EXAMINING THE RELATIONSHIP BETWEEN TRADING VOLUME, MARKET RETURN VOLATILITY AND U.S. AGGREGATE MUTUAL FUND FLOW

A thesis submitted for the degree of Doctor of Philosophy

by

Hayan Omran

Department of Economics and Finance

Brunel University London

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Abstract

This thesis consists of three studies which cover topics in the trading volume-market return volatility linkage, stock market return-aggregate mutual fund flow relationship as well as market return volatility-aggregate mutual fund flow interaction. Chapter 2 investigates the issue of volume-volatility linkage in the US market for the period 1990-2012 (S&P 500) and 1992-2012 (Dow Jones). We construct four sub-samples depending on three different structural points (the Asian Financial Crisis, the Dot-Com Bubble and the 2007 Financial Crisis).

By employing univariate and bivariate GARCH processes, we find positive (negative) bidirectional linkages between these two aforementioned variables in various cases of the estimation, while a mixed one is observed in the remainder of these cases.

Chapter 3 examines the issue of temporal ordering of the range-based stock market return (S&P 500 index) and aggregate mutual fund flow in the U.S. market for the period 1998-2012. We construct nine sub-samples represented by three fundamental cases of the whole data set. In addition, we take into consideration three essential indicators when splitting the whole data set, which are the 2000 Dot-Com Bubble, the 2007 Financial Crisis as well as the 2009 European Sovereign Debt Crisis.

We examine the dynamics of the return-flow interaction by employing bivariate VAR model with various specifications of GARCH approach. Our principal findings display a bidirectional mixed feedback between stock market return and aggregate mutual fund flow for the majority of the sub-samples obtained. Nevertheless, we provide limited evidence of a positive bi-directional causality between return and flow.

Chapter 4 investigates the dynamic relation between S&P 500 return volatility and U.S. aggregate mutual fund flow for the period spanning between 1998 and 2012. We assess the

dynamics of the volatility-flow linkage by employing a bivariate VAR model with the GARCH approach which allows for long memory in the mean and the variance equations.

In addition to the sub-samples obtained in chapter 3, we generate two measurements of volatility. Our baseline results indicate a variety of bidirectional mixed causalities between market return volatility and aggregate mutual fund flow in several sub-samples. In addition, we observe a negative/positive bi-directional relationship between volatility and flow in the rest of the sub-periods.

Summarizing, a range of our findings are in line with the empirical underpinnings that most likely predict a significant linkage between the aforementioned variables. Finally, most of the bidirectional effects are found to be quite robust to the dynamics of the various GARCH processes employed in this thesis.

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Declaration

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Chapter One

Introduction

In this thesis, chapter 2 examines the issue of volume-volatility linkage in the US market for the period 1990-2012 (S&P 500) and 1992-2012 (Dow Jones). Trading volume and return volatility are inextricably linked, and are jointly and simultaneously determined by the same market dynamics with respect to the theoretical and empirical standpoints. The aim of this chapter is to investigate the empirical interaction between volume and volatility and whether this relation could be positive, negative or even mixed bi-directional causality.

A related theoretical issue in the prior literature has presented the dynamics of return volatility. Bessembinder and Seguin (1993) have clarified that volume data possessed by a specific type of dealers might help explaining the linkage between volume and market return volatility. Furthermore, Bekaert and Harvey (2000) have found an insignificant impact of the foreign activity's measurement on the volatility.

Darrat et al. (2003) have not noticed a contemporaneous linkage between volume and volatility by examining the huge majority of DJIA stocks. Wang (2007) has stated that trading through domestic and foreign investor groups is not prominently affecting volatility. Moreover, Chen and Daigler (2008) have indicated that the volume-volatility link is the main method in order to measure the importance and rate of information flow.

On the other hand, Karanasos and Kartsaklas (2009) -as an example of prior empirical studies- have noticed a positive effect of foreign volume on volatility after the Asian Financial Crisis and a reverse impact in the pre-crisis period.

We analyze the volume and volatility dynamics of S&P 500 and Dow Jones stock indices respectively. We estimate the two main parameters in the two aforesaid variables by employing the VAR model with various cases of GARCH approach including the issue of exogenous variables (the lagged values of one variable obtained in the mean equation of the other variable).

In addition, we employ a long span of daily data (1990-2012 for S&P 500, 1992-2012 for Dow Jones respectively) and four sub-sample periods by taking into account the case of Asian Financial Crisis, the Dot-Com Bubble and the 2007 Financial Crisis respectively.

Our results show a positive/negative bidirectional linkage between volume and volatility in various cases of estimation, while a mixed one is observed in the other cases of this estimation (volume affects volatility positively (or negatively) whereas the reverse impact is of the opposite sign).

Chapter 3 examines the issue of temporal ordering of the range-based stock market return (S&P 500 index) and aggregate mutual fund flow in the U.S. market for the period 1998-2012. The unprecedented growth of mutual funds has raised the need for an improved understanding of the linkage between mutual fund flow and stock market return.

The literature on the dynamic relationship between return and flow is mixed. Alexakis et al. (2005) have reported statistical evidence of a mixed bi-directional causality between stock market return and mutual fund flow. Braverman et al. (2005) have presented that this linkage is significantly negative. Some other studies such as Fortune (1998), Mosebach and Najand (1999), and Cha and Kim (2010) have observed a positive relation between market return and mutual fund flow.

A mutual fund is a particular type of an investment vehicle or an institutional device through which investors pool their savings that are to be invested under the guidance of a group of experts in an enormous variety of corporate securities' portfolios in such a way, which does not only minimize risk, but also ensures safety and stable investment's return (see, for instance, Dave (1992) and Mehru (2004) and the references therein).

There are various advantages of mutual fund investments which simplify money management by professionals at the lowest cost and ensure dilution in the transaction costs as a result of the large scale of the economies of operation. Furthermore, they provide flexibility for investors to change the investment objective and they are also convenient for investors to both invest and track their invested capital performance.

The main objective of this chapter is to detect whether flow has a positive/negative impact on return and vice versa, how the potential changes in market price return might affect the mutual fund flow, and whether the lagged values of flow could affect return and vice versa. We assess the return-flow linkage by estimating the main parameters in the two aforementioned variables. For this purpose, we apply bivariate VAR model (examining the impact of the lagged values of one variable included in the mean equation of the other variable) with several cases of the GARCH approach and its' long memory extensions.

In addition, we obtain our required sample of aggregate mutual fund flow consisting of 1,774,367 daily observations and including only the U.S. domestic mutual fund flow existing at any time during the period spanning from February 3rd 1998 to March 20th 2012. We impose some selection criteria depending on Morningstar Category Classifications. This selected process has rendered a final sample of 1,700 domestic mutual funds on average with 3,538 daily observations.

Furthermore, taking into consideration three fundamental indicators which are the 2000 Dot-Com Bubble, the 2007 Financial Crisis and the 2009 European Sovereign Debt Crisis, we divide the whole dataset into three different cases. Whilst two sub-samples (A and B) are obtained in the first case, five ups-and-downs (UDs) sub-samples are included in the second section. In addition, the third case consists of two cyclical (CYs) sub-samples.

We detect a bidirectional mixed feedback between stock market return and aggregate mutual fund flow for the majority of the sub-samples obtained. In particular, flow affects return negatively whereas the reverse impact is of its opposite sign. Nevertheless, we provide limited evidence of a positive bi-directional causality between return and flow in the remainder of these aforementioned sub-samples.

Chapter 4 investigates the dynamic interaction between market return volatility (S&P 500 index) and U.S. aggregate mutual fund flow for the period spanning between 1998 and 2012. The purpose of this chapter is to investigate the empirical relationship between volatility and flow, to detect the impact of the lagged values of volatility on flow and vice versa, and to inspect whether the linkage between these two variables might be changed through employing various measures of volatility.

The study of Cao et al. (2008) is considered as the fundamental paper, which has presented direct evidence on the relation between market volatility and aggregate fund flow. They have employed a dataset for a sample of 859 daily mutual funds covering the period between 1998 and 2003 under the VAR framework, in order to examine the dynamic interaction between aggregate mutual fund flow and market return volatility.

Among others, they find that concurrent and lagged flow has a negative impact on daily market volatility. They also provide evidence of a negative contemporaneous interaction between innovations in market volatility and fund flow. Moreover, the market volatility is negatively affected by the shock in fund flow. That is, an outflow shock induces higher market volatility, whereas an inflow induces lower volatility.

Cao et al. (2008) suggest that the first lag of market volatility has a negative impact on daily fund flow and that the mutual fund investors might time market volatility at the aggregate fund level. Finally, their results display -from morning to afternoon- a strong relationship between outflow and intraday volatility, whilst the interaction between inflow and intraday volatility becomes weaker.

In order to examine the interaction between market volatility and aggregate fund flow, we estimate the main parameters in these two mentioned variables by applying bivariate VAR-GARCH process which allows for long memory in the mean and the variance equations.

In addition to the sub-periods mentioned previously in chapter 3, we employ two fundamental measurements with respect to market return volatility, which are Rogers-Satchell (RS) volatility as well as Garman-Klass-Yang-Zhang (GKYZ) volatility.

Our findings observe a variety of bidirectional mixed causalities between market return volatility and aggregate mutual fund flow in various sub-samples. More specifically, flow has a positive impact on volatility whilst volatility affects flow negatively. Moreover, we detect negative (or positive) bi-directional linkages between volatility and flow in the other sub-samples.

A range of our findings are in line with the empirical underpinnings that most likely predict a significant causal linkage between the aforementioned variables. Most of the bidirectional effects are found to be quite robust to the dynamics of the various GARCH processes employed in this study.

Finally, chapter 5 presents the main conclusions of this thesis.

Chapter Two

An Empirical Analysis Of The Dynamic Relationship Between Trading Volume And Stock Market Volatility

2.1. Introduction

From a theoretical and empirical standpoints, trading volume and return volatility are inextricably linked, and are jointly and simultaneously determined by the same market dynamics. Chen and Daigler (2008) have concentrated on four substantial information theories in observing the volume-volatility linkage. The first theory is the Mixture of Distributions Hypothesis (MDH), which has indicated that the association of volume-volatility case is the main method of measurement the importance and rate of information flow. In addition, the second theory is the Sequential Arrival of Information Hypothesis (SAIH), where information has been realized by various groups of traders at different times with a positive persistent volume-volatility correlation. The third theory is dispersion of beliefs hypothesis, which has demonstrated that with macroeconomic announcements, the same information that are received by every trader at the same time can be interpreted by different kinds of these dealers in different ways. The fourth theory is the noise trader hypothesis, which has assumed that -on a consistent basic- a significant mispricing could be achieved by the noise traders who are sufficiently dominant in the market.

A related theoretical issue in the prior literature has observed the dynamics of return volatility. As Bessembinder and Seguin (1993) have clarified that volume data possessed by a specific type of dealers might help explaining the linkage between volume and volatility. Bekaert and Harvey (2000) have found an insignificant impact of the foreign activity's measurement on the volatility. Moreover, and by examining the huge majority of DJIA stocks, Darrat et al. (2003) have not noticed a contemporaneous linkage between volume and volatility.

Wang (2007) has stated that trading through domestic and foreign investor groups is not prominently affecting volatility. Chen and Daigler (2008) have indicated that the volume-volatility link is the main method in order to measure the importance and rate of information flow. On the other hand, Karanasos and Kartsaklas (2009) -as an example of prior empirical studies- have noticed a positive effect of foreign volume on volatility after the Asian financial crisis and a reverse impact in the pre-crisis period.

This study has three principal objectives. Firstly, it analyzes the volume and volatility dynamics of S&P 500 and Dow Jones stock indices respectively. We estimate the two main parameters in the two variables by employing univariate GARCH-M (1,1) model. The second objective is estimating the bivariate VAR-BEKK GARCH (1,1) processes with lagged values of one variable included in the mean equation of the other variable. Estimating the bivariate VAR-CCC GARCH (1,1) models with lagged values of one variable obtained in the mean equation of the other variable is the third objective of this study.

Our contribution in this paper can be classified as follows. Firstly, we utilize a long span of daily data (1990-2012 for S&P 500, 1992-2012 for Dow Jones respectively) and four sub-sample periods (we take into account the case of Asian Financial Crisis, the Dot-Com Bubble and the 2007 Financial Crisis respectively). Moreover, we employ three different GACRH models (univariate GACRH-M, bivariate VAR-BEKK GARCH and bivariate VAR-CCC GARCH processes respectively).

Our major findings are as follows. In the case of S&P 500, we find a negative bidirectional linkage between volume and volatility in the whole sample, second (spanning between 13th January 1999 and 3rd April 2000) and third (covering the period 1st February 2002 to 25th July 2007) sub-samples using the three aforementioned models. A mixed bidirectional feedback between volume and volatility is found in the fourth sub-sample (between 21st October 2009 and 20th March 2012) through employing these models, and in the first sub-sample (spanning from 7th August 1990 to 27th October 1997) using univariate GARCH-M (1,1) and bivariate VAR-BEKK GARCH (1,1) models respectively. While –for the various samples- volatility affects volume negatively in the case of bivariate VAR-CCC GARCH (1,1) process, there is no significant impact of volume on volatility in the first sub-sample.

In the case of Dow Jones, a mixed bidirectional linkage between volume and volatility is found in the whole sample and the second sub-sample through employing univariate GARCH-M (1,1) model. We find positive bidirectional causality between these two variables in the first and fourth sub-samples respectively. Whereas volatility affects volume negatively, there is no impact of volume and volatility in the third sub-sample.

Using bivariate VAR-BEKK GARCH (1,1) model, a positive bidirectional relation between volume and volatility in the first, second and fourth sub-samples is captured respectively. Whilst we investigate a negative bidirectional linkage in the third sub-sample, we find a mixed bidirectional feedback between volume and volatility in the whole sample. Using bivariate VAR-CCC GARCH (1,1) process, we observe a mixed bidirectional feedback between volume and volatility in the whole sample and the fourth sub-sample. In addition, positive bidirectional causality is detected in the first and second sub-samples. On the contrary, we find a negative bidirectional linkage between volume and volatility in the third sub-sample of Dow Jones. In most of these cases above, we conclude that our results are robust to the GARCH specifications.

The remainder of this article is organized as follows. Section 2 discusses the theoretical and empirical backgrounds concerning the linkage between volume and volatility. Section 3 shows different measurements of these two variables. Section 4 introduces the data, describes the methods of constructing both of trading volume and 'Garman and Klass' volatility, and determines the structural breaks for the four sub-samples. Section 5 discusses our model specifications, reports and describes our empirical results. Section 6 concludes this paper.

2.2. Literature Review

2.2.1. Theoretical Background

The Various Impacts of Volume on Volatility

Lamoureux and Lastrapes (1990) have proved that the prediction of return volatility (or absolute price changes) would be improved by the pattern of information included in trading volume. Bessembinder and Seguin (1993) have examined the symmetric impact of volume on volatility through separating volume into its (expected and unexpected) components. They have found that the effect of unexpected volume shocks on volatility is larger than the impact of expected shocks. Furthermore, the negative unexpected volume shocks have a smaller effect on volatility in comparison with the impact of positive shocks on volatility. These results have been obtained by allowing each of these two components to have a distinct impact on price volatility.

Within studying the impact of exterior speculative activity on returns' volatility through twenty emerging markets, Bekaert and Harvey (2000) have stated the growing activity of the foreign investment within the extent of the net capital flows, country funds, the lifting of legal restrictions and the introduction of ADRs. They have stated that the volatility has been insignificantly affected by the measurement of the foreign activity. By boosting the market's capacity to obtain all the information-induced trading, Kawaller et al. (2001) have argued that the imbalances between liquidity demanders and liquidity suppliers have been mitigated through the remarkable increase in the whole non information-based trading. Furthermore, a marketplace with a smaller population of liquidity providers would be more volatile than that one with a larger population, and vice versa.

According to Dvořák (2001), the amount of foreign trading is considered as another measurement of external activity. While the vehicles for exterior speculators are ADRs and country funds, the alternative measurement of the activity of foreign speculative is the effective volume of foreign trading. The destabilizing impact of the activity of foreign investors is not substantially to be implemented by those investors even when they are considered as irrational and noisy foreigners. Dvořák (2001) has pointed out that the participation of foreign investors is highly recommended either where foreign investors destabilize markets less than local ones, or when liquidity is being supplied to the domestic markets through external trading activity. By increasing the investor base in the emerging markets, external purchases tend to reduce volatility within the first few years after the market liberalization with attempts to manage stable stock markets significantly.

On the extreme contrast, volatility is being increased and the investor base is being declined by foreign sales (Wang (2007)). Wang (2007) has also stated that investor base is not changed by trading through domestic and foreign investor groups, and as a result, this does not have a prominent impact on volatility respectively.

With respect to the Asian Financial Crisis, Karanasos and Kartsaklas (2009) have distinguished between the volume trading before this crisis and that trading after it. Depending on that, they have found a negative impact of foreign volume on volatility in the pre-crisis period, whilst after this crisis, this impact has turned to be positive. This result has been coordinated with that view which is in the emerging markets-when foreigners are buying into domestic markets and particularly in the first few years after the liberalization of the market- volatility is being increased by foreign sales whereas foreign purchases inclined to lower volatility. This implied that, the causal effect of domestic volume on volatility is totally missed before the Asian Crisis, whereas in the post-crisis period, the effect of total and domestic volume on volatility is obviously positive.

Theoretical Explanations beyond the Volume-Volatility Linkage

Whilst the volume-volatility relation is fundamentally underlying on the theories which in turn are related to the accessible information, Chen and Daigler (2008) have concentrated on four substantial information theories in illustrating the case of volume-volatility linkage.

The first theory is the Mixture of Distributions Hypothesis (MDH), this theory indicates that the association of volume-volatility case is the main method of measurement the importance and rate of information flow. The explanation of the relationship between volume and volatility -which has eventually been known as the MDH- has been firstly attempted by the theoretical work of Clark (1973) who has clarified that the changing variance of the changes in prices can be represented by volume. The modified mixture of distribution hypothesis (MMDH) has contained the liquidity requirements of the dealers as well as the informational asymmetries across groups. This adjusted hypothesis has been improved by Andersen (1996) where the autocorrelation stochastic volatility process is considered as the model of the information flow.

The second theory is the Sequential Arrival of Information Hypothesis (SAIH) where information has been recognized by various groups of traders at different times with a positive persistent volume-volatility correlation. This hypothesis has been developed by Copeland (1976, 1977) where the degree of volume-volatility association is dictated by the importance of information possessed by the dealer and also by the type of the dissemination of information. Phillips and Weiner (1994) and Ito et al. (1998) have demonstrated that the private information possessed by the Japanese oil traders and cash foreign exchange traders has a significant impact on the price and volatility of these specified markets. That notion which implied that volume data possessed by a specific type of dealers might help explain the relationship between volume and volatility, it has been suggested by Bessembinder and

Seguin (1993). In addition to this theory's principal factor which is the arrival of information to various types of traders at different speeds, another pertinent factor has been indicated by French and Roll (1986) which is the information asymmetry as public vs. private information is considered as a popularized example of the kind of information.

The third theory is dispersion of beliefs hypothesis, this theory has been developed by Harris and Raviv (1993) as well as Shalen (1993). They have demonstrated that with macroeconomic announcements, the same information that are received by every trader at the same time can be interpreted by different kinds of these dealers in different ways. According to Shalen (1993), a bigger difference in the beliefs -when different qualities of information are possessed by specific traders- can be considered as an association with an exceptional volume and excess volatility to those dealers. In particular, this model associates volatility with the dispersion of beliefs of uninformed traders.

The fourth theory is the noise trader hypothesis which has assumed that -on a consistent basic- a significant mispricing might be achieved by the noise traders who are sufficiently dominant in the market. Nevertheless, the lead-lag framework is the essential structure where this hypothesis should be examined through. This theory has been developed by DeLong et al. (1990a, 1990b) who have argued that destabilize pricing is the major cause of the response of informed traders to the noise trading instead of trading on fundamentals, and this result has been essentially raised by noise traders.

In a brief, the study of Chen and Daigler (2008) has theoretically concentrated on examining the behavior of two fundamental kinds of dealers in relation to their impact on volatility who are the informed institutional clearing members and the uninformed general public respectively. It has concluded that the institutional dealers can directly obtain information on order flow and possess more identical beliefs in comparison with the general public. This study has emphasized on a specific idea that whether -at the early stage of information transfer- the general public are being leaded by the institutional dealers, and then whether -at the subsequent stage of information assimilation- these institutional traders react to pricing errors issued by noise traders.

Causal Relation between Volume and Volatility

Smirlock and Starks (1988) have investigated -at the firm level- a significant causal relation between the absolute price changes (volatility) and trading volume. Furthermore, no causal volume-volatility relation has been found by Darrat et al. (2003) through using 30 DJIA stocks with 5-minute intraday data. By reviewing the approach of Chuang et al. (2012) which has been applied to ten Asian stock markets, they have documented a positive bi-directional trading volume-return volatility causality in Hong Kong, Korea, Singapore, China, Indonesia and Thailand respectively.

Contemporaneous Relation between Volume and Volatility

Tauchen and Pitts (1983) have showed that volume and volatility are contemporaneously proportional to each other where volume is being considered as a function of the number of the changes at the prices, and the information flow's variance is the major cause of increasing the correlation between these two variables. On the other hand, Karpoff (1987) has documented a significant and positive correlation between the volatility of stock prices' return and the trading volume within both the equity and futures markets. Another positive and contemporaneous relation has been obtained by Lamoureux and Lastrapes (1990) between the conditional volatility and trading volume in the U.S. stock returns through depending on a modern dynamic framework.

The evidence that the contemporaneous linkage between volume and volatility is being affected by the dispersion of traders' beliefs (the third hypothesis in the theoretical background) has been provided by Daigler and Wiley (1999). It has been shown that the exaggeration in price movements which might cause greater volatility is the main feature of the uninformed traders, and possessing various qualities of information is the essential reason for the different responses to changeable volatilities that are possessed by several groups of traders.

By examining the contemporaneous linkage between volume and volatility, Ghysels et al. (2000) have found through their literature a positive and significant relation between these two specified variables. Furthermore, Darrat et al. (2003) have showed no contemporaneous linkage between these specified variables, and this result has been included through examining the huge majority of DJIA stocks. With regard to the approach of Chuang et al. (2012), a negative contemporaneous linkage between trading volume and return volatility has been existed in Taiwan and Japan.

2.2.2. Empirical Background

An empirical evidence which has been found by Daigler and Wiley (1999) has indicated that the uninformed general public is the main reason for the positivity of the volume-volatility relationship. They have also stated that information arrival occurs separately from the activity of the liquidity providers. This result above has been obtained through using different types of ARIMA model as well as daily data for five distinct financial future contracts which are belonging to the Chicago Board of Trade and covering the period between June 1986 and June 1988.

Nevertheless, a dynamic relation between volume and volatility of nine stock market indices has been examined by Chen et al. (2001) using daily data for the period between 1973 and 2000. Their results have been obtained through using EGARCH models which have showed

–for all nine markets- a positive linkage between the trading volume and the absolute value of the stock price change (volatility).

Darrat et al. (2003) have investigated the contemporaneous correlation between return's volatility and trading volume through using 5-minute intraday day between April 1998 and June 1998. They have measured return's volatility by the EGARCH-M (1,1) model for all the 30 stocks which compose the DJIA stock market. Whereas the vast majority of the DJIA stocks (27 stocks) have showed no positive contemporaneous correlation between trading volume and return volatility, only three out of these 30 stocks have demonstrated a statistically significant and a positive correlation between these two variables. Moreover, they have pointed out that in a major number of these 30 DJIA stocks, a significant lead-lag linkage between trading volume and return's volatility has been detected.

Karanasos and Kartsaklas (2009) have investigated the temporal relation between the turnover volume and range-based volatility for the period 1995-2005. By using daily data through the Korean market, they have examined the dynamics of these two variables through using bivariate dual long-memory GARCH (1,1) processes and their respective uncertainties as well. With respect to the Asian financial crisis (1997), they have distinguished volume trading before this crisis from volume trading after it. In the pre-crisis period, no causal impact of domestic volume on volatility has been found, whereas there was a negative effect for foreign volume on volatility. On the contrary, they have showed a positive impact of both domestic and foreign volumes on volatility in the post-crisis period.

Through using daily trading volume and market price index for ten Asian stock markets spanning the period between 1998 and 2007, Chuang et al. (2012) have investigated the causal relation between trading volume and return volatility. They have found -by using a bivariate GARCH (1,1) model- a negative contemporaneous linkage between trading volume and return's volatility in Taiwan and Japan, but a positive relation in Indonesia, Singapore, Hong Kong, Thailand, Korea and China. All these results have been robust across all these samples of stock markets.

2.3. Measurement of Volume and Volatility In Literature

Measurement of Volume

Campbell et al. (1993) have produced the trading volume (the type which will be employed in this chapter) by using the moving average of the previous one hundred turnover by volume as follows:

$$y_{vt} = \frac{VLM_t}{\frac{1}{100}\sum_{i=1}^{100} VLM_{t-i}}$$

Another measurement of trading volume has been introduced by Bhaumik et al. (2011) that - by fitting a linear trend (t) and subtracting the fitted values for the original series- they have formed a trend-stationary time series of volume as follows:

$$y_t^{(v)} = \tilde{y}_t^{(v)} - (\hat{a} - \hat{b}t)$$

Where $\tilde{y}_t^{(v)}$ denotes the fitted values of the original series and v denotes volume. A reasonable compromise between computational ease and effectiveness is provided by this linear detrending procedure.

