubbert, Engineering and Hotelling production modelling theories on oil production – (1989 – 2013) – A case of the United Kingdom

	uk1	uk2	uk3	uk4	uk5	uk6	uk7	uk8
LOG(Q_HUBBERT)	0.91		0.76				0.70	0.74
	(21.56)		(9.89)				(6.81)	(9.30)
LOG(OIL_PRICE)			-0.12	-0.15	-0.13	-0.13	-0.12	-0.16
			(-2.35)	(-1.2)	(-0.91)	(-0.88)	(-2.39)	(-2.40)
LOG(INTEREST RATE)			-0.11	0.41	0.44	0.42	-0.05	-0.01
			(-0.95)	(1.47)	(1.42)	(1.41)	(-0.38)	(-0.06)
LOG(Q_ENGINEERING)		1.15		0.63	0.60	0.61		
		(16.27)		(1.90)	(1.64)	(1.64)		
LOG(CAPEX)					-0.03	(-0.61)	-0.03	
					(-0.27)		(-0.93)	
LOG(OPEX)						-0.05		0.13
						(1.41)		(0.94)
constant	0.58	0.95	2.16	2.96	3.02	3.13	2.48	1.97
	(2.13)	(1.01)	(3.10)	(1.57)	(1.53)		(3.19)	(2.71)
No of observation	25	15	25	15	15	15	25	25
R-squared	0.95	0.95	0.96	0.96	0.96	0.96	0.96	0.96
Adjusted R-squared	0.95	0.95	0.96	0.95	0.95	0.95	0.96	0.96
Durbin-Watson stat	0.77	0.61	0.87	0.78	0.86	0.80	0.84	0.85

Note: Data is for 1989–2013. T-stats are in parentheses.

Table 11: Estimation of Effect of Hubbert, Engineering and Hotelling production modelling theories on oil production – (1989 – 2013) – A case of the United Kingdom

	uk9	uk10	uk11	uk12	uk13
LOG(Q_HUBBERT_MODEL)		0.62		0.66	
		(5.92)		(7.71)	
LOG(OIL_PRICE)	0.05	-0.02	-0.19	-0.14	-0.20
	(0.47)	(-0.28)	(-2.26)	(-3.18)	(-2.47)
LOG(Interest rate)	0.25	-0.26	0.37	0.28	0.43
	(1.18)	(-1.96)	(1.91)	(1.57)	(2.25)
LOG(Q_ENGINEERING_MODEL)	0.11		-0.09		
	(0.39)		(-0.30)		
LOG(Cumm CAPEX)			-0.90	0.18	-1.01
			(-3.73)	(2.88)	(-4.62)
LOG(Cost per bbl)	-0.48	-0.23			
	(-3.21)	(-1.93)			
LOG($QEng_t$ /$QEng_{t-1}$)					7.00
					(0.97)
constant	6.48	3.61	9.95	1.65	10.27
	(3.67)	(3.62)	(4.39)	(2.58)	(8.62)
No of observation	15	25	15	24	14
R-squared	0.98	0.97	0.98	0.97	0.98
Adjusted R-squared	0.97	0.96	0.98	0.97	0.98
Durbin-Watson stat	1.21	0.96	1.63	1.30	1.75

Note: Data is for 1989–2013. T-stats are in parentheses.

Table 12: Estimation of Effect of Hubbert, Engineering and Hotelling production modelling theories on oil production – (1989 – 2013) – A case of Nigeria

	nig1	nig2	nig3	nig4	nig5	nig6	nig7	nig8
LOG(Q_HUBBERT)	0.99		0.70				3.10	8.94
	(0.04)		(3.39)				(0.69)	(3.18)
LOG(OIL_PRICE)			-0.01	0.13	0.02	0.15	0.001	0.02
			(-0.52)	(1.75)	(0.11)	(1.92)	(0.01)	(0.26)
LOG(INTEREST RATE)			-0.17	-0.44	-0.32	-0.12	-0.41	0.11
			(-1.68)	(2.58)	(-1.07)	(-0.42)	(-2.22)	(.79)
LOG(Q_ENGINEERING)		-0.22		0.61	-0.70	-0.22		
		(-2.65)		(1.89)	(-0.39)	(-0.32)		
LOG(CAPEX)					-0.26		-0.23	
					(-0.78)		(-1.44)	
LOG(OPEX)						-0.29		-0.61
						(-1.54)		(-4.66)
constant	0.04	7.95	2.36	3.60	11.66	8.01	-12.96	-53.02
	(0.06)	(17.5)	(1.56)	(1.93)	(1.08)	(2.13)	(-0.44)	(-2.82)
No of observation	25	15	25	15	11	11	11	11
R-squared	0.85	0.35	0.87	0.62	0.61	0.67	0.63	0.87
Adjusted R-squared	0.84	0.30	0.85	0.52	0.35	0.49	0.38	0.78
Durbin Watson stat	1.31	1.28	1.34	1.71	1.36	1.58	1.37	1.98

Note: Data is for 1989–2013. T-stats are in parentheses.