Measurement of Volatility

Two principal measurements of volatility have been showed in the related literature. The first type has been constructed by employing the classic range-based estimator of Garman and Klass (1980) as follows (the type will be employed in this chapter):

$$y_{gt} = \frac{1}{2}u^2 - (2ln2 - 1)c^2, t \in Z,$$

Where y_{gt} denotes volatility, c and u represent the differences in the natural logarithms of the closing and opening, and of the high and low prices respectively.

The second approach of volatility has been implied by Parkinson (1980) who has created the first advanced volatility estimator. High and low prices have been used instead of using closing prices. This estimator assumes continuous trading in addition to underestimating this volatility as potential movements when the market is shut are ignored.

Parkinson (1980) has constructed this specific type of volatility as follows:

$$\sigma_P = \sqrt{\frac{F}{N}}\sqrt{\frac{1}{4Ln(2)}\sum_{i=1}^{N}\left(Ln(\frac{h_i}{l_i})\right)^2}$$

Where σ_P denotes Parkinson volatility, F is the number of closing prices in a year, N is the number of historical prices used in the volatility estimate, h_i is the high price and l_i is the low price respectively.

2.4. Data Description and Sub-Samples

The data set which will be employed in this study comprises:

- 5,174 daily trading volume and prices of Dow Jones Stock Market Index, which spanning the period from 21st of May 1992 to 20th of March 2012.
- 5,641 daily trading volume and prices of S&P 500 Stock Market Index, which covering the period from 7th of August 1990 to 20th of March 2012 respectively.

The data above has been obtained through the New York Stock Exchange (NYSE), the NYSE is considered as the largest equities-based exchange in the world that based on the total market capitalization of its listed securities in the U.S.A. since 1792.

Turnover Volume

Since Dow Jones is recording the trading volume of thirty different domestic investors and S&P 500 is obviously recording the trading volume of five hundred domestic companies (investors) on a daily basis, we have constructed the domestic volume through adding all the different trading volumes of domestic investors. The turnover -which is by definition the ratio of the value of shares traded to the value of shares outstanding (see, for instance, Bollerslev and Jubinski (1999) and Campbell et al. (1993), and the references therein)- has been considered as the volume's measurement.

By incorporating the procedure that has been used by Campbell et al. (1993), we have formed the trend-stationary time series of turnover (y_{vt}) through using the moving average of the previous one hundred days. It has also been mentioned in the empirical literature of Lobato and Velasco (2000) who have considered the trading volume as non-stationary several detrending procedures for the data of volume (figure 2.1 for S&P 500 and figure 2.2 for Dow Jones respectively):

$$y_{vt} = \frac{VLM_t}{\frac{1}{100}\sum_{i=1}^{100} VLM_{t-i}}$$

Volume is being represented by VLM. The change in the long run movement of the trading volume has been captured by the time series produced by this metric (Brooks (1998) and Fung and Patterson (1999)). A compromise between computational ease and effectiveness has been reasonably provided by the moving average procedure.

Figure.2.1. Volume of S&P 500

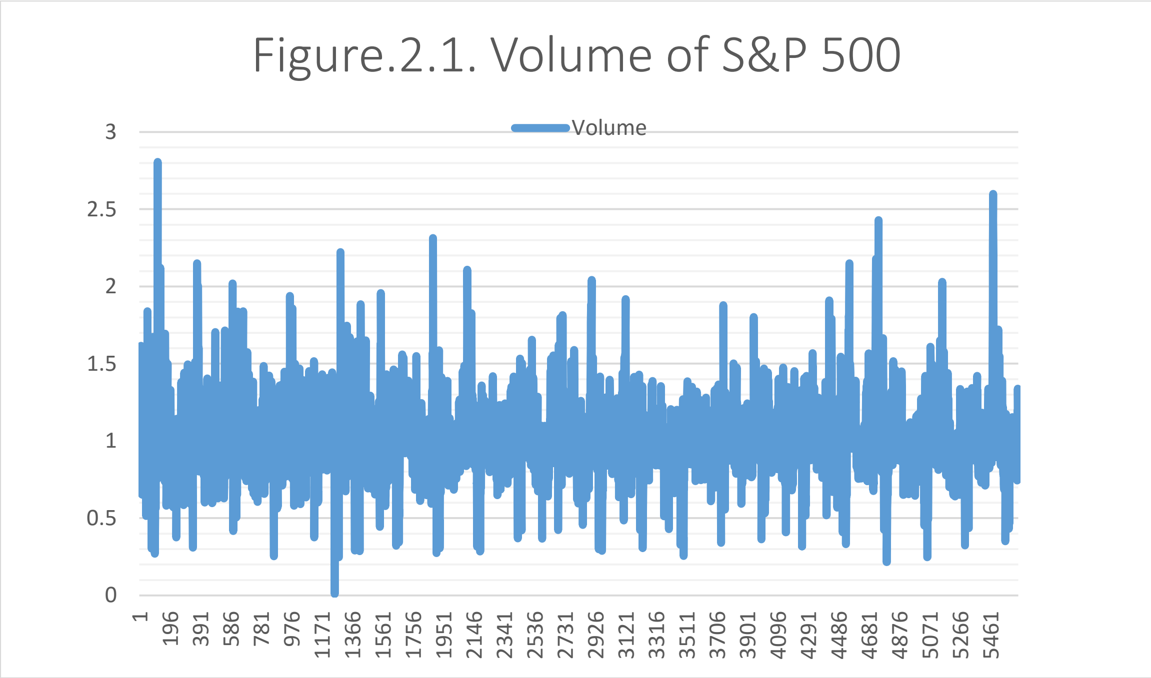
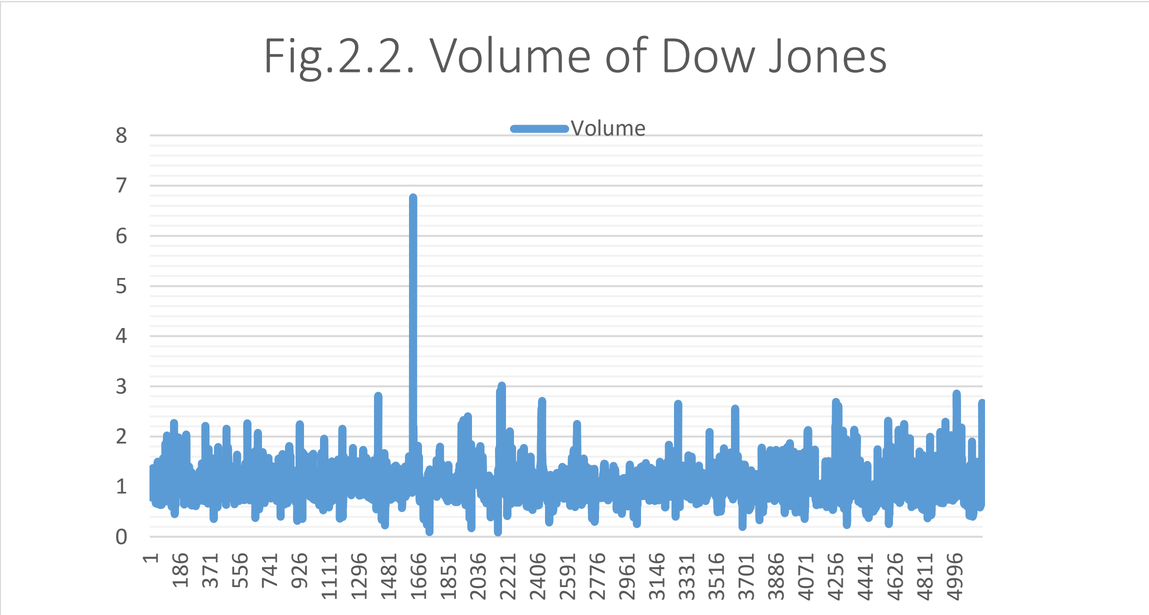


Figure.2.2. Volume of Dow Jones



The Measurement of Stock Volatility

A daily measurement of price volatility has been generated through using the daily opening, closing, high and low prices of both S&P 500 and Dow Jones indices separately.

The different information from the available data of the daily price which was used by several alternative measures is the main point on how to choose one of these various measures in case of obtaining this volatility of the daily prices.

Based on the fundamental equivalent results provided by the range-based and high-frequency integrated volatility and which have been mentioned in the conclusion of Chen et al. (2006), in addition to the high frequency data that has introduced the microstructure biases which should be avoided, we have constructed the daily volatility (y_{gt}) through employing the classic range-based estimator of Garman and Klass (1980) as follows (figure 2.3 for S&P 500 and figure 2.4 for Dow Jones respectively):

$$y_{gt} = \frac{1}{2}u^2 - (2ln2 - 1)c^2, t \in Z,$$

Where c and u represent the differences in the natural logarithms of the closing and opening, and of the high and low prices respectively.

According to Wiggins (1992), the traditional close-to-close estimators normally present more bias and less efficient than that estimator of Garman and Klass (1980). Chen and Daigler (2004) have showed that the high frequency data which has constructed the realized volatility might possess the potential biases that produced by the market microstructure factors - especially through studying the values of cash index- such as stale prices, the uneven time spacing of trading and bid-ask bounce. These problems have been circumvented through the range-based estimator of Garman and Klass (1980) which has been pointed out by Alizadeh et al. (2002).

Figure.2.3. Volatility of S&P 500

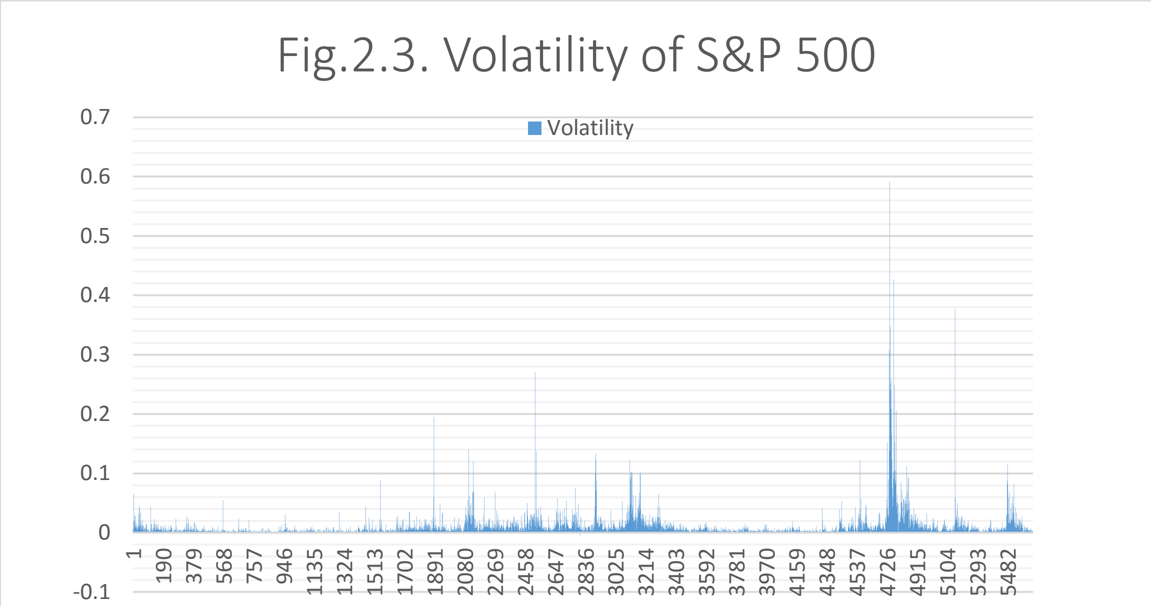
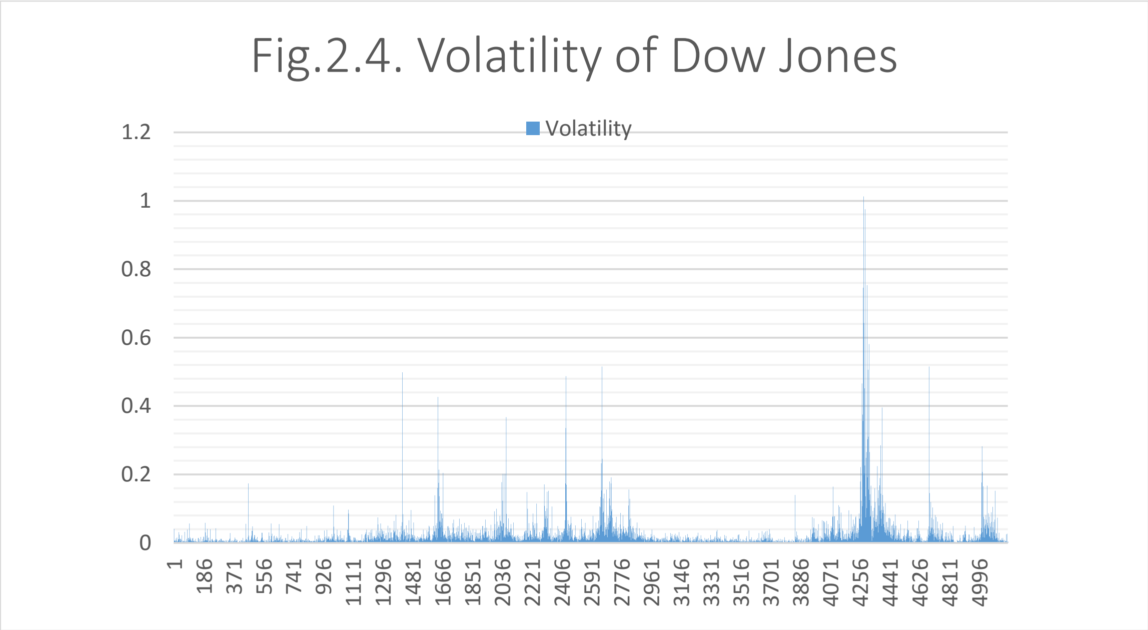


Figure.2.4. Volatility of Dow Jones



Sub-Samples and Structural Breaks

Bai and Perron (2003) –under the most general conditions on the data and the errors respectively- have addressed the problem of testing for multiple structural changes in the context of least squares. They have also found that identification the number and location of the multiple brakes is accompanied with the testing for the presence of these breaks.

Nevertheless, the confidence intervals for the break dates have been formed by Bai and Perron (2003) under diversified hypotheses about both the structure of the data and the errors across segment, which is introducing the possibility of estimating models for various dates of break within the 95 percent confidence interval and evaluating whether or not our inferences are indeed robust to these alternative break dates. Following that one which is responsible for minimizing the sum of squared residuals, our non-reported results definitely seem to be invariant to break dates.

We have addressed three main change-points associated with three principal financial crises. The first change-point has been chosen on the 28th of October 1997. The second break-point has been investigated on the 4th of April 2000, while the last point has been chosen on the 26th of July 2007. The Asian Financial Crisis in 1997 has been considered as the primary association of the first change-point in volatility. This crisis has begun after Thai government has been forced to float the baht (the local currency), so this has been considered as the financial collapse of the Thai currency.

Additionally, the bankrupt of Thailand has effectively started before the collapse of baht because of the burden of foreign debt which has been acquired by Thailand. The classification of the countries affected by this crisis could be presented as follows:

- Vietnam, Brunei, Taiwan, Singapore and China were the least suffered countries.

- Laos, Malaysia, Hong Kong and Philippines have been hurt by this recession more than those countries mentioned above.
- Thailand, Indonesia and South Korea have been the most affected countries through this crisis.

It has been mentioned that a significant uncertainty –which related to the emerging markets in Asia as well as in South America and Russia- has been detected in October 1998, after that, recovery in the economics and the market had begun back.

The second break-point has been associated with the Dot-Com Bubble which has been accompanied by the increase of technology industry and Internet sites that has been resulted in the bursting of this bubble and significantly noticed in the peak of the heavy technology NASDAQ Composite Index in more than double of its actual value just one year before. The remarkable case is that this bubble has been caused by the combination of several factors, the usual definition of this combination is the period of speculation and investment in Internet firms between 1995 and 2001.

During the year 1995, the dot-com companies –which have been considered as the potential consumers- have represented the beginning of a sharp jump in the growth of Internet users. With their attempts of dominating the market, many of these companies have started engaging in unusual business practices like engagement in the policy of growth over profit through assuming that their profits might increase if they would have built up their customer base successfully. Because of the burning of the majority of these dot-com companies through their venture capital, while many of them have never made any profit, the deflation of this bubble has been considered speedily by the end of the year 2001.

The third change-point has been associated with the Global Financial Crisis which has been considered by many economists as the worst financial crisis since the Great Depression in the

1930s. The combination of the housing bubble and the credit boom has been the fundamental cause of this crisis. Both the ratio of house price to rental income and the spreads on credit instruments –which at all times have been extremes- were considered as the most common features of this crisis.

This bubble has been admitted through two quite disparate points of view. While the first one has adopted the case of fundamental mispricing in capital markets –where the weak risk premia and the long-term volatility have been resulted in a false belief that current low levels of the future short-term volatility would stay on these levels-. However, the low credit spreads and inflated prices of risky assets have been implied by this mispricing. The second point of view is that the Federal Reserve has made some essential mistakes and particularly through the Fed's decision to keep the rate of Federal Funds very low for too long period of time. In addition to the failure in controlling the underwriting standards in the mortgage markets which were definitely poor such as no verification of income, negative amortization, no down-payments and so on.

This crisis has been gradually declined through the government interventions which have based on a compatible set of principles such as the substantial definition of the market failure, this intervention has implied through using effective tools, minimizing the costs for the tax payers and moral hazard has not been essentially created by this intervention.

Finally, we have divided the whole data set into four sub-samples depending on these crises mentioned above:

i) The first sample between 21st of May 1992 and 27th of October 1997 for Dow Jones, and between 7th of August 1990 and 27th of October 1997 for S&P 500 respectively.

ii) The second sample between 13th of January 1999 and 3rd of April 2000 for Dow Jones and S&P 500 separately.

iii) The third sample between 1st of February 2002 and 25th of July 2007 for both Dow Jones and S&P 500.

iv) The fourth point between 21st of October 2009 and 20th of March 2012 for Dow Jones and S&P 500 respectively.

It is noteworthy to mention that we have set three different series of dummy variables during those three aforementioned financial crises. The first dummy variable has taken the value 1 for the period around the Asian Financial Crisis and zero otherwise. However, the second dummy variable has involved the value 1 during the Dot-Com Bubble and zero otherwise. The third dummy variable has included the value 1 for the period over the 2007 Financial Crisis and zero otherwise.

2.5. The Econometric Models and Empirical Results

Case.1. Univariate GARCH-M (1,1) Model

One of the beneficial points of using the GARCH-M model is that the case of GARCH models are considerably more common than those ARCH models. This notion is mainly related to the correlation between rewarding a higher return by obtaining an additional risk which most models used in finance suppose.

The mean equation of the GARCH-M process can be written as follows:

$$\Phi_{11}(L)y_{1t} = \mu_1 + \delta_1 g\left(h_{1,t}\right) + \Phi_{12}(L)y_{2t} + \varepsilon_{1t} \qquad VLM \quad 2.1$$

$$\Phi_{22}(L)y_{2t} = \mu_2 + \delta_2 g\left(h_{2,t}\right) + \Phi_{21}(L)y_{1t} + \varepsilon_{2t} \qquad VLT \quad 2.2$$

$$\varepsilon_t|\Omega_{t-1} \sim N\ (0, H_t)$$

The definition $g(h_{1,t}) = h_{1,t}\ and\ g(h_{2,t}) = h_{2,t}$ has originally introduced by Engle et al. (1987) and that was related to the assumption which implied that the proportional changes in the conditional standard deviation could appear less than those changes in the mean. The innovation vector $\varepsilon_t|\Omega_{t-1} \sim N\ (0, H_t)$ is normally distributed with H_t being the corresponding variance-covariance matrix and Ω_{t-1} is the information set available at the time $t-1$, where

$$H_{\mathbf{t}} = \begin{bmatrix} h_{1t} & h_{12,t} \\ h_{21,t} & h_{2t} \end{bmatrix}$$

$h_{it}, i = 1{,}2$ denotes the conditional variance of volume and volatility respectively. $h_{12,t}$ denotes the conditional covariance of the two variables.

The case of $\delta\ > 0$ implies that $h_{1t}\ and\ h_{2t}$ has a positive effect on $y_{1t}\ and\ y_{2t}$ respectively, and vice versa in the case of $\delta\ < 0$.

The lag polynomials $\Phi_{11}(L)$ and $\Phi_{22}(L)$ indicate respectively the response of volume and volatility to their own lags, whereas $\Phi_{12}(L)$ and $\Phi_{21}(L)$ measure respectively the causality from volatility to volume and vice versa. In other words, the own effects are captured by $\Phi_{11}(L)$ and $\Phi_{22}(L)$, whereas the cross effects are captured by $\Phi_{12}(L)$ and $\Phi_{21}(L)$ polynomials for volume and volatility respectively.

The lag polynomials $\Phi_{11}(L)$ and $\Phi_{22}(L)$ are given by:

$$\Phi_{11}(L) = 1 - \sum_{l=1}^{l_{11}} \phi_{11}^{l}\, L^{l}, \qquad VLM \quad 2.3$$

$$\Phi_{22}(L) = 1 - \sum_{l=1}^{l_{22}} \phi_{22}^{l}\, L^{l} \qquad VLT \quad 2.4$$

Whereas, the lag polynomials $\Phi_{12}(L)$ and $\Phi_{21}(L)$ could be written as follows:

$$\Phi_{12}(L) = \sum_{l=1}^{l_{12}} \phi_{12}^{l} L^{l}, \qquad 2.5$$

$$\Phi_{21}(L) = \sum_{l=1}^{l_{21}} \phi_{21}^{l} L^{l} \qquad 2.6$$

The estimation of the different formulations has been obtained through EVIEWS. A range of initial values have been used through our estimations to check for the robustness of these procedures and also to investigate whether or not this estimation procedure has converged to a global maximum.

The Case of S&P 500

Mean Equation (Volume as a Dependent Variable)

With the Univariate GARCH-M Model, we have examined the case of volume as a dependent variable with its' lagged values to investigate which of those lagged values have a significant impact on the volume itself.

By examining the effect of the first five lagged values of volume on the volume itself, the results –with respect to the obtained various samples- have been reported as equation (2.3) in Table (2.1) for S&P 500.

As an example, the equation (2.3) for the first sub-sample can be written as follows:

$$\Phi_{11}(L) = 1 - (\phi_{11}^{1} L^{1} + \phi_{11}^{4} L^{4} + \phi_{11}^{5} L^{5})$$

Mean Equation (Volatility as a Dependent Variable)

With also the Univariate GARCH-M Model, the case of volatility has been studied as a dependent variable with its' lagged values to show which of these lagged values have a significant effect on the volatility itself.

By examining the effect of the first six lagged values of volatility on the volatility itself, the results –with respect to the previous mentioned samples- have been reported as equation (2.4) in Table (2.1) for S&P 500.

As an example, the equation (2.4) for the first sub-sample can be written as follows:

$$\Phi_{22}(L) = 1 - (\phi_{22}^{1}L^{1} + \phi_{22}^{2}L^{2} + \phi_{22}^{3}L^{3} + \phi_{22}^{4}L^{4} + \phi_{22}^{5}L^{5})$$

[1]Table 2.1. Mean Equations: AR Lags (Own Effects)

Samples	Eq. (2.3): Volume	Eq. (2.4): Volatility
Whole Sample	**1,2,3,5**	**1,2,3,4,5**
First Sub-Sample	**1,4,5**	**1,2,3,4,5**
Second Sub-Sample	**1,2,5**	**1,2,5**
Third Sub-Sample	**1,2,5**	**1,2,3,5,6**
Fourth Sub-Sample	**1,2**	**1,6**

Notes: This table reports coefficient estimates of the own effects $\Phi_{11}(L)$ and $\Phi_{22}(L)$.

[1] Regarding the case of obtaining three different series of dummy variables, we have found that only the seventh and tenth lags of the second dummy variable (during the Dot-Com Bubble) have a significant impact on the volume when considering volume as a dependent variable.

Regarding the case of volatility as a dependent variable, we have found that the eighth lag of the first dummy variable (during the Asian Financial Crisis) and the first lag of the second dummy variable (previously mentioned) have just a significant impact on the volatility itself.

Cross Effects (The Volume-Volatility Linkage)

Table (2.2) reports coefficient estimates of the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. Following equation (2.5), $\Phi_{12}(L)$ for the whole sample can be written as follows:

$\phi_{12}(L) = \phi_{12}^{7} L^{7}$, as the seventh lag of volatility in the mean equation has a significant and negative impact on volume as a dependent variable.

In addition, $\Phi_{21}(L)$ for the whole sample could be represented with regards to the equation (2.6) as follows (Table 2.2):

$\phi_{21}(L) = \phi_{21}^{1} L^{1}$, as the first lag of volume in the mean equation has a significant and negative effect on volatility as a dependent variable.

Table 2.2. Mean Equations: AR Lags (Cross Effects)

Samples	$\Phi_{12}(L)$	$\Phi_{21}(L)$
Whole Sample	**7**	**1**
First Sub-Sample	**7**	**8**
Second Sub-Sample	**10**	**5**
Third Sub-Sample	**9**	**10**
Fourth Sub-Sample	**2**	**6**

Notes: This table reports coefficient estimates of the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$.

As seen in Table (2.3), there is a negative bidirectional linkage between volatility and volume in the total, second and third sub-samples respectively. Nevertheless, a bidirectional mixed feedback is realized between volume and volatility in the first (prior to the Asian Financial Crisis) and fourth (after the Global Financial Crisis) sub-samples respectively. More specifically, volume affects volatility positively whereas the reverse impact is of the opposite sign.

That is, the evidence for the whole sample suggests that the negative effect of volume on volatility reflects the negative causal relation between volatility and volume in the second and third sub-samples, and vice versa when examining the impact of volatility on volume. These results are in line with the theoretical argument pointed out by Chuang et al. (2012) in the case of Taiwan and Japan.

It is also noteworthy that the negative impact of volatility on volume in the whole sample is reflected by a symmetrical negative impact for all the four sub-samples. On the other hand, the negative effect of volume on volatility in the whole sample is being only noticed in the second and third sub-samples.

Table 2.3. The Volume-Volatility Linkage (GARCH-M)

Samples	Effect of Volatility on Volume	Effect of Volume on Volatility
Whole Sample	**Negative**	**Negative**
First Sub-Sample	**Negative**	**Positive**
Second Sub-Sample	**Negative**	**Negative**
Third Sub-Sample	**Negative**	**Negative**
Fourth Sub-Sample	**Negative**	**Positive**

Table (2.4) reports parameter estimates of the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. The $\Phi_{12}(L)$ and $\Phi_{21}(L)$ columns report results for the return and flow equations respectively.

Table 2.4. The Coefficients of The Volume-Volatility Link (GARCH-M)

Samples	Φ_{12}	Φ_{21}
Whole Sample	**-0.25 (0.12)****	**-0.01 (0.00)****
First Sub-Sample	**-1.52 (0.88)***	**0.01 (0.00)****
Second Sub-Sample	**-2.03 (0.78)*****	**-0.01 (0.00)***
Third Sub-Sample	**-0.61 (0.34)***	**-0.01 (0.00)*****
Fourth Sub-Sample	**-1.61 (0.46)*****	**0.01 (0.00)*****

Notes: This table reports estimates of the parameters for the Φ_{12} and Φ_{21} respectively.

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Mean and Variance Equations (GARCH-M (1,1) Coefficients)

The analyzing dynamic adjustments of the conditional variances of both volume and volatility can be seen in Table (2.5) for S&P 500.

We will assume that H_t follow univariate GARCH (1,1) processes as follows:

$$h_{it} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1}, i = 1,2, \qquad 2.7$$

and that $h_{12,t} = 0$. Note that $\omega_i > 0$, $\alpha_i > 0$, and $\beta_i \geq 0$ in order for $h_{it} > 0$ for all t. Moreover, $\alpha_i + \beta_i < 1$ for the unconditional variance to exist.

We can note that the sum of the coefficients of ARCH parameter (α) and GARCH parameter (β) for the total sample and the other four sub-samples respectively is less than one, except the case of volatility as a dependent variable in the total sample which is a bit more than one (persistent). Additionally, all the coefficients of ARCH parameter (α) and GARCH parameter (β) are positive and significant in all different sub-samples.

Table 2.5. GARCH-M Coefficients (S&P)

Samples	$h_{1,t}$ **(VLM)**	$h_{2,t}$ **(VLT)**
Whole Sample		
$\boldsymbol{\delta_i}$	**-1.42 (0.48)***	**-8.93 (2.93)***
$\boldsymbol{\alpha_i}$	**0.14 (0.02)***	**0.22 (0.05)***
$\boldsymbol{\beta_i}$	**0.30 (0.08)***	**0.81 (0.03)***
First Sub-Sample		
$\boldsymbol{\delta_i}$	**7.52 (1.00)***	**-18.58 (10.95)***
$\boldsymbol{\alpha_i}$	**0.01 (0.01)***	**0.28 (0.16)***
$\boldsymbol{\beta_i}$	**0.76 (0.10)***	**0.68 (0.14)***
Second Sub-Sample		
$\boldsymbol{\delta_i}$	**0.65 (0.66)***	**-8.72 (30.37)***
$\boldsymbol{\alpha_i}$	**0.28 (0.10)***	**0.04 (0.01)***
$\boldsymbol{\beta_i}$	**0.52 (0.13)***	**0.95 (0.02)***
Third Sub-Sample		
$\boldsymbol{\delta_i}$	**14.78 (1.80)***	**10.70 (6.30)***
$\boldsymbol{\alpha_i}$	**0.01 (0.01)***	**0.10 (0.05)***
$\boldsymbol{\beta_i}$	**0.92 (0.05)***	**0.89 (0.03)***
Fourth Sub-Sample		
$\boldsymbol{\delta_i}$	**1.73 (0.94)***	**-2.35 (1.30)***
$\boldsymbol{\alpha_i}$	**0.12 (0.04)***	**0.46 (0.17)***
$\boldsymbol{\beta_i}$	**0.70 (0.10)***	**0.42(0.07)***

Notes: This table reports estimates of the parameters for the GARCH-M ($\boldsymbol{\delta_i}$), ARCH ($\boldsymbol{\alpha_i}$) and GARCH ($\boldsymbol{\beta_i}$).

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Regarding the case of Dow Jones, it is discussed in details in Appendix (2.A.).

Case.2. Bivariate VAR-BEKK GARCH (1,1) Model

In order to capture the potential interactions between volume and volatility, volume y_{1t} and volatility y_{2t} follow a bivariate VAR model as follows:

$$\mathcal{Y}_{1t} = \mu_{1t} + \Phi_{11}(L)y_{1t} + \Phi_{12}(L)y_{2t} + \varepsilon_{1,t}, \varepsilon_{1,t} \sim (0, h_{1t}) \qquad VLM \quad 2.8$$

$$\mathcal{Y}_{2t} = \mu_{2t} + \Phi_{22}(L)y_{2t} + \Phi_{21}(L)y_{1t} + \varepsilon_{2,t}, \varepsilon_{2,t} \sim (0, h_{2t}) \qquad VLT \quad 2.9$$

$$\varepsilon_t | \Omega_{t-1} \sim N(0, H_t)$$

where $\Phi_{ij}(L) = \sum_{l=1}^{l_{ij}} \phi_{ij}^{l} L^{l}$ and (L) denotes the lag operator.

Moreover, the bivariate vector of innovations ε_t is conditionally normal with mean zero and variance-covariance matrix $H_{\mathbf{t}}$ and Ω_{t-1} is the information set available at the time $t-1$.

That is $\varepsilon_t|\Omega_{t-1} \sim N\ (0, H_{\mathrm{t}})$ where

$$H_{\mathbf{t}} = \begin{bmatrix} h_{1t} & h_{12,t} \\ h_{21,t} & h_{2t} \end{bmatrix}. \quad 2.10$$

$\varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t})'$, $h_{it}, i = 1,2$ denotes the conditional variance of volume and volatility respectively. $h_{12,t}$ denotes the conditional covariance of the two variables.

The bivariate normal distribution is denoted by N and a time-varying 2×2 positive definite conditional variance matrix is denoted by H_t.

With the bivariate VAR Model, we examine the case of volume with its' lagged values to investigate which of those lagged values have a significant impact on the volume itself. The lag polynomial $\Phi_{11}(L)$ is given by equation (2.3).

In addition, by applying the bivariate VAR Model, the case of volatility is examined with its' lagged values to show which of these lagged values have a significant effect on the volatility itself. The lag polynomial $\Phi_{22}(L)$ is given by equation (2.4).

The bi-directional correlation between volume and volatility is represented by the lag polynomials $\Phi_{12}(L)$ and $\Phi_{21}(L)$. Whereas the polynomial $\Phi_{12}(L)$ in equation (2.5) represents the effect of volatility on volume in the mean equation, the polynomial $\Phi_{21}(L)$ in equation (2.6) represents the effect of volume on volatility in the mean equation.

The motivation behind using this model in the case of examining the volume-volatility correlation is that both volume and volatility react to the same daily information, and also having the non-zero conditional covariance on the obtainable set of information (Bera et al. (1997)).

A general form of H_t for the VEC GARCH (1,1) model can be expressed as follows:

$$vech\ (H_t) = vech\ (C) + Avech\ (\varepsilon_{t-1}\varepsilon'_{t-1}) + Bvech\ (H_{t-1}) \quad 2.11$$

Where A and B are 3×3 matrices, C is the 2×2 positive definite symmetric matrix and the $vech$ operation stacks the lower triangular elements of the symmetric matrix in a column.

In addition, there is a difficulty in estimating the parameterization given in the equation (2.11) that H_t could not be assured without imposing the nonlinear parametric restrictions. This implies that the examined straightforward assumption should specify that the conditional variance can depend only on its' lagged values and lagged squared residuals. Whereas this assumption attains to make A and B as diagonal matrices, the $vech\ (H_t)$ of the GARCH (1,1) model can be written as:

$$vech\ (H_t) = \begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_{12} \\ c_2 \end{bmatrix} + \begin{bmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{12} & 0 \\ 0 & 0 & \alpha_{22} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon^2_{2,t-1} \end{bmatrix}$$

$$+ \begin{bmatrix} \beta_{11} & 0 & 0 \\ 0 & \beta_{12} & 0 \\ 0 & 0 & \beta_{22} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix} \quad 2.12$$

This is the diagonal VEC form which is considered as the representative of H_t, while the two necessary conditions for this H_t to be positive definite can be expressed as follows:

$$c_1 > 0, c_2 > 0, c_1 c_2 - c_{12}^2 > 0$$

$$\alpha_1 > 0, \alpha_2 > 0, \alpha_1\alpha_2 - \alpha_{12}^2 > 0$$

Another "positive definite" parameterization which is definitely guaranteed to be positive definite has been suggested by Engle and Kroner (1995) as follows (the Bivariate GARCH-BEKK Model):

$$H_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B \quad 2.13$$

More specifically, equation (2.13) could be written as follows:

$$\begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}' \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$
$$+ \begin{bmatrix} \alpha_{11} & 0 \\ 0 & \alpha_{22} \end{bmatrix}' \begin{bmatrix} \varepsilon^2{}_{1,t-1} & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}\varepsilon_{1,t-1} & \varepsilon^2{}_{2,t-1} \end{bmatrix} \begin{bmatrix} \alpha_{11} & 0 \\ 0 & \alpha_{22} \end{bmatrix}$$
$$+ \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix}' \begin{bmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix} \quad 2.14$$

Where,

$$h_{11,t} = \alpha_{11}^2\varepsilon^2{}_{1,t-1} + b_{11}^2 h_{11,t-1} \quad 2.15$$

$$h_{22,t} = \alpha_{22}^2\varepsilon^2{}_{2,t-1} + b_{22}^2 h_{22,t-1} \quad 2.16$$

$$h_{12,t} = \alpha_{11}\alpha_{22}\varepsilon_{1,t-1}\varepsilon_{2,t-1} + b_{11}b_{22}h_{12,t-1} \quad 2.17$$

In the diagonal BEKK model, the covariance stationary condition is that $a_{11}^2 + b_{11}^2 < 1$ and $a_{22}^2 + b_{22}^2 < 1$. The stationary properties –in the case of diagonal models- are determined solely by the diagonal elements of the A and B matrices.

Myers and Thompson (1989) have clarified that in the case of zero A and B parameters, then H_t transfers to the constant conditional covariance as proposed in the following matrix:

$$H_t = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

The essential reason for choosing to work on BEKK model instead of VEC model is the difficulty in ensuring whether or not the H matrix is always positive definite.

The Case of S&P 500

Mean Equation

With the bivariate VAR model, we will examine the response of volume and volatility to their own lags (the effect of the lagged values of volume (volatility) on the volume (volatility) in the mean equation). In addition, we will examine the causality from volatility to volume and vice versa (the effect of the lagged values of volatility obtained in the mean equation of volume and vice versa).

Own Effects

Table (2.6) reports coefficient estimates of the own effects $\Phi_{11}(L)$ and $\Phi_{22}(L)$. The $\Phi_{11}(L)$ and $\Phi_{22}(L)$ columns report results for the volume and volatility equations respectively.

For example, the lag polynomial $\Phi_{11}(L)$ for the first sub-sample might be written with regards to the equation (2.3) as follows (see Table 2.6):

$$\Phi_{11}(L) = 1 - (\phi_{11}^{1}L^{1} + \phi_{11}^{4}L^{4} + \phi_{11}^{5}L^{5})$$

As an example, and by applying equation (2.4), the lag polynomial $\Phi_{22}(L)$ for the fourth sub-sample might be written as follows (see Table 2.6):

$$\Phi_{22}(L) = 1 - (\phi_{22}^{6}L^{6} + \phi_{22}^{7}L^{7})$$

Table 2.6. Mean Equations: AR Lags (Own Effects)

Samples	Eq. (2.3): $\Phi_{11}(L)$	Eq. (2.4): $\Phi_{22}(L)$
Whole Sample	**1,2,3,4**	**1,2,3,4,5**
First Sub-Sample	**1,4,5**	**1,2,3**
Second Sub-Sample	**1,4,5**	**5**
Third Sub-Sample	**1,4,5**	**1,2,3,4,5**
Fourth Sub-Sample	**1,5**	**6,7**

Notes: This table reports coefficient estimates of the own effects $\Phi_{11}(L)$ and $\Phi_{22}(L)$.

Cross Effects (The Volume-Volatility Linkage)

We have employed the bivariate VAR model with lagged values of one variable which have been obtained in the mean equation of the other variable in case of testing for a causal relation between these two variables.

Table (2.7) reports coefficient estimates of the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. For instance and by following equation (2.5), the lag polynomial $\Phi_{12}(L)$ for the whole sample is as follows:

$$\Phi_{12}(L) = \phi_{12}^{7} L^{7}$$

Moreover, by applying equation (2.6), the lag polynomial for the fourth sub-sample could be written as follows (see Table 2.7):

$$\Phi_{21}(L) = \phi_{21}^{3} L^{3}$$

Table 2.7. Mean Equations: AR Lags (Cross Effects)

Samples	$\Phi_{12}(L)$	$\Phi_{21}(L)$
Whole Sample	**7**	**1**
First Sub-Sample	**1**	**3**
Second Sub-Sample	**7**	**3**
Third Sub-Sample	**9**	**2**
Fourth Sub-Sample	**4**	**3**

Notes: This table reports coefficient estimates of the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$.

As presented in Table (2.8), there is a negative bidirectional relation between volume and volatility in the total, second and third sub-samples respectively. This result is consistent with the finding reported by Karanasos and Kartsaklas (2009) when examining the impact of foreign volume on volatility in the pre-Asian Financial Crisis period.

Nevertheless, a bidirectional mixed feedback is being realized between volatility and volume in the first and fourth sub-samples respectively where volatility (volume) affects volume (volatility) negatively (positively).

That is, the evidence for the whole sample implies that the negative linkage between volume and volatility reflects the negative causal relation between volatility and volume in the second and third sub-samples.

It is also recognized that the negative impact of volatility on volume in the whole sample is being reflected by a negative effect for all the four sub-samples. Nevertheless, the negative impact of volume on volatility in the whole sample is being only noticed in the second and third sub-samples. It is worthy to mention that identical results have been observed when employing both univariate GARCH-M and bivariate VAR-BEKK GARCH models.

Table 2.8. The Volume-Volatility Linkage (VAR-BEKK GARCH)

Samples	Effect of Volatility on Volume	Effect of Volume on Volatility
Whole Sample	Negative	Negative
First Sub-Sample	Negative	Positive
Second Sub-Sample	Negative	Negative
Third Sub-Sample	Negative	Negative
Fourth Sub-Sample	Negative	Positive

Moreover, Table (2.9) reports parameter estimates of the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. The $\Phi_{12}(L)$ and $\Phi_{21}(L)$ columns report results for the volume and volatility equations respectively.

Table 2.9. The Coefficients of The Volume-Volatility Link (VAR-BEKK GARCH)

Samples	$\Phi_{12}(L)$	$\Phi_{21}(L)$
Whole Sample	**-0.30 (0.16)*** **	**-0.01 (0.00)*** **
First Sub-Sample	**-1.98 (0.91)** **	**0.01 (0.00)* **
Second Sub-Sample	**-1.44 (0.71)** **	**-0.01 (0.00)* **
Third Sub-Sample	**-0.78 (0.31)** **	**-0.01 (0.00)* **
Fourth Sub-Sample	**-0.90 (0.27)*** **	**0.01 (0.00)*** **

Notes: This table reports estimates of the parameters for the Φ_{12} and Φ_{21} respectively.

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Variance Equation

Table (2.10) reports estimates of ARCH and GARCH parameters. Following equations (2.15) and (2.16), we notice that the sum of the squared values of ARCH parameter (α) and GARCH parameter (β) for the total sample and all the other sub-samples respectively is less than one. Additionally, the ARCH and GARCH coefficients are positive and significant in all various cases.

Table 2.10. Variance Equations: Bivariate BEKK GARCH Coefficients (S&P)

Samples	$h_{11,t}$ **(Volume)**	$h_{22,t}$ **(Volatility)**
Whole Sample		
α_i	**0.35 (0.03)*** **	**0.37 (0.04)*** **
β_i	**0.70 (0.04)*** **	**0.90 (0.01)*** **
First Sub-Sample		
α_i	**0.33 (0.17)***	**0.48 (0.10)*** **
β_i	**0.41 (0.06)*** **	**0.83 (0.03)*** **
Second Sub-Sample		
α_i	**0.37 (0.09)*** **	**0.17 (0.06)*** **
β_i	**0.79 (0.10)*** **	**0.96 (0.03)*** **
Third Sub-Sample		
α_i	**0.37 (0.04)*** **	**0.33 (0.05)*** **
β_i	**0.71 (0.06)*** **	**0.94 (0.01)*** **
Fourth Sub-Sample		
α_i	**0.37 (0.07)*** **	**0.33 (0.56)*** **
β_i	**0.77 (0.07)*** **	**0.66 (0.09)*** **

Notes: This table reports estimates of the parameters for the ARCH ($\boldsymbol{\alpha_i}$) and GARCH ($\boldsymbol{\beta_i}$).

***, and * stand for significance at the 1% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Regarding the case of Dow Jones, it is discussed in details in Appendix (2.B.).

Case.3. Bivariate VAR-CCC GARCH (1,1) Model

We employ a bivariate VAR-CCC GARCH process to examine the dual volume-volatility linkage. Bollerslev (1986) has developed the GARCH (1,1) model through allowing the conditional variance to depend on the past conditional variances. Whilst the autoregressive components capture the persistence in the conditional variance of volume and volatility, the past squared residual components capture the information shocks to volume and volatility.

With the bivariate VAR Model, we examine the case of volume with its' lagged values to investigate which of those lagged values have a significant impact on the volume itself. The lag polynomial $\Phi_{11}(L)$ is given by equation (2.3).

In addition, by applying the bivariate VAR Model, the case of volatility is examined with its' lagged values to show which of these lagged values have a significant effect on the volatility itself. The lag polynomial $\Phi_{22}(L)$ is given by equation (2.4).

The bi-directional correlation between volume and volatility is represented by the lag polynomials $\Phi_{12}(L)$ and $\Phi_{21}(L)$. Whereas the polynomial $\Phi_{12}(L)$ in equation (2.5) represents the effect of volatility on volume in the mean equation, the polynomial $\Phi_{21}(L)$ in equation (2.6) represents the effect of volume on volatility in the mean equation.

Also, we will assume that H_t follows the bivariate constant conditional correlation (ccc) GARCH (1,1) model of Bollerslev et al. (1992). That is, h_{it} is given by equation (2.7)

And that $h_{12,t}$ is given by: $h_{12,t} = \rho\sqrt{h_{1t}}\sqrt{h_{2t}}$, 2.18

where ρ denotes the ccc.

The Case of S&P 500

Mean Equation

With the bivariate VAR model, we will examine the case of own effects (the effect of the lagged values of volume (volatility) on the volume (volatility) in the mean equation). In addition, we will examine the case of cross effects (the effect of the lagged values of volatility obtained in the mean equation of volume and vice versa).

Own Effects

Table (2.11) reports coefficient estimates of the own effects $\Phi_{11}(L)$ and $\Phi_{22}(L)$. The $\Phi_{11}(L)$ and $\Phi_{22}(L)$ columns report results for the volume and volatility equations respectively.

For example, the lag polynomial $\Phi_{11}(L)$ for the first sub-sample might be written with regards to the equation (2.3) as follows (see Table 2.11):

$$\Phi_{11}(L) = 1 - (\phi_{11}^{1}L^{1} + \phi_{11}^{5}L^{5})$$

As an example, and by applying equation (2.4), the lag polynomial $\Phi_{22}(L)$ for the fourth sub-sample might be written as follows (see Table 2.11):

$$\Phi_{22}(L) = 1 - (\phi_{22}^{1}L^{1} + \phi_{22}^{2}L^{2} + \phi_{22}^{6}L^{6})$$

Table 2.11. Mean Equations: AR Lags (Own Effects)

Samples	**Eq. (2.3):**$\Phi_{11}(L)$	**Eq. (2.4):**$\Phi_{22}(L)$
Whole Sample	**1,2,3**	**1,2,3,4,5**
First Sub-Sample	**1,5**	**1,2,3,5**
Second Sub-Sample	**1,2,5**	**1,2,3,5**
Third Sub-Sample	**1,2,5**	**1,2,3,5**
Fourth Sub-Sample	**1,2,5**	**1,2,6**

Notes: This table reports coefficient estimates of the own effects $\Phi_{11}(L)$ and $\Phi_{22}(L)$.

Cross Effects (The Volume-Volatility Linkage)

We employ the bivariate VAR model with lagged values of one variable which have been obtained in the mean equation of the other variable in case of testing for a causal relation between these two variables.

Table (2.12) reports coefficient estimates of the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. For instance and by following equation (2.5), the lag polynomial $\Phi_{12}(L)$ for the first sub-sample is as follows:

$$\Phi_{12}(L) = \phi_{12}^{7}L^{7}$$

Moreover, by applying equation (2.6), the lag polynomial for the fourth sub-sample could be written as follows (see Table 2.12):

$$\Phi_{21}(L) = \phi_{21}^{3} L^{3}$$

Table 2.12. Mean Equations: AR Lags (Cross Effects)

Samples	$\Phi_{12}(L)$	$\Phi_{21}(L)$
Whole Sample	**7**	**1**
First Sub-Sample	**7**	**-**
Second Sub-Sample	**9**	**3**
Third Sub-Sample	**9**	**3**
Fourth Sub-Sample	**9**	**3**

Notes: This table reports coefficient estimates of the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$.

As seen in Table (2.13), there is a negative bidirectional linkage between volatility and volume in the total sample, second and third sub-samples respectively. Additionally, a bidirectional mixed feedback is being realized between volume and volatility in the fourth sub-sample. Whilst there is a negative impact of volatility on volume in the first sub-sample, no effect of volume on volatility has been noticed. This finding is in line with the study obtained by Bekaert and Harvey (2000).

That is, the evidence for the whole sample suggests that the negative effect of volatility on volume reflects the negative causal relation between volume and volatility in the second and third sub-samples, and vice versa when examining the impact of volume on volatility. Whereas the negative effect of volume on volatility in the whole sample is being only recognized in the second and third sub-samples, the negative impact of volatility on volume in the whole sample is being reflected in all the four sub-samples respectively. We have captured one exceptional issue in comparison between these findings and the aforesaid results of univariate GARCH-M and bivariate VAR-BEKK GARCH processes. Whereas volume affects volatility positively in the first sub-sample, there is no impact of volume on volatility in the case of employing bivariate VAR-CCC GARCH process.

Table 2.13. The Volume-Volatility Link (VAR-CCC GARCH)

Samples	Effect of Volatility on Volume	Effect of Volume on Volatility
Whole Sample	Negative	Negative
First Sub-Sample	Negative	Zero
Second Sub-Sample	Negative	Negative
Third Sub-Sample	Negative	Negative
Fourth Sub-Sample	Negative	Positive

Moreover, Table (2.14) reports parameter estimates of the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. The $\Phi_{12}(L)$ and $\Phi_{21}(L)$ columns report results for the volume and volatility equations respectively.

Table 2.14. The Coefficients of The Volume-Volatility Link (VAR-CCC GARCH)

Samples	$\Phi_{12}(L)$	$\Phi_{21}(L)$
Whole Sample	-0.30 (0.12)**	-0.01 (0.00)**
First Sub-Sample	-1.65 (0.68)**	-
Second Sub-Sample	-0.88 (0.51)*	-0.01 (0.00)*
Third Sub-Sample	-0.86 (0.29)***	-0.01 (0.00)***
Fourth Sub-Sample	-0.49 (0.29)*	0.01 (0.00)***

Notes: This table reports estimates of the parameters for the Φ_{12} and Φ_{21} respectively.

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Variance Equation

Table (2.15) reports estimates of ARCH, GARCH and CCC parameters. Following equation (2.7), we notice that the sum of the coefficients of ARCH parameter (α) and GARCH parameter (β) for the total sample and all the other sub-samples respectively is less than one.

Additionally, the ARCH and GARCH coefficients are positive and significant in all various cases.

With regards to equation (2.18), the conditional correlation between volume and volatility in the whole sample is (0.14). In the four sub-samples, the estimated values of ρ for volume-volatility (0.36, 0.47, 0.40 and 0.38) are higher than the corresponding values for the whole sample. In other words, volume-volatility correlation in the four sub-samples is higher than the correlation in the whole sample.