Table 13: Estimation of Effect of Hubbert, Engineering and Hotelling production modelling theories on oil production – (1989 – 2013) – A case of Nigeria

	nig9	nig10	nig11	nig12	nig13	nig14
LOG(Q_HUBBERT_MODEL)		6.18		-13.85		
		(4.70)		(-1.80)		
LOG(OIL_PRICE)	0.10	0.002	0.08	0.11	0.012	0.07
	(1.84)	(0.06)	(0.79)	(1.06)	(0.40)	(0.76)
LOG(Interest)	0.08	0.10	-0.50	-0.17	0.14	-0.52
	(0.36)	(1.22)	(-2.58)	(-0.87)	(1.43)	(2.62)
LOG(Q_ENGINEERING_MODEL)	-0.56		1.20		-0.04	
	(-1.23)		(2.17)		(-0.16)	
LOG(Cumm CAPEX)			0.11	0.32	0.15	0.09
			(1.19)	(1.52)	(5.36)	(1.03)
LOG(Cost per bbl)	-0.33	-0.41			-0.37	
	(-3.31)	(-8.41)			(-8.24)	
LOG(QHub t /QHubt-1)						44.09
						(2.16)
constant	9.83	-34.51	0.25	98.98	6.73	6.64
	(3.94)	(-3.92)	(0.94)	(1.94)	(5.34)	(14.12)
No of observation	12	12	12	12	12	12
R-squared	0.83	0.95	0.64	0.58	0.97	0.64
Adjusted R-squared	0.73	0.92	0.43	0.35	0.95	0.43
Durbin-Watson stat	1.70	2.08	1.87	1.72	2.25	1.87

Note: Data is for 1989–2013. T-stats are in parentheses.

Taking into account studies such as Reynolds Kolodziej (2007); Reynolds and Pippenger (2009), and Nigeria's involvement with OPEC, it was expected that the Hubbert curve might not fit Nigeria's production model. However, the Hubbert theory does in fact detect a rough curve in Nigeria's production, and the empirical analysis shows significant support of the theory. The linear relationship evident in the results for both country models is in agreement with (Pickering 2008).

Results from this chapter show the significance of the use of various model approaches on the design of oil production activities over the life of a field or producing region. The measure of this impact affects matters such as production forecasting, where for example in the case of the UK we see a conservative prediction of depletion from the Hubbert model in comparison to the engineering model. Where the Hubbert model shows a significant relationship with the actual production, its conservative forecast immediately comes to the fore as an important factor with respect to investment and revenue generation. The significance of the model relationship with actual production volumes lends weight to the level of impact a forecasting or production modelling approach can have on a country's production profile. Subsequently, it may be concluded that the level of import of the Hubbert model to the UK future production is not insignificant. When we look at the case of Nigeria, disregarding the effect and possible impact following the expectation of weak Hubbert theory support could result in exploration and production planning operations which do not capture the true nature of the true impact of the Hubbert theory on the country's future production profile.

5.4 Conclusions

The combination of the various modelling approaches and theories in an empirical analysis has shed some light on the individual limits of prevailing production modelling theories. The empirical analysis carried out on the models highlights how each model relates in comparison to the actual production profiles of Nigeria and the United Kingdom. Though the observation points in some of the models cases fall within the minimum allowed for a statistical study, they are the norm in oil industry E&P studies. The explanatory power in each production modelling approach is varied but however significant, and the results show that when used together a clearer view of their impact emerges, therefore providing more information than would be available if taken individually.

Country Model Results

	Hotelling Theory Production model 1	***Hubbert Theory Production model 2***	***Decline curve Analysis Production model 3***
UK ***(Empirical analysis)***	Weak support *(Significant)*	Strong support *(Significant)*	Depletion profile *(Significant)*
Nigeria ***(Empirical analysis)***	Strong support *(Significant)*	Weak support *(Significant)*	No depletion detected *(Not conclusive)*

Table 14: Production Model Theories - Country Model Results of Theory tests

The results of this study have shed light on the impact of various factors on oil production these two institutionally similar countries. They have shown the impact of the economic and geological constraints on oil supply by these leading producers. These factors have been measured individually using the Hubbert model, which examines supply via reserves and depletion, the decline curve analysis, which models production using decline profile, the Hotelling theory which takes into account cost, interest rate and oil price and additional factors such as OPEC. In order to quantify their relationship and measure their level of impact, the results were compared within an empirical model which combined them. Using this method, this study has been able to test, quantify and compare prevailing industry production models on leading oil producing countries. The results show that this combination of the economic and technical modelling approaches may provide even more insight on petroleum production performance. These results support the findings highlighted in Sorrel et al (2010); Jakobsson et al (2012).