Table 2.15. Variance equations: Bivariate GARCH And CCC Coefficients (S&P)

Samples	h_{1t} **(Volume)**	h_{2t} **(Volatility)**
Whole Sample		
α_i	**0.14 (0.03)*** **	**0.24 (0.05)*** **
β_i	**0.34 (0.08)*** **	**0.74 (0.02)*** **
ρ	**0.14 (0.01)*** **	-
First Sub-Sample		
α_i	**0.22 (0.12)*** **	**0.29 (0.07)*** **
β_i	**0.64 (0.10)*** **	**0.67 (0.06)*** **
ρ	**0.36 (0.02)*** **	-
Second Sub-Sample		
α_i	**0.11 (0.06)* **	**0.01 (0.03)** **
β_i	**0.65 (0.14)*** **	**0.91 (0.05)*** **
ρ	**0.47 (0.05)*** **	-
Third Sub-Sample		
α_i	**0.15 (0.04)*** **	**0.11 (0.04)*** **
β_i	**0.38 (0.14)*** **	**0.88 (0.02)*** **
ρ	**0.40 (0.03)*** **	-
Fourth Sub-Sample		
α_i	**0.19 (0.06)*** **	**0.01 (0.01)* **
β_i	**0.37 (0.14)*** **	**0.98 (0.01)*** **
ρ	**0.38 (0.04)*** **	-

Notes: This table reports estimates of the parameters for the ARCH ($\boldsymbol{\alpha_i}$), GARCH ($\boldsymbol{\beta_i}$) and ccc ($\boldsymbol{\rho}$).

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Regarding the case of Dow Jones, it is discussed in details in Appendix (2.C.).

2.6. Conclusion

In this paper, we have simultaneously investigated the dynamics and interactions of the volume-volatility link. We have been able to highlight different keys of behavioral features which were presented across the various univariate and bivariate formulations.

We have considered several changes according to different chosen samples and discussed how these changes would affect the linkages amongst these two variables.

In particular, we have taken into account the case of structural breaks and employed different specifications of the univariate and bivariate processes in order to obtain all the changeable results.

Our contribution in this paper has been considered as follows: choosing a long span of daily data (1990-2012 for S&P 500, 1992-2012 for Dow Jones respectively) and four sub-sample periods, using three different GARCH models (univariate GARCH-M, bivariate VAR-BEKK GARCH and bivariate VAR-CCC GARCH processes respectively).

We have detected a mixed bidirectional feedback between volume and volatility, for instance, the first sub-sample of S&P 500 'spanning between 7th August 1990 and 27th October 1997' by employing univariate GARCH-M model. However, a negative (positive) bidirectional linkage has been observed in the second sub-sample of S&P 500 'covering the period 13th January 1999 to 3rd April 2000' by using bivariate VAR-BEKK GARCH process (the first sub-sample of Dow Jones by employing bivariate VAR-CCC GARCH model).

Finally, Most of the bidirectional effects are found to be quite robust to the dynamics of the various GARCH processes employed in this paper.

Appendix 2

Appendix 2.A

In this section, we will present the results of univariate GARCH-M (1,1) process with regards to the case of Dow Jones.

Volume as a Dependent Variable

Following equation (2.3), we examine the effect of the first seven lagged values of volume on the volume itself. The results –with respect to the acquired various sub-samples- are reported in Table (A.2.1.).

For instance, the equation (2.3) for the first sub-sample can be written as follows:

$$\Phi_{11}(L) = 1 - (\phi_{11}^{1}L^{1} + \phi_{11}^{2}L^{2} + \phi_{11}^{4}L^{4})$$

Volatility as a Dependent Variable

Employing equation (2.4), we examine the impact of the first eleven lagged values of volatility on the volatility itself. The results –with respect to the previous mentioned samples- are reported in Table (A.2.1.).

As an example, the equation (2.4) for the first sub-sample might be written as follows:

$$\Phi_{22}(L) = 1 - (\phi_{22}^{2}L^{2} + \phi_{22}^{5}L^{5} + \phi_{22}^{6}L^{6})$$

[2]Table A.2.1. Mean Equations: AR Lags (Own Effects)

Samples	Eq. (2.3): Volume	Eq. (2.4): Volatility
Whole Sample	**1,2,3,4,5**	**1,2,3,4,5**
First Sub-Sample	**1,2,4**	**2,5,6**
Second Sub-Sample	**1,4,5**	**9**
Third Sub-Sample	**1,2,3,5**	**11**
Fourth Sub-Sample	**2,3,7**	**2,6,7**

Notes: This table reports coefficient estimates of the own effects $\Phi_{11}(L)$ and $\Phi_{22}(L)$.

Cross Effects (The Volume-Volatility Linkage)

With respect to the $\Phi_{12}(L)$ and $\Phi_{21}(L)$ lag polynomials which have been shown in the Equation (2.5) and Equation (2.6) respectively, $\Phi_{12}(L)$ for the total sample of Dow Jones can be written as follows (see Table A.2.2.):

$\Phi_{12}(L) = \phi_{12}^{1}L^{1}$, as the first lag of volatility in the mean equation has a negative and significant effect on volume as a dependent variable.

Moreover, $\Phi_{21}(L)$ for the total sample of Dow Jones can also be written as follows:

$\Phi_{21}(L) = \phi_{21}^{4}L^{4}$, as the fourth lag of volume in the mean equation has a positive and significant impact on volatility as a dependent variable.

[2] Regarding the case of dummy variables, in the case of volume is a dependent variable, we have found that only the nineteenth lag of the second dummy variable has an impact on the volume.

In the case of volatility is a dependent variable, we have recognized that also the eighth lag of the first dummy variable and the third lag of the second dummy variable have significant effects on the volatility.

Table A.2.2. Mean Equations: AR Lags (Cross Effects)

Samples	$\Phi_{12}(L)$	$\Phi_{21}(L)$
Whole Sample	**1**	**4**
First Sub-Sample	**1**	**4**
Second Sub-Sample	**1**	**5**
Third Sub-Sample	**8**	**-**
Fourth Sub-Sample	**1**	**4**

Notes: This table reports coefficient estimates of the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$.

As seen in Table (A.2.3.) in the whole and second samples, there is a mixed bidirectional feedback between volume and volatility. In particular, volatility affects volume negatively (positively) in the whole sample (second sub-sample), whereas the reverse impact is of the opposite sign.

We also notice that there is a positive bidirectional feedback between volume and volatility in the first and fourth sub-samples respectively. These results are consistent with the theoretical underpinnings showed by Chuang et al. (2012) in the case of Hong Kong, Korea, Singapore, China, Indonesia and Thailand respectively.

While there is a negative impact of volatility on volume in the third sub-sample, no effect of volume on volatility is being noticed.

That is, the evidence for the whole sample suggests that the positive effect of volume on volatility reflects this positive impact in the first and fourth sub-samples.

In addition, it is also noticeable that the negative impact of volatility on volume in the whole sample is being only reflected in the third sub-sample. In comparison with the results presented in table 2.3., we can realize some principal differences. Whereas the effect of volatility on volume in the case of S&P is negative in the first, second and fourth sub-samples, this impact is positive in the case of Dow Jones. In addition, volume affects volatility negatively in the whole sample in the case of S&P, while this impact is positive in the case of Dow Jones. Finally, we observe no effect of volume on volatility in the third sub-sample for Dow Jones, while this impact is negative in the case of S&P.

Table A.2.3. The Volume-Volatility Link (GARCH-M)

Samples	Effect of Volatility on Volume	Effect of Volume on Volatility
Whole Sample	**Negative**	**Positive**
First Sub-Sample	**Positive**	**Positive**
Second Sub-Sample	**Positive**	**Negative**
Third Sub-Sample	**Negative**	**Zero**
Fourth Sub-Sample	**Positive**	**Positive**

Table (A.2.4.) reports parameter estimates of the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. The $\Phi_{12}(L)$ and $\Phi_{21}(L)$ columns report results for the return and flow equations respectively.

Table A.2.4. The Coefficients of The Volume-Volatility Link (GARCH-M)

Samples	Φ_{12}	Φ_{21}
Whole Sample	**-0.25 (0.08)*** **	**0.01 (0.00)***
First Sub-Sample	**1.40 (0.35)*** **	**0.01 (0.00)*** **
Second Sub-Sample	**0.97 (0.38)** **	**-0.01 (0.00)*** **
Third Sub-Sample	**-0.36 (0.15)** **	**-**
Fourth Sub-Sample	**0.89 (0.20)*** **	**0.01 (0.00)***

Notes: This table reports estimates of the parameters for the Φ_{12} and Φ_{21} respectively.

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Mean And Variance Equations (GARCH-M (1,1) Coefficients)

In the case of Dow Jones, the sum of these coefficients is less than one as volume is the dependent variable for all chosen samples, and in the fourth sub-sample as volatility is the dependent variable. While, this sum is a bit more than one for the whole sample and the first three sub-samples as volatility is a dependent variable (persistent).

Moreover, all the coefficients of ARCH parameter (α) and GARCH parameter (β) are significant and positive in all different samples (see Table A.2.5.).

Table A.2.5. GARCH-M Coefficients (Dow Jones)

Samples	$h_{1,t}$ **(VLM)**	$h_{2,t}$ **(VLT)**
Whole Sample		
δ_i	**-0.84 (0.44)***	**-2.90 (1.63)***
α_i	**0.14 (0.02)*****	**0.19 (0.04)*****
β_i	**0.28 (0.09)*****	**0.83 (0.03)*****
First Sub-Sample		
δ_i	**1.38 (1.40)***	**1.49 (0.46)*****
α_i	**0.07 (0.03)***	**0.14 (0.64)****
β_i	**0.52 (0.28)***	**0.84 (0.07)****
Second Sub-Sample		
δ_i	**1.05 (0.93)***	**10.67 (4.84)****
α_i	**0.16 (0.05)*****	**0.07 (0.04)***
β_i	**0.78 (0.09)*****	**0.94 (0.04)*****
Third Sub-Sample		
δ_i	**-0.06 (0.03)****	**0.01 (0.00)*****
α_i	**0.15 (0.04)*****	**0.23 (0.10)****
β_i	**0.34 (0.13)***	**0.78 (0.06)*****
Fourth Sub-Sample		
δ_i	**-0.10 (0.06)***	**-0.20 (0.00)****
α_i	**0.13 (0.05)*****	**0.28 (0.25)***
β_i	**0.40 (0.19)****	**0.70 (0.09)*****

Notes: This table reports estimates of the parameters for the GARCH-M ($\boldsymbol{\delta}_i$), ARCH ($\boldsymbol{\alpha}_i$) and GARCH ($\boldsymbol{\beta}_i$).

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Appendix 2.B

In this section, we will present the results of bivariate VAR-BEKK GARCH (1,1) model with regards to the case of Dow Jones.

Mean Equation (Bivariate VAR Model)

With the bivariate VAR model, we will examine the response of volume and volatility to their own lags (the effect of the lagged values of volume (volatility) on the volume (volatility) in the mean equation). In addition, we will examine the causality from volatility to volume and vice versa (the effect of the lagged values of volatility obtained in the mean equation of volume and vice versa).

Own Effects

Table (B.2.1.) reports coefficient estimates of the own effects $\Phi_{11}(L)$ and $\Phi_{22}(L)$. The $\Phi_{11}(L)$ and $\Phi_{22}(L)$ columns report results for the volume and volatility equations respectively.

For example, the lag polynomial $\Phi_{11}(L)$ for the first sub-sample might be written with regards to the equation (2.3) as follows (see Table B.2.1.):

$$\Phi_{11}(L) = 1 - (\phi_{11}^{1}L^{1} + \phi_{11}^{2}L^{2} + \phi_{11}^{4}L^{4})$$

As an example, and by applying equation (2.4), the lag polynomial $\Phi_{22}(L)$ for the fourth sub-sample might be written as follows (see Table B.2.1.):

$$\Phi_{22}(L) = 1 - (\phi_{22}^{2}L^{2} + \phi_{22}^{6}L^{6} + \phi_{22}^{7}L^{7})$$

Table B.2.1. Mean Equations: AR Lags (Own Effects)

Samples	**Eq. (2.3):** $\Phi_{11}(L)$	**Eq. (2.4):** $\Phi_{22}(L)$
Whole Sample	**1,2,3,4,5**	**1,2,3,4,5**
First Sub-Sample	**1,2,4**	**1,2,5**
Second Sub-Sample	**1,3,5**	**9**
Third Sub-Sample	**1,2,5**	**1,2,3,4,5**
Fourth Sub-Sample	**1,3,5**	**2,6,7**

Notes: This table reports coefficient estimates of the own effects $\Phi_{11}(L)$ and $\Phi_{22}(L)$.

Cross Effects (The Volume-Volatility Linkage)

We have employed the bivariate VAR model with lagged values of one variable which have been obtained in the mean equation of the other variable in case of testing for a causal relation between these two variables.

Table (B.2.2.) reports coefficient estimates of the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. For instance and by following equation (2.5), the lag polynomial $\Phi_{12}(L)$ for the whole sample is as follows:

$$\Phi_{12}(L) = \phi_{12}^{5} L^{5}$$

Moreover, by applying equation (2.6), the lag polynomial for the fourth sub-sample could be written as follows (see Table B.2.2.):

$$\Phi_{21}(L) = \phi_{21}^{4} L^{4}$$

Table B.2.2. Mean Equations: AR Lags (Cross Effects)

Samples	$\Phi_{12}(L)$	$\Phi_{21}(L)$
Whole Sample	**5**	**4**
First Sub-Sample	**1**	**4**
Second Sub-Sample	**1**	**4**
Third Sub-Sample	**5**	**2**
Fourth Sub-Sample	**2**	**4**

Notes: This table reports coefficient estimates of the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$.

As seen in Table (B.2.3.) in the whole sample, there is a mixed bidirectional feedback between volatility and volume. In particular, volatility affects volume negatively in the whole sample whereas the reverse impact is of the opposite sign.

We can also notice that there is a positive bidirectional linkage between volatility and volume in the first, second and fourth sub-samples respectively. This result is in line with the empirical evidence pointed out by Chen et al. (2001).

Additionally, there is a negative bidirectional feedback between volume and volatility in the third sub-sample.

On the other hand, it is also noteworthy that the negative impact of volatility on volume in the whole sample is being only reflected in the third sub-sample. That is, the evidence for the whole sample indicates that the positive effect of volume on volatility reflects a positive impact in the first, second and fourth sub-samples respectively.

In comparison with the results presented in table 2.8., some principal differences could be observed. Whereas the effect of volatility on volume in the case of S&P is negative in the first, second and fourth sub-samples, this impact is positive in the case of Dow Jones. In addition, volume affects volatility negatively in the whole sample and the second sub-sample in the case of S&P, while this impact is positive in the case of Dow Jones.

Table B.2.3. The Volume-Volatility Link (VAR-BEKK GARCH)

Samples	Effect of Volatility on Volume	Effect of Volume on Volatility
Whole Sample	Negative	Positive
First Sub-Sample	Positive	Positive
Second Sub-Sample	Positive	Positive
Third Sub-Sample	Negative	Negative
Fourth Sub-Sample	Positive	Positive

In addition, Table (B.2.4.) reports parameter estimates of the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. The $\Phi_{12}(L)$ and $\Phi_{21}(L)$ columns report results for the volume and volatility equations respectively.

Table B.2.4. The Coefficients of The Volume-Volatility Link (VAR-BEKK GARCH)

Samples	$\Phi_{12}(L)$	$\Phi_{21}(L)$
Whole Sample	**-0.20 (0.08)****	**0.01 (0.00)****
First Sub-Sample	**1.75 (0.36)*****	**0.01 (0.00)*****
Second Sub-Sample	**0.80 (0.42)***	**0.01 (0.00)*****
Third Sub-Sample	**-0.40 (0.17)****	**-0.01 (0.00)****
Fourth Sub-Sample	**0.74 (0.25)*****	**0.01 (0.00)***

Notes: This table reports estimates of the parameters for the Φ_{12} and Φ_{21} respectively.

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Variance Equation (Bivariate BEKK GARCH (1,1) Model)

Table (B.2.5.) reports estimates of ARCH and GARCH parameters. Following equations (2.15) and (2.16), we notice that the sum of the squared values of ARCH parameter (α) and GARCH parameter (β) for the total sample and all the other sub-samples respectively is less than one. Additionally, the ARCH and GARCH coefficients are positive and significant in all various cases.

Table B.2.5. Variance Equations: Bivariate BEKK GARCH Coefficients (Dow Jones)

Samples	$h_{11,t}$ **(Volume)**	$h_{22,t}$ **(Volatility)**
Whole Sample		
$\boldsymbol{\alpha_i}$	**0.26 (0.03)*** **	**0.37 (0.05)*** **
$\boldsymbol{\beta_i}$	**0.87 (0.03)*** **	**0.91 (0.02)*** **
First Sub-Sample		
$\boldsymbol{\alpha_i}$	**0.15 (0.04)*** **	**0.29 (0.20)*** **
$\boldsymbol{\beta_i}$	**0.91 (0.07)*** **	**0.88 (0.04)* **
Second Sub-Sample		
$\boldsymbol{\alpha_i}$	**0.40 (0.07)*** **	**0.25 (0.05)*** **
$\boldsymbol{\beta_i}$	**0.88 (0.06)*** **	**0.93 (0.02)*** **
Third Sub-Sample		
$\boldsymbol{\alpha_i}$	**0.39 (0.05)*** **	**0.36 (0.06)*** **
$\boldsymbol{\beta_i}$	**0.82 (0.17)** **	**0.91 (0.02)*** **
Fourth Sub-Sample		
$\boldsymbol{\alpha_i}$	**0.42 (0.06)*** **	**0.46 (0.18)*** **
$\boldsymbol{\beta_i}$	**0.85 (0.31)*** **	**0.87 (0.06)*** **

Notes: This table reports estimates of the parameters for the ARCH ($\boldsymbol{\alpha_i}$) and GARCH ($\boldsymbol{\beta_i}$).

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Appendix 2.C

In this section, we will present the results of bivariate VAR-CCC GARCH (1,1) process with regards to the case of Dow Jones.

Mean Equation (Bivariate VAR Model)

With the bivariate VAR model, we will examine the case of own effects (the effect of the lagged values of volume (volatility) on the volume (volatility) in the mean equation). In addition, we will examine the case of cross effects (the effect of the lagged values of volatility obtained in the mean equation of volume and vice versa).

Own Effects

Table (C.2.1.) reports coefficient estimates of the own effects $\Phi_{11}(L)$ and $\Phi_{22}(L)$. The $\Phi_{11}(L)$ and $\Phi_{22}(L)$ columns report results for the volume and volatility equations respectively.

For example, the lag polynomial $\Phi_{11}(L)$ for the second sub-sample might be written with regards to the equation (2.3) as follows (see Table C.2.1.):

$$\Phi_{11}(L) = 1 - (\phi_{11}^{1}L^{1} + \phi_{11}^{3}L^{3})$$

As an example, and by applying equation (2.4), the lag polynomial $\Phi_{22}(L)$ for the second sub-sample might be written as follows (see Table C.2.1.):

$$\Phi_{22}(L) = 1 - \phi_{22}^{9}L^{9}$$

Table C.2.1. Mean Equations: AR Lags (Own Effects)

Samples	**Eq. (2.3):**$\Phi_{11}(L)$	**Eq. (2.4):**$\Phi_{22}(L)$
Whole Sample	**1,2,3**	**1,2,3,4,5**
First Sub-Sample	**1,2,4**	**1,2,5,6**
Second Sub-Sample	**1,3**	**9**
Third Sub-Sample	**1,2,3**	**1,2,3,4**
Fourth Sub-Sample	**1,3**	**2,6,7**

Notes: This table reports coefficient estimates of the own effects $\Phi_{11}(L)$ and $\Phi_{22}(L)$.

Cross Effects (The Volume-Volatility Linkage)

We have employed the bivariate VAR model with lagged values of one variable which have been obtained in the mean equation of the other variable in case of testing for a causal relation between these two variables.

Table (C.2.2.) reports coefficient estimates of the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. For instance and by following equation (2.5), the lag polynomial $\Phi_{12}(L)$ for the whole sample is as follows:

$$\Phi_{12}(L) = \phi_{12}^{5} L^{5}$$

Moreover, by applying equation (2.6), the lag polynomial for the first sub-sample could be written as follows (see Table C.2.2.): $\Phi_{21}(L) = \phi_{21}^{4} L^{4}$

Table C.2.2. Mean Equations: AR Lags (Cross Effects)

Samples	$\Phi_{12}(L)$	$\Phi_{21}(L)$
Whole Sample	**5**	**4**
First Sub-Sample	**1**	**4**
Second Sub-Sample	**1**	**4**
Third Sub-Sample	**5**	**2**
Fourth Sub-Sample	**2**	**10**

Notes: This table reports coefficient estimates of the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$.

As seen in Table (C.2.3.) in the whole sample and the fourth sub-sample, there is a mixed bidirectional feedback between volume and volatility. In particular, volatility affects volume negatively in the whole sample and the fourth sub-sample respectively whereas the reverse impact is of the opposite sign.

We also notice that there is a positive bidirectional feedback between volume and volatility in the first and second sub-samples respectively. This result is consistent with the theoretical background presented by Ghysels et al. (2000).

On the other hand, there is a negative bidirectional feedback between these two variables in the third sub-sample.

Those results in Table (C.2.3.) imply the evidence for the whole sample which suggests that the positive effect of volume on volatility reflects the same positive impact in the first, second and fourth sub-samples respectively. Moreover, it is also taken into account that the negative impact of volatility on volume in the whole sample is being reflected in the third and fourth sub-samples respectively.

In comparison with the results presented in table 2.13., we can observe some principal differences. Whereas the effect of volatility on volume in the case of S&P is negative in the first and second sub-samples, this impact is positive in the case of Dow Jones. In addition, volume affects volatility negatively in the whole sample and the second sub-sample in the case of S&P, while this impact is positive in the case of Dow Jones. Finally, we observe no effect of volume on volatility in the first sub-sample for S&P, while this impact is positive in the case of Dow Jones.

Table C.2.3. The Volume-Volatility Link (VAR-CCC GARCH)

Samples	Effect of Volatility on Volume	Effect of Volume on Volatility
Whole Sample	**Negative**	**Positive**
First Sub-Sample	**Positive**	**Positive**
Second Sub-Sample	**Positive**	**Positive**
Third Sub-Sample	**Negative**	**Negative**
Fourth Sub-Sample	**Negative**	**Positive**

Moreover, Table (C.2.4.) reports parameter estimates of the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. The $\Phi_{12}(L)$ and $\Phi_{21}(L)$ columns report results for the volume and volatility equations respectively.

Table C.2.4. The Coefficients of The Volume-Volatility Link (VAR-CCC GARCH)

Samples	$\Phi_{12}(L)$	$\Phi_{21}(L)$
Whole Sample	**-0.19 (0.09)****	**0.01 (0.00)****
First Sub-Sample	**1.02 (0.33)*****	**0.01 (0.00)*****
Second Sub-Sample	**0.71 (0.36)****	**0.01 (0.00)*****
Third Sub-Sample	**-0.48 (0.17)*****	**-0.01 (0.00)****
Fourth Sub-Sample	**-0.44 (0.21)****	**0.01 (0.00)***

Notes: This table reports estimates of the parameters for the Φ_{12} and Φ_{21} respectively.

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Variance Equation (Bivariate CCC GARCH (1,1) Model)

Table (C.2.5.) reports estimates of ARCH, GARCH and CCC parameters. Following equation (2.7), we notice that the sum of the coefficients of ARCH parameter (α) and GARCH parameter (β) for the total sample and all the other sub-samples respectively is less than one. Additionally, the ARCH and GARCH coefficients are positive and significant in all various cases.

With regards to equation (2.18), the conditional correlation between volume and volatility in the whole sample and first sub-sample is (0.34). In the second and fourth sub-samples, the estimated values of ρ for volume-volatility (0.80 and 0.75) are higher than the corresponding values for the whole sample. On the contrary, the conditional correlation's estimated value for the third sub-sample (0.31) is lower than the corresponding value for the whole sample/first sub-sample (0.34).

Table C.2.5. Variance equations: Bivariate GARCH And CCC Coefficients: (Dow Jones)

Samples	h_{1t} **(Volume)**	h_{2t} **(Volatility)**
Whole Sample		
α_i	**0.13 (0.02)***	**0.19 (0.05)***
β_i	**0.34 (0.10)***	**0.80 (0.03)***
ρ	**0.34 (0.02)***	-
First Sub-Sample		
α_i	**0.07 (0.03)**	**0.15 (0.06)**
β_i	**0.53 (0.28)***	**0.54 (0.18)***
ρ	**0.34 (0.03)***	-
Second Sub-Sample		
α_i	**0.15 (0.06)***	**0.06 (0.03)**
β_i	**0.76 (0.12)***	**0.93 (0.05)***
ρ	**0.80 (0.05)***	-
Third Sub-Sample		
α_i	**0.14 (0.04)***	**0.12 (0.06)**
β_i	**0.31 (0.16)***	**0.84 (0.04)***
ρ	**0.31 (0.03)***	-
Fourth Sub-Sample		
α_i	**0.26 (0.03)**	**0.23 (0.12)**
β_i	**0.72 (0.18)***	**0.76 (0.09)***
ρ	**0.75 (0.04)***	-

Notes: This table reports estimates of the parameters for the ARCH ($\boldsymbol{\alpha_i}$), GARCH ($\boldsymbol{\beta_i}$) and ccc ($\boldsymbol{\rho}$).

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Chapter Three

Examining the Interaction Between Aggregate Mutual Fund Flow and Stock Market Return: Evidence From U.S. Market

3.1. Introduction

In the past few decades, various empirical studies have been undertaken in order to determine the key factors which drive the growth in the capital market. In this context, the linkage between stock market return and mutual fund flow has constituted a substantial part of the total problem of this growth. Capital market growth greatly needs investment flow for the purpose of financing investment projects. Moreover, the flow of mutual funds might be significantly considered in this direction. It is defined as the financial market which could contribute to the country's real economic growth.

Financial economists and researchers have appointed two fundamental channels through which the improvement in the country's financial system could affect the process of economic growth. First, the capital accumulation channel induces the financial sector development that could lead to economic growth. Second, the total factor productivity channel indicates that an effective financial system can facilitate the adoption of modern technology in order to boost development of the technology- and knowledge- intensive industries, which could be achieved through the provision of functional credit facilities as well as the other financial services.

As a result, mutual funds, local institutions and foreign investors play a vital role in this direction. Thus, the function of domestic mutual funds' inflow could not be overemphasized in contributing to aforementioned channels. Mutual funds could strengthen the capital accumulation channel through mobilizing the country's small savings herewith providing avenues for massive development investments.

A mutual fund is considered as an instrument of investing money. It is defined as an established fund in the form of a trust for the purpose of raising money through the sale of units to the public under one or more schemes for security investments including money market instruments. Furthermore, a mutual fund pools the savings of a group of investors who share the same financial purpose. This group of investors buys units of a specific mutual fund scheme which has a defined strategy and investment objective. Consequently, the fund's manager utilizes the allocated money to purchase securities such as bonds and stocks which constitute the fund's portfolio. The income generated through these investments and other capital appreciations recognized by the scheme is proportionally shared by its' unit holders with regards to the number to units obtained by investors.

Mishra et al. (2009) have stated that the mutual fund is considered as the most appropriate investment for the public because it offers a suitable opportunity to invest in both diversified and professionally managed portfolios of securities with a relatively low cost.

A mutual fund is a particular type of an investment vehicle or an institutional device through which investors pool their savings that are to be invested under the guidance of a group of experts in an enormous variety of corporate securities' portfolios in such a way which does not only minimize risk, but also ensures safety and stable investment's return (see, for instance, Dave (1992) and Mehru (2004) and the references therein). There are various advantages of mutual funds' investment. It simplifies money management by professionals at the lowest cost and ensures dilution in the transaction costs as a result of the large scale of the economies of operation. However, it provides flexibility for investors to change the investment objective and it is also convenient for investors to both invest their money and track the invested money's performance.

The literature on the dynamic relation between mutual fund flow and stock market return is mixed. Alexakis et al. (2005) have reported a statistical evidence of a mixed bi-directional causality between stock market return and mutual fund flow. Braverman et al. (2005) have observed that this linkage is significantly negative (this study is in line with our results for the majority of sub-samples obtained). Some other studies such as Fortune (1998), Mosebach and Najand (1999), and Cha and Kim (2010) have presented a positive relation between market return and mutual fund flow (this finding is consistent with the result of the sub-sample (UD5) for all various cases of the GARCH processes).

It is noteworthy to mention that we have observed two principal hypotheses in detecting the impact of mutual fund flow on security return which are the information revelation hypothesis and the price pressures hypothesis. The information revelation hypothesis has addressed that since the market responds to information revelation, then prices will be moving in the same direction as the fund flow and as a result, returns will be positively correlated with security returns. In addition, the price pressures hypothesis has presented that if mutual fund flow might exert price pressures, then security return should exhibit reversals as price should be returned to the fundamental levels after the sentiment or pressure wave has passed. Furthermore, another important hypothesis has been detected in the case of mutual fund flow affected by security return. The feedback-trader hypothesis has addressed the question of whether or not mutual fund investors move money into a market as a response to recent performance in this market.

This paper has two fundamental objectives. The first objective is analysing the return-flow linkage of the U.S. stock market (S&P 500 index). We estimate the main parameters in the two aforesaid variables by applying bivariate VAR model (examining the impact of the lagged values of one variable obtained in the mean equation of the other variable) with a univariate GARCH and univariate FIGARCH processes. The second objective is employing the bivariate VAR model (examining the effect of the lagged values of flow included in the mean equation of return and vice versa) with bivariate CCC GARCH as well as bivariate CCC FIGARCH models.

Our contribution in this study might be classified as follows. We obtain our required mutual funds flow' sample consisting of 1,774,367 daily observations and including only the U.S. domestic mutual funds flow existing at any time during the period spanning from February 3rd 1998 to March 20th 2012. In addition, we impose some selection criteria depending on Morningstar Category Classifications. This selected process has rendered a final sample of 3,538 daily observations. Furthermore, we run our statistical analysis on the whole sample through dividing it into three different cases. Whilst two sub-samples (A and B) are obtained in the first case, the second section includes five ups-and-downs (UDs) sub-samples. In addition, the third case consists of two cyclical (CYs) sub-samples. In the way of splitting our sample, we also take into account three major events in the financial markets, which are the 2000 Dot-Com Bubble, the 2007 Financial Crisis and the 2009 European Sovereign Debt Crisis.

Our major findings are as follows. Firstly, by employing both the univariate and bivariate CCC GARCH processes, there is a bidirectional mixed feedback between stock market return and aggregate mutual fund flow for the majority of the sub-samples obtained. In particular, flow affects return negatively whereas the reverse impact is of the opposite sign. That is, the evidence from the bivariate VAR model with both the univariate and bivariate CCC GARCH models suggests that the causal negative (positive) effect from flow (return) to return (flow) for the whole sample reflects the causal relation between flow and return in the sub-sample (B) which covers the period spanning from 26th July 2007 to 20th March 2012 (during both the 2007 Financial Crisis and the 2009 European Sovereign Debt Crisis). Moreover, a positive bi-directional causality between return and flow has been realized for the sub-sample

(B) as well as the sub-sample (UD5), which spans from 10th March 2009 until 20th March 2012 (during the 2009 European Sovereign Debt Crisis).

Secondly, by employing the bivariate VAR model with both the univariate and bivariate CCC FIGARCH processes, we find a bidirectional mixed feedback between stock market return and aggregate mutual fund flow in all the aforementioned sub-samples with one exception case (a positive causality) in the sub-sample (UD5). In particular, return affects flow positively whereas the reverse impact is of the opposite sign.

Hence, it is imperative to examine the return-flow causality in a developed market economy such as the U.S.A. This paper is organized as follows. The second section reviews the existing literature. The third section introduces the data, and describes the method of constructing both aggregate mutual fund flow and stock market return. The fourth section shows the various sub-samples. The fifth section displays the econometric approach. Empirical findings are reported and discussed in the sixth section. The seventh section concludes the paper.

3.2. Literature Review

The existing literature has basically focused on examining the interaction between mutual fund flows and stock market returns in the case of developed countries, and only a few studies have concerned the case of emerging market economies. Warther (1995) is considered as the pioneer of the study of aggregate mutual fund cash flows and security returns. This study has found a high correlation between concurrent unexpected cash flows into mutual funds and aggregate security returns, but no relation between these returns and concurrent expected flows. Using monthly data spanning the period January 1984 to December 1992 for stock, bond and money market funds as well as time series regressions, it has reported a negative linkage between subsequent flows and returns, but a positive relation between subsequent returns and flows. However, Warther (1995) has also found an evidence of a positive correlation between the fund flows and the returns of the securities held by the funds, and another evidence of a non-relation between these fund flows and the returns of other types of the securities. He has rejected both sides of a feedback trading model, arguing that security's returns neither lead nor lag mutual fund's flows.

Warther (1995) has presented two principal types of theory for the purpose of detecting the impact of mutual fund flow on security return. The first hypothesis is information revelation by mutual fund flow. Information revelation is considered as another explanation for a potential linkage between mutual fund flow and security price movements. If mutual fund investors possess information, then their trades will be associated with new information. Since the market responds to this information revelation, then prices will be moving in the same direction as the fund flow and as a result, returns will be positively correlated with security returns. In this scenario, the market is responding efficiently to new information rather than reacting to fund flow because of price pressure.

On the other hand, the second hypothesis is price pressures and investor sentiment. Since investor sentiment is considered as an essential force in the markets and if flow into mutual funds is a good measure of this sentiment, then security returns should have a significant and positive correlation with flow into mutual funds. With regards to the price-pressure hypothesis, if mutual fund flow might exert price pressures, then security return should exhibit reversals as price should be returned to the fundamental levels after the sentiment or pressure wave has passed.

Warther (1995) has mentioned another important hypothesis in terms of mutual fund flow affected by security return. The feedback-trader hypothesis has addressed the question of whether or not mutual fund investors move money into a market as a response to recent performance in this market.

Moreover, Potter (1996) has investigated the lead-lag linkage between fund flows and returns for several categories of equity funds. By using Granger causality tests, this study has provided an evidence that flow into aggressive growth funds could be predicted by using the stock returns, but the same has not applied in the case of income funds. Remolona et al. (1997) have examined the correlation between the market performance and the net flows into the various mutual fund groups. They have used four macroeconomic variables which were considered as instruments for stock and bond excess returns: the consumer price index, capacity utilization, domestic employment and the Federal Reserve’s target federal funds rate. Their findings were strongly consistent with those of Warther (1995) in that market returns are highly correlated with aggregate mutual fund flows. The analysis of their instrumental variables has suggested weak effects of short-term returns on mutual fund flows.

Furthermore, Fortune (1998) has found an evidence on mixed causal relationship between mutual fund flows and market returns. He has stated that some mutual fund flows have an impact on future security returns, while future fund flows would be affected by security returns. Additionally, he has provided –by implying VAR models with seven variables as well as monthly data for the period January 1984 through December 1996- a strong evidence of positive linkage between contemporaneous returns and mutual fund flows. Surprisingly, these results are in sharp contrast with the conclusions of Warther (1995), Potter (1996) and Remolona et al. (1997) respectively that no significant effect of past security returns on fund flows has been detected.

The relationship between aggregate mutual fund flows and stock and bond returns has also been investigated by Edwards and Zhang (1998). They have employed two statistical procedures in order to identify a causal relationship: the instrumental variables method and Granger causality analysis. They have also examined aggregate monthly flows of U.S. bond mutual funds for the period January 1976 through February 1996 and aggregate monthly flows of U.S. equity mutual funds for the period January 1961 through February 1996. While flows into bond and stock funds have not affected either bond or stock returns for the period 1971-1981 (when stock returns have been considerably depressed by widespread redemptions from equity mutual funds), the bond and stock returns have significantly affected the magnitude of flows into both bond and stock funds.

Meanwhile, Potter and Schneeweis (1998) have stated that security market returns are essential in case of predicting flows into growth funds and aggressive growth funds. Their result has rejected the hypothesis that 'equity fund flows lead security returns. Fant (1999) has examined the relationship between the components of aggregate equity mutual fund flows and stock market returns. These components are new sales, redemptions, exchanges-in and exchanges-out. From a Granger causality perspective and aggregate monthly fund flow data from January 1984 through December 1995, he has showed an instantaneous feedback between returns and exchanges-in and-out in a given month, as well as an evidence of feedback from returns to exchanges-out. He has also reported that flow-return linkage which was documented by Warther (1995) has been solely existed between exchanges and returns.

On the other hand, Mosebach and Najand (1999) have applied Engle and Granger error correlation model, followed by a state space procedure to investigate the long-run equilibrium linkage between the S&P 500 index and the net flow of funds into equity mutual funds. Using monthly data from January 1984 through July 1998, they have provided an evidence of a causal relation between the stock market and the net inflow of funds. Additionally, their results have showed that the level of the stock market in the previous month has had a significant impact on the net flow of funds invested in the stock market. They have also reported a bi-directional causality between flow of funds into the market and the level of the stock market, and this has been resulted in that a current robust equity market motivates much more investment in this market.

Edelen and Warner (2001) have examined the relationship between unexpected aggregate flow into U.S. equity funds and market returns. Using high frequency daily data for a sample of 424 U.S. equity funds and for the period 2nd February 1998 through 30th June 1999, they have mainly reported a concurrent flow-return linkage, but flow has also followed returns with a one-day lag. The lagged response of flow has indicated either a positive feedback trading or a common response of both flow and returns to new information. Furthermore, they have reported a very strong association between the previous days' return and funds flow. This association has suggested that the reaction of funds flow to returns or to the information driving returns has essentially happened with a one-day lag, as well as investors generally require an overnight period in order to react. In addition, they have provided an evidence of a significant correlation between concurrent market returns and aggregate mutual fund flow at a daily frequency. This concurrent relation has indicated that both institutional trading and funds flow have affected returns.

Meanwhile, Cha and Lee (2001) have contradicted the results reported by Edelen and Warner (2001) with regards to positive feedback. Using a sample of monthly data covering the period from January 1984 to December 1999 and Granger causality tests, they have not provided an evidence for the price-pressure impact that equity fund flows -in the presence of market fundamentals- have directly affected stock market prices. Instead, they have stated that the performance of the stock market has a direct impact on the equity fund flows. In general, investors change their demand for stocks through their attempt to forecast the fundamentals of firms.

The impact of market returns on aggregate fund flows has been also examined by Papadamou and Siriopoulos (2002) through using a similar methodology to Warther (1995). Using monthly data covering the period from January 1998 to March 2002, their result has showed a small positive concurrent linkage between market returns and unexpected net flows. While, they have stated a low correlation between market returns and fund flows. Nevertheless, the interaction between index fund flows and asset prices has been analysed by Goetzmann and Massa (2003). Using daily fund flow's data and performing a Geweke-Meese-Dent (GMD) test, they have indicated a strong contemporaneous correlation between S&P market returns and fund flow, while no correlation between flows and overnight returns has been reported.

Using quarterly data for a 44-year time period and spanning from the first quarter of 1952 through the last quarter of 1995, Boyer and Zheng (2004) have explored the correlation between stock market returns and mutual fund flows. They have indicated that the contemporaneous relation between mutual fund flows and return is significant and positive. They have employed the VAR approach to study the lead-lag relation between quarterly mutual fund flows and stock returns. Their finding has suggested that mutual fund sector might exert price pressure on the market through its' demand for stocks.

Furthermore, by employing weekly data covering the period January 10, 1992 through August 31, 2001, Indro (2004) has examined the correlation between investor sentiment and net aggregate equity fund flow. He has showed that net aggregate equity fund flow in the current week is higher when individual investors have become more bullish in both the previous and current weeks. He has also provided an evidence that the relationship between investor sentiment and net aggregate equity fund flow has remained strong even after accounting for the effects of inflation and risk premium. Moreover, he has suggested that the behaviour of equity fund investors is influenced not only by investor sentiment, but also by economic fundamentals.

Alexakis et al. (2005) have investigated the same-day interaction between stock market returns and mutual fund flows in Greece. They have examined the possibility of a causality mechanism through which stock returns may affect mutual fund flows and vice versa. By running the error correlation model, they have reported a statistical evidence of a mixed bi-directional causality between stock returns and mutual fund flows. Furthermore, they have also showed that –by implying the Cointegration regression- mutual fund flows induce stock market returns to either increase or decline. As a result, inflows and outflows of cash into and out of equity funds seem to cause ascending and descending stock returns in the Greek stock market.

Braverman et al. (2005) have examined the relationship between aggregate new flows into and out of the funds and the subsequent returns. By employing a statistical test based on bootstrapping technique and using monthly data of aggregate US mutual funds spanning the years 1984-2003, they have provided an evidence that this linkage is significantly negative. This negative correlation has induced mutual fund investors –as a group- to recognize a higher long-term accumulated return on ‘buy and hold’ position in these funds than long-term accumulated return. Furthermore, using different categories of funds, bonds and money market funds, they have found a similar negative relation between the lagged flows into and out of these funds and future returns.

Cha and Kim (2005) have investigated both the short-run and long-run dynamic relationship between mutual fund flows and security returns. They have employed several empirical methods including Granger causality tests, SURECM, DSUR analyses, DOLS and iterative SURDAF tests. By using system approach, they have examined various asset classes including stock, bond and money markets. They have provided an empirical evidence of a positive long-run relation between mutual fund flows and security returns. Furthermore, their findings have indicated that the security performance in the U.S. financial market seems to be the most important element in the case of explaining mutual fund flows. This study is consistent with our empirical result for the fifth ups-and-downs sub-sample.

Furthermore, Cha and Kim (2006) have employed both a single equation method including error correlation model and the Granger causality test to examine the interaction between stock market prices and aggregate mutual fund flows. They have provided an evidence that stock market returns lead stock fund flows in the U.S. financial market. By using daily data provided by the Korean Stock Exchange (KSE) and covering the 1996-2003 period, Oh and Parwada (2007) have analysed the linkages between stock market returns and mutual fund flows which were measured as stock sales, purchases and net trading volumes.

The results from bivariate VAR model have showed a significant negative correlation between returns and net trading volumes, but a significant positive correlation has been observed in the case of both stock sales and purchases. This finding has suggested that negative feedback trading has been indicated at an aggregate level, which is inconsistent with the U.S. mutual fund's finding of Edelen and Warner (2001). However, the result of Granger causality tests has affirmed the hypothesis that 'flow does not Granger-cause return' for both stock sales and net trading volumes, but rejecting it in the case of stock purchases as this test has revealed that standardized purchase flow might contain information about returns.

In addition, another result has rejected the hypothesis that 'return does not Granger-cause flow' for all the flows' measures at high levels of statistical significance, which is consistent with that result obtained by Cha and Kim (2006). This result has suggested that –in Korean equity mutual funds- stock market returns could move mutual fund flows.

Rakowski and Wang (2009) have analysed the dynamic relationship between mutual fund flows and market returns. They have obtained daily data for both mutual fund flows and returns covering the period March 2000 to October 2006. They have distinguished between contrarian and momentum traders. While contrarian investors act –when flows are preceded by negative returns- as if they are buying funds that have previously suffered a price decline but selling those funds whose prices have increased, momentum traders are those who are chasing hot funds, and –as a result- flows will be positively related to the lagged returns. This study is in line with our empirical findings for all various cases of GARCH processes. By applying a Vector Auto Regression (VAR) approach, they have showed a significant negative impact from the lagged return on the current day's flow. This has implied that less mutual fund investors are following a strategy consistent with momentum behaviour than with short-term contrarian behaviour. They have also provided an evidence of a positive interaction between daily returns and lagged flows. According to their point of view, this positive link could be due to either a temporary price pressure effect or a permanent information impact.

For comparison, they have also run a VAR model on monthly returns and mutual fund flows. They have found a very few statistically significant lead-lag interactions between mutual fund flows and returns. In contrast to the daily results, the monthly results have showed almost no relation between future returns and mutual fund flows. They have also observed a significant positive autocorrelation of daily returns but a significant negative autocorrelation of daily flows at short lags. On the contrary, a significant positive autocorrelation of fund flows at monthly intervals has been reported. They have concluded that the level of active management, investment objectives and fund's marketing policies might explain the variation in the dynamics of daily mutual fund flows.

In addition, Cha and Kim (2010) have stated that investors move their capital to the securities which yield higher returns in order to rebalance their investment portfolios. They have combined information from money and bond markets with information from the stock market in a system method. This study has focused on the linkage between aggregate mutual fund flows and stock market returns at the macro level. They have found a high positive relationship between these two components.

Many theoretical approaches have supported their result such as investor sentiment, the information revelation and the price pressure theory. As one of the most essential factors affecting the mutual fund market is the investor sentiment, the information revelation hypothesis has indicated that purchases of well-informed investors may consider as a signal to other less-informed investors to buy a specified mutual fund. This idea was supported by the assumption that the market would react to the available information rather than responding to the flow of mutual funds. Additionally, increasing in mutual funds' inflows stimulates the stock prices to go up and this is resulted in a higher demand to hold stocks. This notion has been implied by the price pressure theory. These three elements have been also discussed by Warther (1995) and Boyer and Zheng (2004).

They have collected their monthly data on aggregate mutual fund flows through the Investment Company Institute (ICI). According to ICI, mutual funds have included money market funds, bond and income funds and equity funds. They have employed a Seemingly Unrelated Regression Error Correlation Model (SURECM), and Granger and Sims causality in a system method. The distinction point of this study was improving the efficiency and providing more economically rational estimates through utilizing information from the money, stock and bond markets.

Jank (2012) has investigated the relation between mutual fund flows and stock market returns. By applying a vector autoregressive (VAR) model with its' residuals and using quarterly data covering the period from 1984:Q1 until 2009:Q4, he has showed a contemporaneous correlation between stock returns and mutual fund flows. Moreover, and by separating the fund flows into their expected and unexpected components in order to provide a direct insight into the linkage between stock returns and fund flows, he has stated that market returns are uncorrelated with expected flows, but are correlated with unexpected flows. In addition, he has focused on the information-response hypothesis which includes two principal implications. Firstly, variables which predict the real economy should be linked to mutual fund flows and secondly, the real economic activity should be also predicted by mutual fund flows if they react to news about the real economy.

Alexakis et al. (2013) have empirically examined the relationship between mutual fund flows and stock market prices in Japan. In particular, they have investigated both the short and long run dynamics between fund units and stock prices. They have employed the crouching error correction model (CECM) and daily data spanning from 1st January 1998 to 31st December 2007 to assess the causality effects between fund flow changes and stock market movements. In the case of negative movements, their finding has indicated a unidirectional causality running from fund flows to stock prices. Whereas for positive movements, it has showed a bi-directional causality between fund units and stock index prices. Overall, they have provided an evidence that both stock prices and unit formation are affected by market microstructure, taxation as well as investors' sentiment.

Aydogan et al. (2014) have examined the dynamic interactions between stock market returns and mutual fund flows for the Turkish capital market. Their data set has covered the period from June 2nd, 2005 through August 31st, 2012. By employing the conventional Engle and Granger and Johansen-Juselius cointegration tests, the empirical results have asserted an existence long-run relationship between all categories of mutual fund flows and stock returns. Furthermore, the statistical evidence obtained through running the vector error correction test 'causality test' has suggested a bidirectional causality between stock returns and all categories of fund flows. In other words, the lagged stock returns might Granger cause the mutual fund flows and vice versa.

As a result, the literature on the dynamic relation between mutual funds flow and stock market returns is mixed. Alexakis et al. (2005) have reported a statistical evidence of a mixed bi-directional causality between stock returns and mutual fund flows. Braverman et al. (2005) have provided an evidence that this linkage is significantly negative. Some other studies (Fortune (1998), Mosebach and Najand (1999) and Cha and Kim (2010)) have provided an evidence of a positive relation between market returns and mutual funds flow.

3.3. Data

Sample of Mutual Funds

We have obtained our daily data on mutual funds flow from Trim Tabs Investment Research of Santa Rosa, CA., which has primarily collected daily data on total net assets (TNAs), net asset values (NAVs) and flow for a sample varying from one daily individual mutual fund to 8,135 daily individual mutual funds. The collection procedures of Trim Tabs' data could be summarized as follows: mutual fund investors send orders for redemption or purchase to the transfer agent or the fund customer service centre on a daily basis. At the time, when a fund receives an order from an investor, then the order –by law- has to be executed at the next calculated net asset value. The day's net asset value is considered as the day's closing prices of securities held by the previous trading day's fund and shares outstanding.

Afterwards, net asset value is reported to both the transfer agent and the National Association of Security Dealers before 5:30 P.M. EST. The transfer agent promptly processes all orders overnight after the net asset value has been calculated and employs this net asset value at the process of computing the change in the fund's receivables, payables, cash and shares outstanding. Consequently, the transfer agent reports back these varied numbers to fund managers in order to be entered into the fund's balance sheet on the subsequent morning. Furthermore, each morning, Trim Tabs receives funds' data of the previous day's net asset values and total net assets (see, for instance, Edelen and Warner (2001) and Cao et al. (2008) and their references therein). Finally, Trim Tabs calculates the net flow of each mutual fund as follows: $Flow_t = TNA_t - NAV_t \frac{TNA_{t-1}}{NAV_{t-1}}$.

We have obtained a data sample containing all the traded mutual funds within the U.S.A. and consisting of 4,829,466 daily mutual funds flow observations. This sample has been received via an Access file with regards to the massive number of daily observations. In order to maintain the data set, we have applied the Queries technique via Access which is considered as the most beneficial method to manipulate the data and compose our target sample. Consequently, we have obtained our required sample consisting of 1,774,367 daily observations and including only the U.S. domestic mutual funds flow existing at any time during the period spanning from February 3rd 1998 to March 20th 2012. In addition, we have applied some selection criteria depending on Morningstar Category Classifications which will be subsequently discussed in details.

Aggregation

This study has utilized data on net flows of U.S. domestic mutual funds, as expressed by the net change of mutual funds flow units. However, we have formed our final aggregated sample by obtaining the sum of mutual funds flow as well as the sum of total net assets on a daily basis. This selected process has rendered a final sample of 3,538 daily observations. This final data set has been converted to an Excel file to start running our statistical analyses. However, in order to eliminate the possible outliers which could occur as a result of recording errors, we have applied a five-standard-deviation filter suggested by Chalmers et al. (2001) to identify a potential error in the total net assets' data series. If the daily change in TNA was more than five standard deviations for each single fund, we hand-checked TNA against alternative sources because a five-standard-deviation change is an extremely rare case.