The importance of petroleum revenue from oil production to both the United Kingdom and Nigeria cannot be underplayed, and it is the nature of the business to attempt to forecast production and depletion rates and dates for future planning. Having investigated all of the above factors and production modelling approaches, this study may be progressed and expanded to investigate the impact of government policy on petroleum revenue based on these production volumes in comparison with each other.

Except in the event of additional recoverable reserve volumes, which may give rise to another cycle within the curve, the United Kingdom is well into the production depletion stages of its reserves. Therefore, optimum planning and execution of oil and gas projects is crucial in the face of rising cost, fluctuating oil prices and low production presents precarious position. The oil business needs to remain profitably attractive whilst ensuring revenue to all stakeholders including the government, so this magnifies the importance of production modelling and forecasting which in turn affects planning and production. The high reserve volumes in the case of Nigeria on the other hand continue to ensure a certain level of interest. And though Nigeria may support the Hotelling theory, this may not be such a strong motivator if oil prices continue to fluctuate and production quotas remain in place, thereby making it difficult for stakeholders to increase production and balance revenue shortfalls.
However, with oil exploration and production revenue being the largest singular industry contributor to UK government revenue and accounting form more than over seventy percent Nigeria's total government revenue take, efficient management government oil policy in both countries is critical.

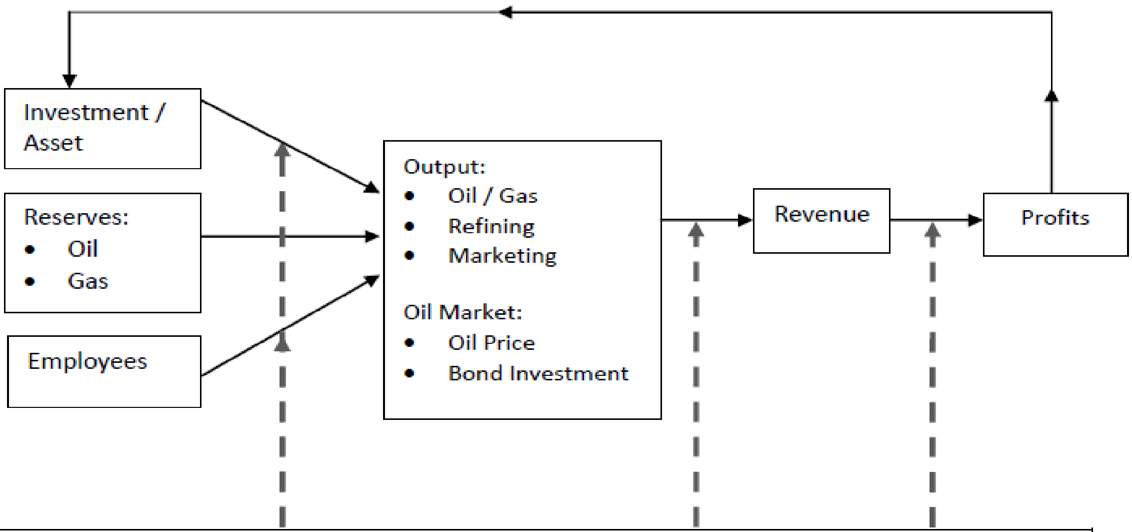
6 CHAPTER 6: CONCLUSIONS

The results of this research have shed light on the impact of various factors on oil production of the United Kingdom and Nigeria which are institutionally similar. These factors have been measured individually using the Hubbert model, which examines supply via reserves and depletion, the decline curve analysis, which models production using decline profile, the Hotelling theory which takes into account cost, interest rate and oil price and additional factors such as OPEC. In order to quantify their relationship and measure their level of impact, the thesis employs the Hotelling model, Hubbert peak oil model and the combined Cobb-Douglas production model. Using this method, this study has been able to test, quantify and compare prevailing industry production models on leading oil producing countries. The results show that this combination of the economic and technical modelling approaches may provide even more insight on petroleum production performance. These results support the findings highlighted in Sorrel et al (2010); Jakobsson et al (2012).