Because the number of mutual funds is inconstant over time (it varies from one daily individual mutual fund to 3,538 daily individual mutual funds), we have normalized the aggregate flow by dividing the sum of daily mutual funds flow by the sum of daily total net assets (aggregate flow is expressed as a percentage of the aggregate total net assets):

$$\%flow_t = \frac{Flow_t}{TNA_t}, Flow_t = \sum_{i=1}^{3538} flow_{it}$$

Classification

With regards to the Morningstar Category Classifications (2012), we have only considered funds which are domestically operated such as U.S. stock, sector stock and balanced asset classifications. We have excluded all other funds that are internationally operated (funds which invest their money to other international stocks).

The U.S. Stock Asset

The U.S. stock asset classifications consist of large value, large blend, large growth, mid-cap value, mid-cap blend, mid-cap growth, small value, small blend and small growth categories. Meanwhile, the categories of sector stock are communications, equity energy, equity precious metals, financial, global real estate, health, industrials, natural resources, real estate, technology and utilities. The balanced asset classification contains solely aggressive allocation category.

Large-value portfolios invest primarily in big U.S. companies which either growing more slowly or less expensive than other large-cap stocks. While large-cap stocks are defined as the stocks in the top 70% of the capitalization of the U.S. equity market, value is defined based on both slow growth (low growth rates for sales, book value, earnings and cash flow) and low valuations (high dividend yields and low price ratios).

On the other hand, large-blend portfolios are fairly representative of the overall U.S. stock market in growth rates, price and size. They tend to invest across the spectrum of U.S. industries and owing to their broad exposure. As a result, these portfolios' returns might be similar to those of S&P 500 index. This blend style is mainly assigned to portfolios where neither value nor growth characteristics predominate.

Furthermore, large-growth portfolios invest fundamentally in big U.S. companies which are designed to grow faster than other large-cap stocks. Most of these portfolios focus mainly on companies in promptly expanding industries. Growth is normally defined based on both high valuations (low dividend yields and high price ratios) and rapid growth (high growth rates for sales, book value, earnings and cash flow).

Whilst some mid-cap value portfolios focus primarily on medium-size companies, the others own a mixture of small-, medium- as well as large-cap stocks. All investors look for U.S. stocks that are growing more slowly or less expensive than the market. As a result, the U.S. mid-cap range for market capitalization typically represents 20% of the overall capitalization of the U.S. equity market.

However, the typical mid-cap blend portfolios invest in U.S. stocks of numerous styles and sizes, giving these portfolios a middle-of-the-road profile. Whereas most of this type of portfolios shy away from high-priced growth stocks, they are never considered as price-conscious which they terminate in value territory.

Nevertheless, some mid-cap growth portfolios focus on midsize firms, while others invest in stocks of all sizes and thus leading to a mid-cap profile. They command relatively higher prices through targeting U.S. companies which are designed to expand faster than other mid-cap stocks.

Small-value portfolios invest primarily in small U.S. companies with growth rates and valuations below the other small-cap peers. Meanwhile, small-cap stocks are defined as the stocks in the bottom 10% of the capitalization of the U.S. equity market.

Furthermore, companies at the smaller end of the market-capitalization range are in favour of small-blend portfolios. Whereas some of this type of portfolios employ a discipline that maily leads to holdings with growth rates and valuations close to the small-cap averages, others aim to own an array of growth and value stocks.

On the other hand, the main focus of small-growth portfolios is on faster-growing firms whose shares are at the lower end of the range of market-capitalization. Both young companies in their early growth stages and firms in up-and-coming industries are in favour of these portfolios. In addition, stocks of these businesses mainly tend to be volatile as a result of they are fast-growing and usually richly valued.

The Sector Stock

Communications portfolios –as a category of sector stock classifications- primarily concentrate on media and telecommunications companies of various kinds. Whilst a few of this kind of portfolios prefer film studios, publishers, online service providers and entertainment firms, most of them buy some combinations of wireless-communications, communications-equipment companies, traditional phone companies as well as cable television.

Meanwhile, equity energy portfolios invest in equity securities of U.S. companies which conduct business fundamentally in energy-related industries. This specific type of business includes and is not limited to companies in coal, exploration, pipelines, refineries, natural gas services, oil and gas services as well as alternative energy.

Mining stocks business is the essential concentration of equity precious-metals portfolios, as some of these portfolios do their own small amounts of gold bullion. Some portfolios have significant exposure to platinum-, silver- as well as base-metal-mining stocks, while others focus merely on gold-mining stocks.

Furthermore, financial portfolios primarily seek capital appreciation through investing in equity securities of U.S. financial services companies. These companies include brokerage firms, insurance companies, banks as well as consumer credit providers.

Portfolios of global real estate invest in both U.S. and non-U.S. real estate securities, in addition to the real estate operating companies. These portfolios primarily purchase securities such as convertible securities, securities that issued by real estate investment trusts and REIT-like entities and debt & equity securities.

Health portfolios concentrate on the health-care and medical industries. Whilst a few portfolios focus on one industry segment such as biotechnology firms and service providers, most of them fundamentally invest in a range of companies through buying everything from pharmaceutical and medical-device makers to hospitals, HMOs as well as nursing homes.

Moreover, industrial portfolios mainly seek capital appreciation through investing in equity securities of U.S. companies which are engaged in services belonging to cyclical industries. This investment includes companies in automotive, construction, machinery, chemicals, transportation, aerospace and defence, paper and environmental services.

Natural-resources portfolios exclusively concentrate on U.S. commodity-based industries such as chemicals, energy and minerals. While some of these portfolios invest across this spectrum in order to offer broad natural-resources exposure, others focus heavily on specific industries.

The principal investment of real estate portfolios is in diversified types of real estate investment trusts. REITs are –by definition- companies that manage and develop real estate properties. There are various types of REITs including mortgage, factory-outlet, hotel, office, apartment, industrial, health-care and shopping centre REITs.

Technology portfolios fundamentally concentrate on investing in the U.S. high-tech companies, specifically in the software, networking, computer, internet and semiconductor stocks. While some focus on a single technology industry, a few invest in biotechnology and medical-device stocks. Meanwhile, utilities portfolios mainly seek capital appreciation through investing in the U.S. equity securities. Public utilities generally include gas, telephone-service alongside electricity providers.

Balanced Asset

Last but not least, aggressive-allocation portfolios primarily seek to provide both income and capital appreciation through investing in three principal areas which are cash, stocks and bonds. These portfolios typically have 10% to 30% of assets in cash and fixed income and the remainder is in equities. The main feature of this type of portfolios is their tending to hold more extensive positions in stocks rather than moderate-allocation portfolios.

Finally, we have obtained data on S&P 500 stock index from Thomson Reuters Database. We have calculated stock market returns as the natural logarithm of daily closing prices (P_t) of S&P 500:

$$Return_t = LN\ P_t - LN\ P_{t-1}$$

3.4. Sub-Samples

The whole data set that has been examined in this chapter is covering the period spanning from 3rd February 1998 to 20th March 2012 (see figure 3.1. for closing price, figure 3.2. for return and figure 3.3. for flow). This whole sample comprises of 3,538 daily observations.

We have run our statistical analysis on this whole sample through dividing it into three different ways. Whilst two sub-samples (A and B) have been obtained in the first case, the second case has included five ups-and-downs (UDs) sub-samples. In addition, the third case has consisted of two cyclical (CYs) sub-samples. We have taken into account three major financial events through splitting the whole data set, which are the 2000 Dot-Com Bubble, the 2007 Financial Crisis as well as the 2009 European Sovereign Debt Crisis.

Whereas the sub-sample (A) of the first case has included 2,369 observations and covered the period 3rd February 1998 to 25th July 2007, the data set spanning from 26th July 2007 to 20th March 2012 has been obtained in the sub-sample (B) with 1,169 daily observations.

Furthermore, the sub-sample (UD1) in the second case has involved the period 3rd February 1998 to 1st September 2000. The second pattern (UD2) has spanned from 5th September 2000 to 7th October 2002. The period from 8th October 2002 until 19th July 2007 has been included in the sub-sample UD3. Whilst the fourth pattern (UD4) has covered the period 20th July 2007 to 9th March 2009, the sub-sample (UD5) has spanned from 10th March 2009 until 20th March 2012.

In addition, the CYs sub-samples that comprised the third case have covered the periods 3rd February 1998 until 7th October 2002 and 8th October 2002 to 9th March 2009 respectively. In other words, the sub-sample (CY1) has involved both the sub-samples (UD1 and UD2), whereas the sub-samples (UD3 and UD4) have comprised the sub-sample (CY2).

The essential purpose of obtaining these nine sub-samples is examining the positive/negative flow-return linkage amongst various periods of time and recognizing the possible changes that might happen to this relationship.

Figure.3.1. Closing Price

Figure.3.2. Stock Market Return

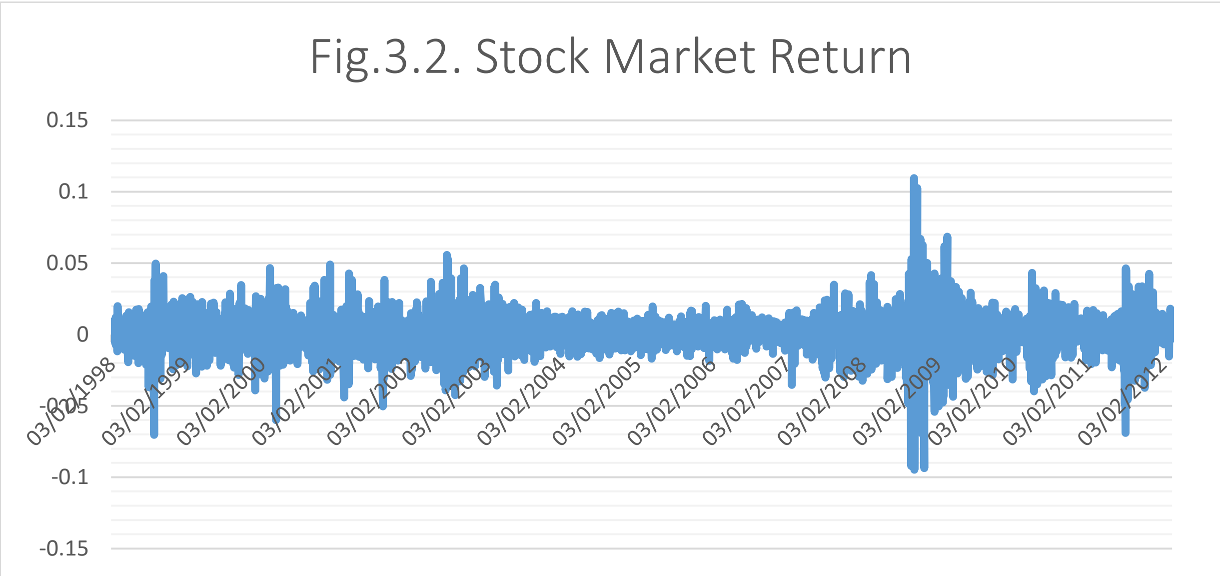
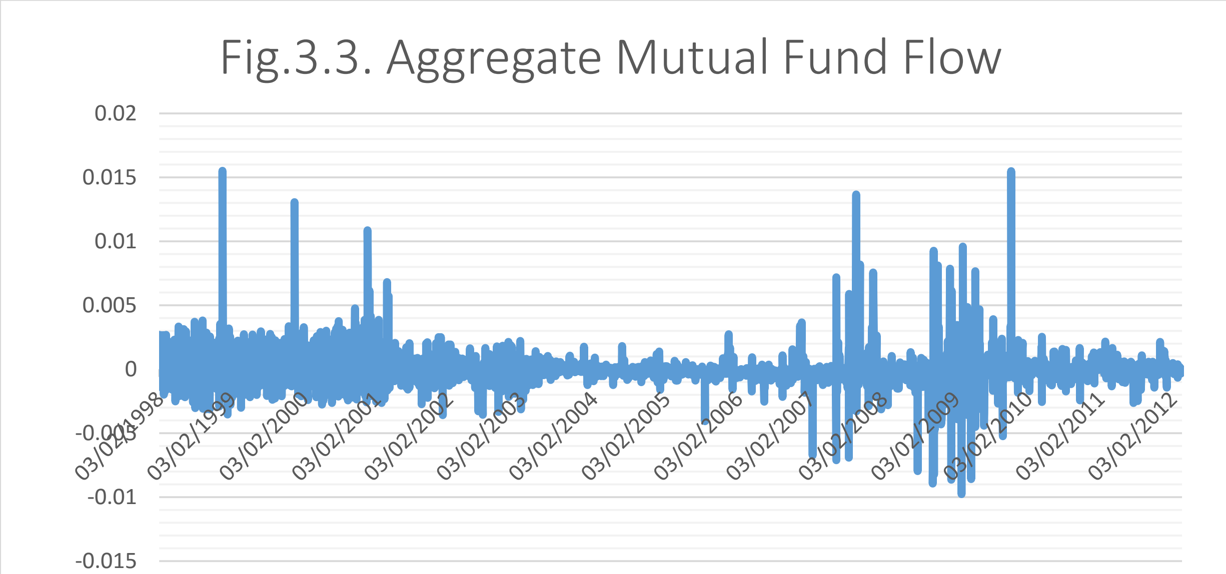


Figure.3.3. Aggregate Mutual Fund Flow



3.5. The Econometric Approach

The estimates of the various formulations have been obtained by quasi maximum likelihood estimation (QMLE) as it has been implemented by James Davidson (2007) in Time Series Modelling (TSM). In order to check for the robustness of our estimates, we have employed a range of starting values to be ensured that the estimation procedures have converged to a global maximum. Furthermore, the minimum value of the information criteria has been considered when choosing the best fitting specification.

The VAR-GARCH (1,1) Models

Mean Equation

In order to capture the potential interactions between return and flow, stock return (y_{1t}) and mutual funds flow (y_{2t}) follow a bivariate VAR model as follows:

$$\mathbf{\Phi}(L)\boldsymbol{y}_t = \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t, \qquad 3.1$$

With $\boldsymbol{y}_t = \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}$, $\boldsymbol{\mu} = \begin{bmatrix} \mu_{1t} \\ \mu_{2t} \end{bmatrix}$, $\boldsymbol{\varepsilon}_t = \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$ and $\mathbf{\Phi}(L) = \begin{bmatrix} (1-\Phi_{11}(L)) & -\Phi_{12}(L) \\ -\Phi_{21}(L) & (1-\Phi_{22}(L)) \end{bmatrix}$, where $\Phi_{ij}(L) = \sum_{l=1}^{l_{ij}} \phi_{ij}^{l}\, L^{l}$ and (L) denotes the lag operator.

The coefficients of the significant lags regarding return and flow are represented by $\Phi_{11}(L)$ and $\Phi_{22}(L)$ respectively, whilst $\Phi_{12}(L)$ and $\Phi_{21}(L)$ are considered as the significant lags of the exogenous variables with regards to return and flow respectively. In other words, the own effects are captured by $\Phi_{11}(L)$ and $\Phi_{22}(L)$, whereas the cross effects are captured by $\Phi_{12}(L)$ and $\Phi_{21}(L)$ polynomials for return and flow respectively.

With the bivariate VAR Model, we examine the case of return as a dependent variable with its' lagged values to investigate which of those lagged values have a significant impact on the return itself.

In particular, by following equation (3.1), the lag polynomial $\Phi_{11}(L)$ can be represented as follows:

$$\Phi_{11}(L) = \sum_{l=1}^{l_{11}} \phi_{11}^{l}\, L^{l}. \qquad 3.2$$

In addition, by applying the bivariate VAR Model, the case of flow has been included as a dependent variable with its' own lagged values to show which of these lagged values have a significant effect on the flow itself.

Following equation (3.1), the lag polynomial $\Phi_{22}(L)$ could be represented as follows:

$$\Phi_{22}(L) = \sum_{l=1}^{l_{22}} \phi_{22}^{l}\, L^{l}. \qquad 3.3$$

The bi-directional causality between return and flow is represented by the lag polynomials $\Phi_{12}(L)$ and $\Phi_{21}(L)$. The polynomial $\Phi_{12}(L)$ in equation (3.1) represents the effect of flow on return in the mean equation, and might be written as follows:

$$\Phi_{12}(L) = \sum_{l=1}^{l_{12}} \phi_{12}^{l} L^{l}. \qquad 3.4$$

The polynomial $\Phi_{21}(L)$ in equation (3.1) captures the impact of return on flow in the mean equation, and could be written as follows:

$$\Phi_{21}(L) = \sum_{l=1}^{l_{21}} \phi_{21}^{l} L^{l}. \qquad 3.5$$

Variance Equation

On the other hand, the bivariate vector of innovations ε_t is conditionally normal with mean zero and variance-covariance matrix $\mathbf{H_t}$. That is $\varepsilon_t|\Omega_{t-1} \sim N\ (\mathbf{0}, \mathbf{H_t})$**:**

$$\mathbf{H_t} = \begin{bmatrix} h_{1t} & h_{12,t} \\ h_{21,t} & h_{2t} \end{bmatrix}, \quad 3.6$$

where $h_{it}, i = 1,2$ denotes the conditional variance of stock return and mutual funds flow respectively. $h_{12,t}$ denotes the conditional covariance of the two variables.

GARCH Models

In this chapter, we will examine four alternative GARCH models. First, we will assume that h_{it} follow univariate GARCH (1,1) processes:

$$h_{it} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1}, i = 1,2, \qquad 3.7$$

And that $h_{12,t} = 0$. Note that $\omega_i > 0$, $\alpha_i > 0$, and $\beta_i \geq 0$ in order for $h_{it} > 0$ for all t. Moreover, $\alpha_i + \beta_i < 1$ for the unconditional variance to exist.

The related literature on GARCH (1,1) model is enormous (for instance, see Engle, 1982, Bollerslev, 1986, Bollerslev et al. 1992, Bollerslev et al. 1994, and the references therein). The ARCH process that introduced by Engle (1982) has explicitly recognized the difference between the conditional variance and the unconditional variance, allowing the conditional variance to change over time as a function of past errors.

Second, we will assume that $\mathbf{H_t}$ follows the bivariate constant conditional correlation (CCC) GARCH (1,1) model of Bollerslev et al. (1992). That is, h_{it} is given by equation (3.7).

And that $h_{12,t}$ is given by:

$$h_{12,t} = \rho\sqrt{h_{1t}}\sqrt{h_{2t}}, \qquad 3.8$$

where ρ denotes the ccc.

We employ a bivariate GARCH model to examine the dual linkage between U.S. mutual funds flow and S&P 500 stocks return. Bollerslev (1986) has developed the GARCH (1,1) model through allowing the conditional variance to depend on the past conditional variances. Whereas the autoregressive components capture the persistence in the conditional variance of flow and return, the past squared residual components capture the information shocks to flow and return.

Bollerslev et al. (1992) have stated that bivariate GARCH model has been empirically shown to reasonably capture the time variation in the volatility of daily stock market returns.

The VAR-FIGARCH (1,d,1) Models

Third, we will assume that h_{it} follow univariate FIGARCH (1,d,1) processes:

$$(1-\beta_i L)h_{it} = \omega_i + [(1-\beta_i L)-(1-c_i L)(1-L)^{d_i}]\varepsilon_{i,t-1}^2, i = 1,2, \quad 3.9$$

where $c_i = \alpha_i + \beta_i$ and d_i is the long memory parameter. Note that if $d_i = 0$, then the above FIGARCH (1,d,1) model reduces to the GARCH (1,1) model in equation (3.7). The sufficient conditions of Bollerslev and Mikkelsen (1996) for the positivity of the conditional variance of a FIGARCH (1,d,1) model: $\omega_i > 0$, $\beta_i - d_i \leq c_i \leq \frac{2-d_i}{3}$, and $d_i\left(c_i - \frac{1-d_i}{2}\right) \leq \beta_i(c_i - \beta_i + d_i)$ should be satisfied for both i (see also Conrad and Haag, 2006, Conrad, 2010, and Karanasos et al. 2016,). We also assume that $h_{12,t} = 0$.

The presence of apparent long-memory in the autocorrelations of absolute returns of diverse financial asset prices have been reported by Dacorogna et al. 1993, Ding et al. 1993, Harvey, 1993, and Delima et al. 1994,. However, the FIGARCH process has been primarily introduced by Baillie et al. (1996). This approach has purposed to improve a more flexible class of processes for the conditional variance which are more capable of representing and explaining the observed temporal dependencies in the financial market's volatility.

In the fourth case (bivariate FIGARCH (1,d,1)), we will assume that $h_{12,t}$ is given by equation (3.8), where h_{it} is given by equation (3.9).

Brunetti and Gilbert (1998) have extended the multivariate GARCH (1,1) model to the multivariate FIGARCH (1,d,1) model by using the constant correlation parameterization. Their choice has fundamentally been motivated by three principal considerations which are: it is considered as the most parsimonious of the available specifications, stationarity is being ensured by restrictions on the diagonal elements of the variance-covariance parameters matrices only and the variance-covariance matrices are positive definite under weak conditions.

3.6. Empirical Results

Case.1. VAR-GARCH (1,1) Model

In this section, we will present the results related to the bivariate VAR-GARCH process in the mean equation (including the cases of own effects and cross effects respectively) as well as the findings related to the variance equation (the GARCH coefficients).

Mean Equation

With the bivariate VAR model, we will examine the case of own effects in addition to examining the case of cross effects (the impact of the lagged values of flow obtained in the mean equation of return and vice versa).

Own Effects

Table (3.1) reports the chosen lags for the own effects $\Phi_{11}(L)$ and $\Phi_{22}(L)$. The $\Phi_{11}(L)$ and $\Phi_{22}(L)$ columns report results for the return and flow equations respectively.

Return as a Dependent Variable

By examining the effect of the first five lagged values of return on the return itself, the results –with respect to the obtained various samples- have been reported as Eq. (3.2) in Table (3.1).

As an example, the equation (3.2) for the sub-sample (A) can be written as follows:

$$\Phi_{11}(L) = \phi_{11}^{5} L^{5}.$$

Table 3.1. Mean Equations: AR Lags (Own Effects)

Samples	Eq. (3.2): $\Phi_{11}(L)$	Eq. (3.3): $\Phi_{22}(L)$
Whole Sample	**1**	**2,3**
<u>Panel A:</u>		
A	**5**	**1,3,4,5,6,7,8**
B	**1**	**2,3,4,5**
<u>Panel B (Ups and Downs Sub-Samples):</u>		
UD1	**3**	**1,2**
UD2	**2**	**3**
UD3	**1**	**1,2,3**
UD4	**1**	**1**
UD5	**3**	**2,3**
<u>Panel C (Cyclical Sub-Samples):</u>		
CY1	**5**	**1**
CY2	**1**	**1**

Notes: This table reports significant lags for the own effects $\Phi_{11}(L)$ and $\Phi_{22}(L)$.

Flow as a Dependent Variable

By examining the effect of the first eight lagged values of flow on the flow itself, the results – with respect to the previous mentioned samples- have been reported as Eq.(3.3) in Table (3.1).

As an example, the equation (3.3) for the sub-sample (A) can be written as follows:

$$\Phi_{22}(L) = \phi_{22}^{1}L^{1} + \phi_{22}^{3}L^{3} + \phi_{22}^{4}L^{4} + \phi_{22}^{5}L^{5} + \phi_{22}^{6}L^{6} + \phi_{22}^{7}L^{7} + \phi_{22}^{8}L^{8}.$$

Cross Effects (The Return-Flow Linkage)

Table (3.2) reports the chosen lags for the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. Following equation (3.4), $\Phi_{12}(L)$ for the sub-sample (UD1) might be written as follows (see Table 3.2):

$$\Phi_{12}(L) = \phi_{12}^{10}L^{10}.$$

In addition, $\Phi_{21}(L)$ for the whole sample could be represented with regards to the equation (3.5) as follows (Table 3.2):

$$\Phi_{21}(L) = \phi_{21}^{5}L^{5}.$$

Table 3.2. Mean Equations: AR Lags (Cross Effects)

Samples	$\Phi_{12}(L)$	$\Phi_{21}(L)$
Whole Sample	**4**	**5**
Panel A:		
A	**10**	**1**
B	**3**	**1**
Panel B:		
UD1	**10**	**1**
UD2	**12**	**1**
UD3	**4**	**1**
UD4	**16**	**1**
UD5	**3**	**1**
Panel C:		
CY1	**10**	**1**
CY2	**14**	**1**

Notes: This table reports significant lags for the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$.

As we can see in Table (3.3), there is a bidirectional mixed feedback between stock market return and aggregate mutual fund flow for each of the whole sample, sub-samples (A), (UD1 up to UD4) as well as the sub-samples (CY1 and CY2). In particular, flow affects return negatively whereas the reverse impact is of the opposite sign. This result is consistent with the study obtained by Alexakis et al. (2005).

Moreover, a positive bi-directional causality between return and flow is realized for sub-sample (B) which covers the time intervals of both the 2007 Financial Crisis and the 2009 European Sovereign Debt Crisis, as well as the sub-sample (UD5) during the period of the 2009 European Sovereign Debt Crisis. This positive relation could be related to market sentiments which stimulate mutual fund investments. This finding is in line with the study presented by Fortune (1998) as well as the theoretical underpinning of Warther (1995).

That is, the evidence from the bivariate VAR model suggests that the negative effect from flow to return for the whole sample comes from the same impact in the sub-sample (A). This aforementioned effect in the sub-sample (A) comes from the negative effect of flow on return in the sub-sample (CY1). Moreover, this impact in the sub-samples (CY1) and (CY2) is consistent with the same negative effect in the sub-samples (UD1) and (UD2), as well as in the sub-samples (UD3) and (UD4) respectively.

Table 3.3. The Return-Flow Link (VAR-GARCH)

Samples	Effect of Flow on Return	Effect of Return on Flow
Whole Sample	**Negative**	**Positive**
<u>Panel A:</u>		
A	**Negative**	**Positive**
B	**Positive**	**Positive**
<u>Panel B:</u>		
UD1	**Negative**	**Positive**
UD2	**Negative**	**Positive**
UD3	**Negative**	**Positive**
UD4	**Negative**	**Positive**
UD5	**Positive**	**Positive**
<u>Panel C:</u>		
CY1	**Negative**	**Positive**
CY2	**Negative**	**Positive**

Table (A.3.1.) in Appendix (3) reports the estimated coefficients for the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. The $\Phi_{12}(L)$ and $\Phi_{21}(L)$ columns report results for the return and flow equations respectively.

Variance Equation

With regards to equation (3.7), the analysing dynamic adjustments of the conditional variances for both return and flow can be seen in Table (3.4). Table (3.4) reports estimates of the ARCH and the GARCH parameters.

We can note that the sum of the coefficients of the ARCH parameter (α) and the GARCH parameter (β) for the total sample and all the other sub-samples respectively is less than one. Additionally, all the ARCH and GARCH coefficients are positive and significant for all sub-samples.

Table 3.4. Variance Equations: GARCH Coefficients (VAR-GARCH)

Samples	h_{1t} (Return)	h_{2t} (Flow)
Whole Sample		
α_i	**0.08 (0.01)***	**0.08 (0.03)***
β_i	**0.91 (0.01)***	**0.91 (0.03)***
Panel A:		
Sub-Sample A		
α_i	**0.06 (0.01)***	**0.11 (0.02)***
β_i	**0.93 (0.01)***	**0.88 (0.03)***
Sub-Sample B		
α_i	**0.10 (0.02)***	**0.10 (0.03)***
β_i	**0.88 (0.01)***	**0.89 (0.03)***
Panel B:		
Sub-Sample UD1		
α_i	**0.06 (0.03)**	**0.14 (0.04)***
β_i	**0.92 (0.03)***	**0.62 (0.10)***
Sub-Sample UD2		
α_i	**0.10 (0.03)***	**0.11 (0.03)***
β_i	**0.85 (0.04)***	**0.88 (0.03)***
Sub-Sample UD3		
α_i	**0.03 (0.03)***	**0.17 (0.10)***
β_i	**0.96 (0.03)***	**0.71 (0.18)***
Sub-Sample UD4		
α_i	**0.09 (0.02)***	**0.06 (0.16)***
β_i	**0.90 (0.03)***	**0.91 (0.15)***
Sub-Sample UD5		
α_i	**0.11 (0.02)***	**0.01 (0.01)***
β_i	**0.86 (0.02)***	**0.98 (0.01)***
Panel C:		
Sub-Sample CY1		
α_i	**0.08 (0.02)***	**0.13 (0.03)***
β_i	**0.89 (0.03)***	**0.83 (0.04)***
Sub-Sample CY2		
α_i	**0.06 (0.01)***	**0.16 (0.05)***
β_i	**0.93 (0.01)***	**0.82 (0.05)***

Notes: This table reports parameters' estimates for the ARCH ($\boldsymbol{\alpha_i}$) and GARCH ($\boldsymbol{\beta_i}$) coefficients.

***, ** and * stand for significance at the 1%, 5% and 10% significant levels.

The numbers in parentheses are standard errors.

Case.2. Bivariate VAR-CCC GARCH (1,1) Model

In this section, we will present the findings related to the bivariate VAR-CCC GARCH model in the mean equation (including the cases of own effects and cross effects respectively) as well as the results related to the variance equation (the GARCH coefficients).

Mean Equation

With the bivariate VAR model, we will examine the case of own effects (the effect of the lagged values of return (flow) on the return (flow) in the mean equation). We will also examine the case of cross effects (the effect of the lagged values of flow obtained in the mean equation of return and vice versa).

Own Effects

Table (3.5) reports the chosen lags for the own effects $\Phi_{11}(L)$ and $\Phi_{22}(L)$. The $\Phi_{11}(L)$ and $\Phi_{22}(L)$ columns report results for the return and flow equations respectively.

For example, the lag polynomial $\Phi_{11}(L)$ for the sub-sample (UD1) might be written with regards to the equation (3.2) as follows (see Table 3.5):

$$\Phi_{11}(L) = \phi_{11}^{3} L^{3} + \phi_{11}^{6} L^{6}.$$

As another example, and by applying equation (3.3), the lag polynomial $\Phi_{22}(L)$ for the sub-sample (UD1) might be written as follows (see Table 3.5):

$$\Phi_{22}(L) = \phi_{22}^{1} L^{1} + \phi_{22}^{2} L^{2}.$$

Table 3.5. Mean Equations: AR Lags (Own Effects)

Samples	Eq. (3.2): $\Phi_{11}(L)$	Eq. (3.3): $\Phi_{22}(L)$
Whole Sample	**1**	**2,3**
Panel A:		
A	**5**	**1,3,4,5,6,7,8**
B	**1**	**2,3**
Panel B:		
UD1	**3,6**	**1,2**
UD2	**2**	**3,4,5**
UD3	**1,5**	**2,3,4,5**
UD4	**1**	**3**
UD5	**1**	**2,3**
Panel C:		
CY1	**5**	**1,2**
CY2	**1,2**	**1,2,3**

Notes: This table reports significant lags for the own effects $\Phi_{11}(L)$ and $\Phi_{22}(L)$.

Cross Effects (The Return-Flow Linkage)

Table (3.6) reports the chosen lags for the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. For instance and by following equation (3.4), the lag polynomial $\Phi_{12}(L)$ for the sub-sample (UD4) is as follows:

$$\Phi_{12}(L) = \phi_{12}^{7} L^{7}.$$

In addition, by applying equation (3.5), the lag polynomial for the sub-sample (UD4) could be written as follows (see Table 3.6):

$$\Phi_{21}(L) = \phi_{21}^{1} L^{1}.$$

Table 3.6. Mean Equations: AR Lags (Cross Effects)

Samples	$\Phi_{12}(L)$	$\Phi_{21}(L)$
Whole Sample	**4**	**5**
Panel A:		
A	**10**	**5**
B	**3**	**3**
Panel B:		
UD1	**4**	**1**
UD2	**3**	**1**
UD3	**4**	**1**
UD4	**7**	**1**
UD5	**3**	**1**
Panel C:		
CY1	**6**	**1**
CY2	**2**	**1**

Notes: This table reports significant lags for the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$.

As seen in Table (3.7), there is a bidirectional mixed feedback between aggregate mutual fund flow and stock market return for the whole sample, sub-samples (A), (UD1 up to UD4) as well as the sub-samples (CY1 and CY2). In particular, flow affects return negatively whereas the reverse impact is of the opposite sign.

That is, the evidence from the bivariate VAR model suggests that the causal negative (positive) effect from flow (return) to return (flow) for the whole sample reflects the causal relation between flow and return in the sub-sample (B).

Moreover, a positive bi-directional causality between return and flow is realized for the sub-sample (B) as well as the sub-sample (UD5). This positive relation might be associated with the hypothesis of information revelation by mutual fund flow presented through the theoretical study of Warther (1995). This finding is consistent with the study reported by Boyer and Zheng (2004).

Thus, in comparison between the univariate GARCH model and bivariate CCC GARCH process, we notice that, qualitatively, the results are similar with regards to the return-flow relationship.

Table 3.7. The Return-Flow Link (VAR-CCC GARCH)

Samples	Effect of Flow on Return	Effect of Return on Flow
Whole sample	Negative	Positive
Panel A:		
A	Negative	Positive
B	Positive	Positive
Panel B:		
UD1	Negative	Positive
UD2	Negative	Positive
UD3	Negative	Positive
UD4	Negative	Positive
UD5	Positive	Positive
Panel C:		
CY1	Negative	Positive
CY2	Negative	Positive

Moreover, Table (A.3.2.) in Appendix (3) reports the estimated coefficients for the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. The $\Phi_{12}(L)$ and $\Phi_{21}(L)$ columns report results for the return and flow equations respectively.

Variance Equation

Table (3.8) reports estimates of ARCH, GARCH and CCC parameters. Following equation (3.7), we notice that the sum of the coefficients of the ARCH parameter (α) and the GARCH parameter (β) for the total sample and all the other sub-samples respectively is less than one. Additionally, the ARCH and GARCH coefficients are positive and significant in all cases.

With regards to equation (3.8), the conditional correlations between return and flow in the whole sample and sub-sample (UD5) are (0.01). In the sub-samples (A), (B), (UD1 up to UD4), (CY1) and (CY2), the estimated values of ρ for return-flow (0.02, 0.07, 0.11, 0.08, 0.05, 0.22, 0.12 and 0.09 respectively) are higher than the corresponding value for the whole sample/ sub-sample (UD5) which is (0.01).

Table 3.8. Variance Equations: GARCH and CCC Coefficients (VAR-CCC GARCH)

Samples	h_{1t} (Return)	h_{2t} (Flow)
Whole Sample		
α_i	0.08 (0.01)***	0.08 (0.03)***
β_i	0.91 (0.01)***	0.91 (0.03)***
ρ	0.01 (0.01)***	-
Panel A:		
Sub-Sample A		
α_i	0.06 (0.01)***	0.12 (0.04)***
β_i	0.93 (0.01)***	0.87 (0.03)***
ρ	0.02 (0.02)**	-
Sub-Sample B		
α_i	0.11 (0.02)***	0.10 (0.05)**
β_i	0.88 (0.01)***	0.89 (0.05)***
ρ	0.07 (0.04)*	-
Panel B:		
Sub-Sample UD1		
α_i	0.06 (0.03)**	0.12 (0.04)***
β_i	0.92 (0.03)***	0.69 (0.11)***
ρ	0.11 (0.05)**	-
Sub-Sample UD2		
α_i	0.10 (0.03)***	0.08 (0.02)***
β_i	0.86 (0.04)***	0.90 (0.02)***
ρ	0.08 (0.05)*	-
Sub-Sample UD3		
α_i	0.04 (0.01)***	0.11 (0.40)**
β_i	0.94 (0.01)***	0.73 (0.54)**
ρ	0.05 (0.03)**	-
Sub-Sample UD4		
α_i	0.09 (0.02)***	0.12 (0.04)***
β_i	0.90 (0.02)***	0.87 (0.04)***
ρ	0.22 (0.08)***	-
Sub-Sample UD5		
α_i	0.11 (0.02)***	0.04 (0.02)*
β_i	0.87 (0.02)***	0.95 (0.03)***
ρ	0.01 (0.03)*	-

Panel C:		
Sub-Sample CY1		
α_i	**0.08 (0.02)*** **	**0.13 (0.03)*** **
β_i	**0.89 (0.03)*** **	**0.84 (0.04)*** **
ρ	**0.12 (0.04)***	**-**
Sub-Sample CY2		
α_i	**0.06 (0.01)*** **	**0.14 (0.05)*** **
β_i	**0.93 (0.01)*** **	**0.84 (0.05)*** **
ρ	**0.09 (0.03)***	**-**

Notes: This table reports parameters' estimates for the ARCH ($\boldsymbol{\alpha_i}$), GARCH ($\boldsymbol{\beta_i}$) and ccc ($\boldsymbol{\rho}$) coefficients.

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Case.3. VAR-FIGARCH (1,d,1) Model

In this section, we will present the findings with respect to the bivariate VAR-FIGARCH model in the mean equation (including the cases of own effects and cross effects respectively) as well as the results related to the variance equation.

Mean Equation

Following the bivariate VAR model, we will examine the case of own effects (the impact of the lagged values of return (flow) on the return (flow) as dependent variables). In addition, we will examine the case of cross effects (the impact of the lagged values of flow obtained in the mean equation of return and vice versa).

Own Effects

We take into consideration two fundamental points when choosing our models which are information criteria and significance of the coefficients. Table (3.9) reports the chosen lags for the own effects $\Phi_{11}(L)$ and $\Phi_{22}(L)$. The $\Phi_{11}(L)$ and $\Phi_{22}(L)$ columns report results for the return and flow equations respectively.

Return as a Dependent Variable

By examining the effect of the first five lagged values of return on the return itself, the results –with respect to the obtained various samples- have been reported as Eq. (3.2) in Table (3.9).

As an example, the lag polynomial $\Phi_{11}(L)$ for the whole sample might be written as follows:

$$\Phi_{11}(L) = \phi_{11}^{1}L^{1} + \phi_{11}^{5}L^{5}.$$

Table 3.9. Mean Equations: AR Lags (Own Effects)

Samples	**Eq. (3.2):** $\Phi_{11}(L)$	**Eq. (3.3):** $\Phi_{22}(L)$
Whole Sample	**1,5**	**2,3,4,5**
<u>Panel A:</u>		
A	**5**	**1,3,4,5,6,7,8**
B	**1**	**2,3,4,5**
<u>Panel B:</u>		
UD1	**3**	**1,2**
UD2	**2**	**3**
UD3	**1**	**1,2,3**
UD4	**1**	**1**
UD5	**3**	**2,3**
<u>Panel C:</u>		
CY1	**5**	**1**
CY2	**1**	**1**

Notes: This table reports significant lags for the own effects $\Phi_{11}(L)$ and $\Phi_{22}(L)$.

Flow as a Dependent Variable

By examining the effect of the first eight lagged values of flow on the flow itself, the results –with respect to the previous mentioned samples- have been reported as Eq.(3.3) in table (3.9).

For instance, the lag polynomial $\Phi_{22}(L)$ for the whole sample could be written as follows:

$$\Phi_{22}(L) = \phi_{22}^{2}L^{2} + \phi_{22}^{3}L^{3} + \phi_{22}^{4}L^{4} + \phi_{22}^{5}L^{5}.$$

Cross Effects (The Return-Flow Linkage)

Table (3.10) reports the chosen lags for the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. Following equation (3.4), $\Phi_{12}(L)$ for the sub-sample (CY1) might be written as follows (see Table 3.10):

$$\Phi_{12}(L) = \phi_{12}^{10}L^{10}.$$

Moreover, by applying equation (3.5), $\Phi_{21}(L)$ for the sub-sample (B) could be represented as follows (see Table 3.10):

$$\Phi_{21}(L) = \phi_{21}^{3}L^{3}.$$

Table 3.10. Mean Equations: AR Lags (Cross Effects)

Samples	$\Phi_{12}(L)$	$\Phi_{21}(L)$
Whole Sample	**4**	**1**
Panel A:		
A	**10**	**5**
B	**9**	**3**
Panel B:		
UD1	**10**	**1**
UD2	**12**	**1**
UD3	**4**	**1**
UD4	**1**	**1**
UD5	**3**	**1**
Panel C:		
CY1	**10**	**1**
CY2	**14**	**1**

Notes: This table reports significant lags for the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$.

The likelihood ratios tests as well as the information criteria have chosen the formulation with the bidirectional feedback between stock markets return and mutual funds flow.

As we can see in Table (3.11), there is a bidirectional mixed feedback between stock market return and aggregate mutual fund flow for all the aforesaid sub-samples except the sub-sample (UD5). In particular, return affects flow positively whereas the reverse impact is of the opposite sign.

Moreover, a positive bi-directional causality between flow and return is noticed in the sub-sample (UD5). This finding is in line with the result presented by Mosebach and Najand (1999).

Table 3.11. The Return-Flow Link (VAR-FIGARCH)

Samples	Effect of Flow on Return	Effect of Return on Flow
Whole Sample	Negative	Positive
Panel A:		
A	Negative	Positive
B	Negative	Positive
Panel B:		
UD1	Negative	Positive
UD2	Negative	Positive
UD3	Negative	Positive
UD4	Negative	Positive
UD5	Positive	Positive
Panel C:		
CY1	Negative	Positive
CY2	Negative	Positive

Table (A.3.3.) in Appendix (3) reports the estimated coefficients for the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. The $\Phi_{12}(L)$ and $\Phi_{21}(L)$ columns report results for the return and flow equations respectively.

Variance Equation

Following equation (3.9), the analysing dynamic adjustments of the conditional variances of both return and flow can be seen in Table (3.12). Table (3.12) reports estimates of the ARCH and the GARCH parameters.

We note that the sum of the coefficients of the ARCH parameter (α) and the GARCH parameter (β) for the total sample and all the other sub-samples respectively is less than one. Additionally, all the ARCH and GARCH coefficients are positive and significant in all various sub-samples. In other words, the GARCH coefficients in all cases have satisfied the sufficient and necessary conditions for the non-negativity of the conditional variances (see, for instance, Conrad and Haag, 2006,).

Table 3.12. Variance Equations: GARCH Coefficients (VAR-FIGARCH)

Samples	h_{1t} **(Return)**	h_{2t} **(Flow)**
Whole Sample		
α_i	**0.09 (0.18)*** **	**0.24 (0.06)*** **
β_i	**0.84 (0.09)*** **	**0.74 (0.06)*** **
Panel A:		
Sub-Sample A		
α_i	**0.11 (0.05)** **	**0.27 (0.06)*** **
β_i	**0.87 (0.06)*** **	**0.65 (0.07)*** **
Sub-Sample B		
α_i	**0.05 (0.10)*** **	**0.02 (0.01)** **
β_i	**0.79 (0.05)*** **	**0.97 (0.02)*** **
Panel B:		
Sub-Sample UD1		
α_i	**0.24 (0.08)*** **	**0.16 (0.09)* **
β_i	**0.31 (0.13)** **	**0.62 (0.10)*** **
Sub-Sample UD2		
α_i	**0.36 (0.20)* **	**0.15 (0.05)*** **
β_i	**0.37 (0.32)** **	**0.84 (0.06)*** **
Sub-Sample UD3		
α_i	**0.08 (0.04)** **	**0.16 (0.14)* **
β_i	**0.90 (0.04)*** **	**0.29 (0.24)** **
Sub-Sample UD4		
α_i	**0.11 (0.08)*** **	**0.18 (0.15)** **
β_i	**0.26 (0.07)*** **	**0.34 (0.12)** **
Sub-Sample UD5		
α_i	**0.32 (0.08)*** **	**0.02 (0.10)** **
β_i	**0.67 (0.07)*** **	**0.96 (0.03)*** **

Panel C:		
Sub-Sample CY1		
α_i	**0.06 (0.07)*** **	**0.22 (0.18)*** **
β_i	**0.34 (0.31)***	**0.72 (0.18)****
Sub-Sample CY2		
α_i	**0.15 (0.04)*** **	**0.06 (0.17)****
β_i	**0.84 (0.04)*** **	**0.84 (0.37)****

Notes: This table reports parameters' estimates for the ARCH ($\boldsymbol{\alpha_i}$) and GARCH ($\boldsymbol{\beta_i}$) coefficients.

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

As presented in Table (3.13), the estimated of d_1 and d_2 have governed the long-run dynamics of the conditional heteroscedasticity. The estimation of univariate FIGARCH processes for all return and flow's samples have realized estimated values of d_1 and d_2 that are significantly different from zero or one. In other words, the conditional variances of the two variables have been characterized by the FIGARCH behavior.

In the whole sample and sub-samples (B), (UD1) and (CY1) of return, we can notice that return has generated similar fractional variance parameters: (0.29, 0.29, 0.27 and 0.29). Moreover, the values of this coefficient for sub-samples (A), (UD3) and CY2 are (0.20, 0.10 and 0.18) are markedly lower than the corresponding value for the whole sample (0.29). However, although these estimated values are relatively small, they are significantly different from zero. Nevertheless, for the sub-samples (UD2), (UD4) and (UD5), the fractional differencing parameters (0.41, 0.39 and 0.35) are higher than the corresponding value for the whole sample (0.29).

Moreover, flow has generated similar fractional differencing parameters for the whole sample, sub-samples (A), (UD4), (UD5) and (CY2): (0.41, 0.39, 0.31, 0.33 and 0.36). Nevertheless, the fractional variance parameter for the sub-sample (CY1) is (0.43) which is higher than the corresponding value for the whole sample. Furthermore, the values of the coefficient for sub-samples (B), (UD1 up to UD3) are (0.26, 0.20, 0.19 and 0.15) which are significantly lower than the corresponding values for the whole sample. Even though these estimated values are relatively small, they are remarkably different from zero.

It is noteworthy that in all cases, these estimated valued are robust to the measures of return and flow obtained respectively. In other words, these two univariate FIGARCH processes have generated very similar fractional differencing parameters.

Table 3.13. Variance Equation: The Coefficients of The FIGARCH Model

Samples	d_1 **(Return)**	d_2 **(Flow)**
Whole Sample	**0.29 (0.08)*****	**0.41 (0.08)*****
Panel A:		
A	**0.20 (0.07)*****	**0.39 (0.07)*****
B	**0.29 (0.10)*****	**0.26 (0.03)****
Panel B:		
UD1	**0.27 (0.07)*****	**0.20 (0.02)****
UD2	**0.41 (0.09)*****	**0.19 (0.08)*****
UD3	**0.10 (0.04)****	**0.15 (0.09)*****
UD4	**0.39 (0.05)****	**0.31 (0.07)*****
UD5	**0.35 (0.08)*****	**0.33 (0.08)*****
Panel C:		
CY1	**0.29 (0.06)*****	**0.43 (0.07)****
CY2	**0.18 (0.04)****	**0.36 (0.08)*****

Notes: This table reports parameters' estimates for the GARCH Long Memory in the variance equation for return and flow respectively.

** and * stand for significance at the 5% and 10% significant levels respectively.

Case.4. Bivariate VAR-CCC FIGARCH (1,d,1) Model

In this section, we will present the findings related to the bivariate VAR-CCC FIGARCH model in the mean equation (including the cases of own effects and cross effects respectively) as well as the results related to the variance equation.

Mean Equation

With the bivariate VAR model, we will examine the case of own effects. We will also examine the case of cross effects (the effect of the lagged values of flow obtained in the mean equation of return and vice versa).

Own Effects

Table (3.14) reports the chosen lags for the own effects $\Phi_{11}(L)$ and $\Phi_{22}(L)$. The $\Phi_{11}(L)$ and $\Phi_{22}(L)$ columns report results for the return and flow equations respectively.

Following equation (3.2), the lag polynomial $\Phi_{11}(L)$ for the sub-sample (UD1) might be written as follows (see Table 3.14):

$$\Phi_{11}(L) = \phi_{11}^{3}L^{3} + \phi_{11}^{7}L^{7}.$$

For example and by applying equation (3.3), the lag polynomial $\Phi_{22}(L)$ for the sub-sample (UD1) might be written as follows (see Table 3.14):

$$\Phi_{22}(L) = \phi_{22}^{1}L^{1} + \phi_{22}^{2}L^{2} + \phi_{22}^{7}L^{7}.$$

Table 3.14. Mean Equations: AR Lags (Own Effects)

Samples	Eq. (3.2): $\Phi_{11}(L)$	Eq. (3.3): $\Phi_{22}(L)$
Whole Sample	**1,5**	**2,3,4,5**
<u>Panel A:</u>		
A	**5**	**1,3,4,5,6,7,8**
B	**1**	**2,3,4,5**
<u>Panel B:</u>		
UD1	**3,7**	**1,2,7**
UD2	**2**	**3,4,5**
UD3	**1**	**2,3,4**
UD4	**1**	**3**
UD5	**1**	**2,3**
<u>Panel C:</u>		
CY1	**5**	**1,2**
CY2	**1,2**	**1,2,3**

Notes: This table reports significant lags for the own effects $\Phi_{11}(L)$ and $\Phi_{22}(L)$.

Cross Effects (The Return-Flow Linkage)

Table (3.15) reports the chosen lags for the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. Following equation (3.4), the lag polynomial $\Phi_{12}(L)$ for the sub-sample (UD3) is as follows:

$$\Phi_{12}(L) = \phi_{12}^{4}L^{4}.$$

As an example and by applying equation (3.5), the lag polynomial $\Phi_{21}(L)$ for the sub-sample (B) could be written as follows (see Table 3.15):

$$\Phi_{21}(L) = \phi_{21}^{3} L^{3}.$$

Table 3.15. Mean Equations: AR Lags (Cross Effects)

Samples	$\Phi_{12}(L)$	$\Phi_{21}(L)$
Whole Sample	**4**	**1**
Panel A:		
A	**10**	**5**
B	**9**	**3**
Panel B:		
UD1	**4**	**1**
UD2	**3**	**1**
UD3	**4**	**1**
UD4	**7**	**1**
UD5	**3**	**1**
Panel C:		
CY1	**6**	**1**
CY2	**2**	**1**

Notes: This table reports significant lags for the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$.

Following the results presented in Table (3.16), there is a bidirectional mixed feedback between aggregate mutual fund flow and stock market return for all the aforementioned sub-samples but one exception which is the sub-sample (UD5). In particular, return affects flow positively whereas the reverse impact is of the opposite sign. This finding is consistent with the study obtained by Aydogan et al. (2014).

Moreover, a positive bi-directional causality between flow and return is only realized in the sub-sample (UD5). This result is in line with the finding reported by Cha and Kim (2010).

Thus, in comparison between the univariate FIGARCH model and bivariate CCC FIGARCH process, we notice that the same results are captured regarding the return-flow relationship.