The chart below highlights factors which affect oil and gas production operations which include production volume, and therefore oil and gas profit chain. The chart has been modified from Wolf (2009) to include the additional factors investigated by the preceding chapters. This study has investigated on a country level reserves, expenditure, and national policy in the guise of OPEC membership; and on a market level it has investigated commodity price and interest rates to provide a measure of their combined effect on actual production volumes.

Figure 18: Oil and Gas Profit chain with key influence factors

Oil and Gas Profit chain with key influence factors



Managerial & Technical capabilities and incentives		
Structural variables		
Firm level	**Country level**	**Market level**
• Non-commercial objectives • Underlying asset quality • Reserves entitlement, management and accounting • Production policy (depletion) • Operational profile • Taxation	• Policy objectives • National petroleum industry structure • Petroleum governance (including licensing, fiscal regime) • State capacity • Stage of economic development	• Commodity prices and margins • Quality / pricing differentials

Source: Wolf, 2009

Using a case study scenario over a period 25 years, annual data for oil production, operating expenditure, oil price, and interest rates on a country aggregate level are used to develop the models for both countries. The results have been compared against the model expectation to estimate their level of support for the economic and engineering production models. Using the results, forecasting models based on the Engineering production and Hubbert production models were developed and extrapolated for both countries to predict depletion in both country cases. Following this an econometric model which combines the theories into a single model was developed to measure which of the production theories is strongest; i.e. which these competing effects best describes the production behaviours in both countries, and how they compare against the reserves constraint in describing production.

The estimated results have highlighted the impact of the geological and economic constraints on oil supply in both countries. The results of the analysis support both the Hotelling theory which best describes the Nigeria production, and the Hubbert model which best describes the United Kingdom production. The results shed light on the limitations of various production modelling theories. They show the Hubbert theory to be the most robust in explaining actual production profiles in both countries, while the Decline Curve theory (Engineering) has the weakest explanatory power. The results of this study indicate support that despite the varying explanatory power of the production model approaches in the two countries, when used in combination these theories can provide a more comprehensive perspective on production behaviour. These results support the findings highlighted in Sorrel et al (2010); Jakobsson et al (2012).

One major contribution of this thesis to literature is the finding of evidence to support our arguments for oil production that is determined by combined factors of Hotelling theory, Hobbert theory and Cobb-Douglas theory. On this basis, we claim that oil production is different from conventional production functions.

Current studies point to some prevalent factors that affect oil and gas production plan and designs (see figure 18). The economic factors which affect hydrocarbon asset development and production can be grouped into four. They are Oil and Gas Fiscal Policy, Domestic/Political Policy, Operational, and Monetary factors. In keeping with the study by Kalstod and Soreide (2009) and Dworkin (1977), we can take policy in this context to be the current position or focus of a government in developing its natural resource, which usually encompasses political and fiscal policies. Though there already exists much in-depth analysis of these economic factors on country economic growth, corporate governance, political stability and socioeconomics, these studies can be expanded to carry out a study of the combination of these factors in comparison with different country scenarios. The study by Al-Kasim et al (2013) sets good basis for the analysis the selected factors such as oil and government policy on fiscal revenue for Nigeria and the United Kingdom.

The limit of this thesis is the sample size which is small. In the future the sample size needs to be enlarged to test our new developed oil production model that combines both Cobb-Douglas production factors and oil reserve constraints.

7 APPENDIX

7.1 Appendix 1: EXCEL Solver Method for Bell curve Extrapolation

Hubbert equation is defined as:

$$f(x) = a \exp\left(-\frac{(x-b)^2}{2c^2}\right)$$

Or it can also be expressed as:

$$f(x) = Mexp^{-\left(\frac{C-x}{W}\right)^2}$$

Where M is the maximum value of curve (which occurs at C), x is value at the year (x), C is centre

Nigeria	Peak year 'C' =	2005
	Max Production 'M' =	959mmbbl
	Width 'W' =	24
UK	Peak year 'C' =	1999
	Max Production 'M' =	980mmbbl
	Width 'W' =	24

Step 1: Create column and solve for Hubbert production using the equation

Step 2: Create Error column → $Q_{hubbert} - Q_{actual}$

Step 3: Create error squared column → $(Q_{hubbert} - Q_{actual})^2$

Step 4: Create cell for sum of errors squared

Step 5: Go to data → Solver (far right)

Step 6: To constrain the curve → set target cell to sum errors square → equal to min of 0 → by changing cells to cells c, m, and w.

This solves the curve to target cells by making sum error squared minimum at 0 – thereby changing c, m and w.

Step 7: Click Solve

Step 8: Keep the solver solution

Step 9: Click OK

Step 10: Drag down to extrapolate for future years

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