Table 3.16. The Return-Flow Link (VAR-CCC FIGARCH)

Samples	Effect of Flow on Return	Effect of Return on Flow
Whole Sample	**Negative**	**Positive**
Panel A:		
A	**Negative**	**Positive**
B	**Negative**	**Positive**
Panel B:		
UD1	**Negative**	**Positive**
UD2	**Negative**	**Positive**
UD3	**Negative**	**Positive**
UD4	**Negative**	**Positive**
UD5	**Positive**	**Positive**
Panel C:		
CY1	**Negative**	**Positive**
CY2	**Negative**	**Positive**

Table (A.3.4.) in Appendix (3) reports the estimated coefficients for the cross effects $\Phi_{12}(L)$ and $\Phi_{21}(L)$. The $\Phi_{12}(L)$ and $\Phi_{21}(L)$ columns report results for the return and flow equations respectively.

Variance Equation

Table (3.17) reports estimates of the ARCH and the GARCH parameters. Following equation (3.9), the analysing dynamic adjustments of the conditional variances of both return and flow can be seen in Table (3.17).

We note that the sum of the coefficients of the ARCH parameter (α) and the GARCH parameter (β) for the total sample and all the other sub-samples respectively is less than one. Additionally, the ARCH and GARCH coefficients are positive and significant in all cases. In other words, the GARCH coefficients in all cases have satisfied the sufficient and necessary conditions for the non-negativity of the conditional variances (see, for instance, Conrad and Haag, 2006,).

Table 3.17. Variance Equations: GARCH Coefficients (VAR-CCC FIGARCH)

Samples	h_{1t} (Return)	h_{2t} (Flow)
Whole Sample		
α_i	0.29 (0.35)**	0.02 (0.01)*
β_i	0.70 (0.35)**	0.79 (0.01)***
Panel A:		
Sub-Sample A		
α_i	0.05 (0.01)***	0.12 (0.24)*
β_i	0.94 (0.07)***	0.87 (0.23)***
Sub-Sample B		
α_i	0.36 (0.06)***	0.02 (0.01)**
β_i	0.63 (0.06)***	0.97 (0.01)***
Panel B:		
Sub-Sample UD1		
α_i	0.23 (0.08)***	0.14 (0.09)***
β_i	0.31 (0.14)**	0.68 (0.10)***
Sub-Sample UD2		
α_i	0.24 (0.19)***	0.13 (0.04)***
β_i	0.73 (0.17)***	0.85 (0.04)***
Sub-Sample UD3		
α_i	0.26 (0.08)***	0.18 (0.14)*
β_i	0.55 (0.12)***	0.32 (0.25)*
Sub-Sample UD4		
α_i	0.27 (0.05)***	0.06 (0.03)**
β_i	0.70 (0.05)***	0.93 (0.02)***
Sub-Sample UD5		
α_i	0.08 (0.02)***	0.28 (0.26)***
β_i	0.90 (0.02)***	0.56 (0.09)***
Panel C:		
Sub-Sample CY1		
α_i	0.07 (0.03)***	0.05 (0.04)**
β_i	0.68 (0.33)**	0.71 (0.17)***
Sub-Sample CY2		
α_i	0.16 (0.04)***	0.10 (0.11)*
β_i	0.83 (0.04)***	0.78 (0.10)***

Notes: This table reports parameters' estimates for the ARCH ($\boldsymbol{\alpha_i}$) and GARCH ($\boldsymbol{\beta_i}$) coefficients.

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

As presented in Table (3.18), the estimated of d_i's $i = 1,2$ have governed the long-run dynamics of the conditional heteroscedasticity. The estimation of bivariate FIGARCH processes for all return and flow's samples have realized estimated values of d_1 and d_2 which are significantly different from zero or one. In other words, the conditional variances of the two variables have been characterized by the GARCH behavior.

In the whole sample, sub-samples (UD1), (UD5) and (CY1) of return, we notice that return has generated similar fractional variance parameters: (0.31, 0.28, 0.34 and 0.30). Moreover, the values of this coefficient for sub-sample (A), (UD3) and (CY2) are given by (0.22, 0.11 and 0.22) which are markedly lower than the corresponding value for the whole sample (0.31). However, although these estimated values are relatively small, they are significantly different from zero. Nevertheless, for the sub-samples (B), (UD2) and (UD4), the fractional differencing parameters (0.39, 0.37 and 0.40) are higher than the corresponding value for the whole sample (0.31).

In addition, flow has generated similar fractional differencing parameters for the whole sample, sub-samples (A), (CY1) and (CY2) which are (0.39, 0.36, 0.41and 0.35). Furthermore, the values of the coefficient for sub-samples (B), (UD1 up to UD5) that are given by (0.22, 0.19, 0.17, 0.13, 0.31 and 0.32) are significantly lower than the corresponding value for the whole sample. Even though these estimated values are relatively small, they are remarkably different from zero.

It is noteworthy that in all cases, these estimated valued are robust to the measures of return and flow obtained respectively. In other words, these two bivariate CCC GARCH and bivariate CCC FIGARCH processes have generated very similar fractional differencing parameters.

With regards to the equation (3.8), the conditional correlation between return and flow for the whole sample and sub-sample (CY1) is (0.21). In the sub-samples (A) and (B), the estimated values of $\boldsymbol{\rho}$ for return-flow (0.30 and 0.25) are higher than the corresponding value for the whole sample (0.21). On the contrary, the conditional correlation's estimated values for the sub-samples (UD1 up to UD5) and (CY2) are given as (0.12, 0.08, 0.05, 0.16, 0.11 and 0.08) which are lower than the corresponding value for the whole sample (0.21).

Table 3.18. Variance Equation: The Coefficients of The FIGARCH Model: Fractional and CCC Parameters

Samples	Return	Flow
Whole Sample		
d_i	**0.31 (0.07)*** **	**0.39 (0.05)** **
$\boldsymbol{\rho}$	**0.21 (0.02)*** **	-
<u>Panel A:</u>		
Sub-Sample A		
d_i	**0.22 (0.02)** **	**0.36 (0.03)** **
$\boldsymbol{\rho}$	**0.30 (0.02)*** **	-
Sub-Sample B		
d_i	**0.39 (0.06)*** **	**0.22 (0.08)*** **
$\boldsymbol{\rho}$	**0.25 (0.04)*** **	-
<u>Panel B:</u>		
Sub-Sample UD1		
d_i	**0.28 (0.07)*** **	**0.19 (0.08)*** **
$\boldsymbol{\rho}$	**0.12 (0.05)** **	-
Sub-Sample UD2		
d_i	**0.37 (0.02)** **	**0.17 (0.06)*** **
$\boldsymbol{\rho}$	**0.08 (0.05)** **	-
Sub-Sample UD3		
d_i	**0.11 (0.08)*** **	**0.13 (0.02)** **
$\boldsymbol{\rho}$	**0.05 (0.03)** **	-
Sub-Sample UD4		
d_i	**0.40 (0.05)*** **	**0.31 (0.09)*** **
$\boldsymbol{\rho}$	**0.16 (0.06)** **	-
Sub-Sample UD5		
d_i	**0.34 (0.04)** **	**0.32 (0.02)** **
$\boldsymbol{\rho}$	**0.11 (0.04)*** **	-

Panel C:		
Sub-Sample CY1		
d_i	**0.30 (0.07)***	**0.41 (0.04)**
$\boldsymbol{\rho}$	**0.21 (0.04)***	**-**
Sub-Sample CY2		
d_i	**0.22 (0.04)**	**0.35 (0.03)**
$\boldsymbol{\rho}$	**0.08 (0.03)***	**-**

Notes: This table reports parameters' estimates for the long-memory (d_i), $i = 1,2$ and ccc ($\boldsymbol{\rho}$) coefficients.

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

3.7. Conclusion

In this paper, we have examined the dynamic interactions between stock market return and aggregate mutual fund flow. The variables under consideration were inextricably linked. We have highlighted various key behavioural features which were presented across the different univariate and bivariate specifications. We have mentioned two principal hypotheses in detecting the effect of mutual fund flow on security return which are the information revelation hypothesis and the price pressures hypothesis. In addition, the feedback-trader hypothesis has been presented in terms of mutual fund flow affected by security return.

We have taken into account the 2000 Dot-Com Bubble, the 2007 Financial Crisis as well as the 2009 European Sovereign Debt Crisis and discussed how these changes have affected the relationships among these two variables. Furthermore, we have employed the bivariate VAR model with various specifications of univariate and bivariate GARCH processes in order to capture all the changeable results.

Our contribution in this study has been considered as follows: obtaining a long span of daily data (from February 3rd 1998 to March 20th 2012), dividing the whole data set into three different cases with nine sub-samples and employing the bivariate VAR model with four different GARCH processes (univariate GARCH, bivariate CCC GARCH, univariate FIGARCH and bivariate CCC FIGARCH models).

We have reported a bidirectional mixed feedback between stock prices return and aggregate mutual fund flow for the majority of the samples obtained. In particular, the lagged values of flow have negatively affected return whereas the reverse impact is of the opposite sign. Nevertheless, we have noticed two exceptional issues with respect to the return-flow linkage. Firstly, by employing both the univariate and bivariate CCC GARCH processes, a positive bi-directional causality between return and flow has been realized for the sub-samples (B) and (UD5) during the 2007 Financial Crisis and the 2009 European Sovereign Debt Crisis.

Secondly, we have observed a positive bi-directional causality between return and flow in the sub-sample (UD5) through employing both the univariate and bivariate CCC FIGARCH models. Last but not least, most of the bidirectional effects have been found to be quite robust to the dynamics of the different GARCH models employed in this paper.

Appendix 3

Table A.3.1. The Coefficients of The Return-Flow Link (Case 1)

Samples	$\Phi_{12}(L)$	$\Phi_{21}(L)$
Whole Sample	**-0.49 (0.20)****	**0.00 (0.00)***
Panel A:		
A	**-0.64 (0.26)****	**0.03 (0.00)***
B	**1.02 (0.45)****	**0.02 (0.00)***
Panel B:		
UD1	**-0.99 (0.36)***	**0.08 (0.00)***
UD2	**-0.21 (0.52)*****	**0.04 (0.00)***
UD3	**-0.66 (0.32)****	**0.01 (0.00)***
UD4	**-1.32 (0.56)****	**0.01 (0.00)*****
UD5	**1.29 (0.58)****	**0.02 (0.00)***
Panel C:		
CY1	**-0.71 (0.30)****	**0.06 (0.00)***
CY2	**-0.51 (0.36)*****	**0.01 (0.00)***

Notes: This table reports parameters' estimates for the Φ_{12} and Φ_{21} respectively.

***, **, and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Table A.3.2. The Coefficients of The Return-Flow Link (Case 2)

Samples	$\Phi_{12}(L)$	$\Phi_{21}(L)$
Whole Sample	**-0.49 (0.21)****	**0.00 (0.00)***
Panel A:		
A	**-0.65 (0.26)****	**0.00 (0.00)*****
B	**0.10 (0.04)****	**0.04 (0.02)*****
Panel B:		
UD1	**-0.59 (0.35)*****	**0.08 (0.00)***
UD2	**-0.84 (0.56)****	**0.04 (0.00)***
UD3	**-0.60 (0.32)*****	**0.01 (0.00)***
UD4	**-1.26 (0.56)****	**0.01 (0.00)*****
UD5	**1.24 (0.47)****	**0.02 (0.00)***
Panel C:		
CY1	**-0.13 (0.28)*****	**0.06 (0.00)***
CY2	**-0.12 (0.31)***	**0.01 (0.00)***

Notes: This table reports parameters' estimates for the Φ_{12} and Φ_{21} respectively.

***, **, and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Table A.3.3. The Coefficients of The Return-Flow Link (Case 3)

Samples	$\Phi_{12}(L)$	$\Phi_{21}(L)$
Whole Sample	**-0.43 (0.22)*** **	**0.02 (0.00)***
<u>Panel A:</u>		
A	**-0.60 (0.26)** **	**0.00 (0.00)** **
B	**-0.97 (0.46)** **	**0.00 (0.00)** **
<u>Panel B:</u>		
UD1	**-1.05 (0.35)***	**0.08 (0.00)***
UD2	**-0.11 (0.56)** **	**0.04 (0.00)***
UD3	**-0.64 (0.32)** **	**0.01 (0.00)***
UD4	**-0.96 (0.53)*** **	**0.02 (0.00)***
UD5	**0.94 (0.52)*** **	**0.01 (0.00)***
<u>Panel C:</u>		
CY1	**-0.78 (0.30)***	**0.06 (0.00)***
CY2	**-0.49 (0.34)** **	**0.02 (0.00)***

Notes: This table reports parameters' estimates for the Φ_{12} and Φ_{21} respectively.

***, **, and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Table A.3.4. The Coefficients of The Return-Flow Link (Case 4)

Samples	$\Phi_{12}(L)$	$\Phi_{21}(L)$
Whole Sample	**-0.44 (0.22)** **	**0.02 (0.00)***
<u>Panel A:</u>		
A	**-0.65 (0.27)** **	**0.00 (0.00)*** **
B	**-0.09 (0.05)** **	**0.04 (0.02)*** **
<u>Panel B:</u>		
UD1	**-0.56 (0.32)*** **	**0.08 (0.00)***
UD2	**-0.87 (0.57)** **	**0.04 (0.00)***
UD3	**-0.66 (0.34)*** **	**0.01 (0.00)***
UD4	**-0.79 (0.54)** **	**0.02 (0.00)***
UD5	**1.44 (0.60)** **	**0.02 (0.00)***
<u>Panel C:</u>		
CY1	**-0.04 (0.29)*** **	**0.06 (0.00)***
CY2	**-0.14 (0.29)** **	**0.02 (0.00)***

Notes: This table reports parameters' estimates for the Φ_{12} and Φ_{21} respectively.

***, **, and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Chapter Four

On the Linkage Between Market Return Volatility and U.S. Aggregate Mutual Fund Flow: Evidence From GARCH Approach

4.1. Introduction

The issue of U.S. aggregate mutual fund flow has been assessed by various studies over the past decades. Three principal papers have examined this case. Firstly, the study of Warther (1995) has observed a positive correlation between equity funds flow and concurrent market returns, but a negative relation between market returns and subsequent fund flow on a monthly basis. Secondly, Edelen and Warner (2001) have showed that there is no association between daily returns and lagged flow, whereas a positive association between daily flow and each of concurrent and previous day's market returns has been detected. Thirdly, Froot et al. (2001) have stated that the documented interaction between U.S. equity funds flow and market returns is uncovered in other emerging markets and developed markets. All these three findings have suggested that market returns are affected by both flow and flow-induced trade, and this impact is not idiosyncratic. In addition, they have not provided evidence of feedback trading by mutual funds' investors on a daily basis, but only a positive feedback in monthly data. As a result, they have exclusively concentrated on the interaction between market returns and funds flow.

The study of Cao et al. (2008) is considered as the first paper which has presented direct evidence on the relation between market volatility and fund flow. They have employed a dataset for a sample of 859 daily mutual funds covering the period between 1998 and 2003 as well as VAR specifications in order to examine the dynamic interaction between aggregate mutual fund flow and market return volatility. One of their primary findings is that concurrent and lagged flow has a negative impact on daily market volatility. They have also provided evidence of a negative contemporaneous interaction between innovations in market volatility and fund flow. Another finding has suggested that market volatility is negatively affected by the shock in fund flow. That is, an outflow shock induces higher market volatility whereas an inflow induces lower volatility. They have also observed that the first lag of market volatility has a negative impact on daily fund flow and provided evidence that mutual fund investors might time market volatility at the aggregate fund level. Furthermore, their findings have suggested –from morning to afternoon- a strong relationship between outflow and intraday volatility, whilst the interaction between inflow and intraday volatility becomes weaker.

Since this chapter is focusing on examining the relationship between aggregate mutual fund flow and market return volatility, it includes two principal objectives. Firstly, we analyse the volatility-flow interaction in the U.S. stock market (S&P 500 index). We estimate the main parameters in the two aforementioned variables by applying bivariate VAR model (examining the impact of the lagged values of one variable obtained in the mean equation of the other variable) with the univariate GARCH and bivariate CCC GARCH processes. Secondly, we employ the univariate ARFIMA-FIGARCH as well as bivariate CCC ARFIMA-FIGARCH models in order to assess this linkage between volatility and flow.

Our contribution in this chapter could be classified as follows. We employ two fundamental measurements with respect to market return volatility which are Rogers-Satchell (RS) volatility as well as Garman-Klass-Yang-Zhang (GKYZ) volatility. We obtain our required sample of aggregate mutual fund flow which includes only the U.S. domestic mutual funds flow covering the period spanning from February 3rd 1998 to March 20th 2012. Moreover, we impose specific selection criteria depending on Morningstar Category Classifications. This selected process has rendered a final sample of 3,538 daily observations.

In addition, we divide the whole sample into three different cases for the purpose of statistical analyses. We take into account three fundamental indicators through splitting the whole data set, which are the 2000 Dot-Com Bubble, the 2007 Financial Crisis and the 2009 European Sovereign Debt Crisis. The first case consists of two sub-samples (A and B), whereas the second section includes five ups-and-downs (UDs) sub-samples. Two cyclical (CYs) sub-samples are obtained in the third case. The sub-sample (A) covers the period 3rd February 1998 to 25th July 2007, whereas the data set spanning from 26th July 2007 to 20th March 2012 is obtained in the sub-sample (B). In addition, the sub-sample (UD1) involves the period 3rd February 1998 to 1st September 2000. The sub-sample (UD2) spans from 5th September 2000 to 7th October 2002. The period from 8th October 2002 until 19th July 2007 is included in the sub-sample (UD3). Whereas the sub-sample (UD4) covers the period 20th July 2007 to 9th March 2009, the sub-sample (UD5) spans from 10th March 2009 until 20th March 2012. Moreover, the sub-samples CYs cover the periods 3rd February 1998 until 7th October 2002, and 8th October 2002 to 9th March 2009 respectively.

By employing the case of RS volatility with respect to the various models of GARCH and ARFIMA-FIGARCH, we detect a bidirectional mixed feedback between market return volatility and aggregate mutual fund flow in the whole sample, sub-samples (UD1) and (CY1). In particular, flow affects volatility positively whereas the reverse effect is of the opposite sign. Moreover, we observe a negative bi-directional causality between volatility and flow in the sub-samples (B), (UD2), (UD4) and (UD5). However, a positive bi-directional causality is noticed in the sub-samples (A), (UD3) and (CY2). That is, the evidence for the total sample suggests that the positive (negative) effect from flow (volatility) to volatility (flow) reflects the causal relation between volatility and flow in the sub-samples (UD1) and (CY1). In brief, the positive impact of flow on volatility in the whole sample comes from the same impact in the sub-samples (A), (UD1) and (UD3). This aforesaid positive effect in the sub-sample (A) comes from sub-samples (UD1) and (UD3). Finally, this effect in the sub-samples (CY1) and (CY2) comes from sub-samples (UD1) and (UD3) respectively.

On the other hand, the following findings are observed by employing the various cases of GARCH and ARFIMA-FIGARCH processes in order to assess the volatility-flow interaction in the case of GKYZ volatility. We detect a negative bi-directional causality between stock market volatility and aggregate mutual fund flow in the whole sample, sub-samples (B), (UD2), (UD4), (UD5) and (CY1). Additionally, there is a bidirectional mixed feedback between volatility and flow in the sub-sample (UD1). In particular, volatility affects flow negatively whereas the reverse impact is of the opposite sign. A positive bi-directional causality is noticed in the sub-samples (A), (UD3) and (CY2). That is, the evidence for the whole sample suggests that the negative effect from volatility (flow) to flow (volatility) reflects the negative interaction between volatility and flow in the sub-samples (B) and (CY1). Summarizing, the negative effect of flow on volatility in the whole sample comes from the same effect in the sub-samples (B), (CY1). This aforesaid negative impact in the sub-sample (B) is consistent with the same impact in the sub-samples (UD4) and (UD5). Finally, this effect in the sub-sample (CY1) comes from sub-sample (UD2).

Finally, we detect one exceptional issue in comparison between the results observed by employing RS volatility and GKYZ volatility. Whereas we observe a bidirectional mixed feedback between volatility and flow in the whole sample and sub-sample (CY1), this relation turned to be negative in the case of GKYZ volatility with respect to the aforesaid samples.

This remainder of this chapter is organized as follows. The second section reviews the existing literature. The third section introduces the data and describes the method of constructing market returns volatility. The fourth section presents the various sub-samples. The fifth section explains the econometric models. Empirical findings are reported and discussed in the sixth section. The seventh section concludes the chapter.

4.2. Literature Review

With the global rapid growth of mutual funds markets, many academicians have been interested in examining the linkages between stock markets return and mutual funds flow. As a result, there is a large literature regarding the issue of mutual funds flow. Numerous studies have evaluated these linkages through using both daily and monthly data at the individual fund level as well as the aggregate market level. Whereas individual fund level studies tend to focus on the relations among micro-level characteristics such as investors' timing ability, redemption policy and indirect costs, the aggregate market-level studies analyse these linkages with a focus on various macro-level variables.

Earlier studies have assessed the micro-level relations between the flow of money into/out of mutual funds and the individual fund' performance. For instance, Ippolito (1992) has stated that mutual funds' investors primarily move cash into funds which have had the best performance in the previous year. Sirri and Tufano (1998) have documented a striking flow-performance linkage through analysing annual funds flow. Edelen (1999) is the first who has examined the linkage between mutual funds flow and their performance at the individual fund level. He has focused on a significant and statistically indirect cost in the form of a negative relationship between investor flow and abnormal return of an individual fund.

Coval and Stafford (2007) have examined the institutional price pressure and asset fire sales through focusing on the stock transactions of mutual funds. They have provided an evidence that transaction prices which normally occur below fundamental value could be leaded by widespread selling of financially distressed mutual funds. Friesen and Sapp (2007) have employed cash flow data for the purpose of analysing the timing ability of mutual funds' investors. In addition, Greene et al. (2007) have stated that the most effective tool in controlling the fund flow's volatility is the redemption fee. By using VAR approach, Rakowski and Wang (2009) have assessed the linkage between market return and short-term mutual funds flow, they found that future fund' returns are positively affected by past fund's flows.

On the other hand, Warther (1995) and Edelen and Warner (2001) have extended previous studies through examining the linkage between market returns and mutual funds flow at the macro-level. Since investors take money out of an individual fund and then invest in another individual fund, this is resulted in that cash into one fund is considered as an expense of another fund. As Warther (1995) has stated, there is a fundamental difference between analyses of the macro-level and the micro-level. Whilst only the aggregate flow into/out of all mutual funds is relevant as flows amongst mutual funds are offsetting at the macro-level, the analysis of the micro-level, by contrast, might help explaining how mutual funds compete against each other for the purpose of expanding their respective market share.

Warther (1995) is the first who has investigated the contemporaneous relationship between market returns and mutual funds flow through employing monthly data at the aggregate market level. He has showed that mutual fund flows are negatively affected by stock market returns, but a positive contemporaneous relationship between unexpected fund flows and stock market returns. It is noteworthy to mention that these findings are in line with the belief which implies that new fund' investors are not necessarily smarter than existing fund' investors.

Moreover, Fortune (1998) has provided strong evidence that there is an impact from realized security returns to subsequent security purchases. This result has obtained through employing the VAR approach. Cha and Lee (2001) have found that the stock market's performance has an influence on equity fund flows. Edelen and Warner (2001) have analysed the equity fund flow-stock market's performance linkage in the context of daily frequency data, they have concluded that returns are affected by institutional trading and fund flows are reflected by this daily contemporaneous relationship.

All these aforementioned studies have assessed the U.S. mutual fund markets. However, Alexakis et al. (2005) have investigated the interaction between stock market returns and fund flows in Greece. Ferreira et al. (2012) have examined the convexity of the performance-flow linkage amongst 28 countries. They have not considered the concurrent interactions between performance and fund flow and they have employed financial and economic factors in an international context for the purpose of explaining their findings. Whereas Alexakis et al. (2013) have studied the causality between stock index prices and fund flows in Japan.

The study of Busse (1999) is the first to employ daily data in the context of mutual funds. With the EGARCH approach, he has provided strong evidence of a negative interaction between conditional market volatility and fund systematic risk levels. He has shed light on a fundamental question of whether or not mutual funds might time market volatility. Using a daily dataset for a sample of 230 domestic equity funds spanning the period between 1985 and 1995, he has showed that mutual funds could time market volatility at an individual fund level. He has also stated that funds could change market exposure when volatility changes. In particular, funds might increase market exposure when market volatility is low.

Since the existing literature has focused on the relation between stock returns and fund flows, the study of Cao et al. (2008) is considered as the first paper which has presented direct evidence on the relation between market volatility and fund flow. They have employed a dataset for a sample of 859 daily mutual funds covering the period between 1998 and 2003 as well as VAR specifications in order to examine the dynamic interaction between aggregate mutual fund flow and market return volatility. One of their primary findings is that concurrent and lagged flow has a negative impact on daily market volatility. They have also provided evidence of a negative contemporaneous interaction between innovations in market volatility and fund flow. Another finding has suggested that market volatility is negatively affected by the shock in fund flow. That is, an outflow shock induces higher market volatility whereas an inflow induces lower volatility. They have also found that the first lag of market volatility has a negative impact on daily fund flow and provided evidence that mutual fund investors might time market volatility at the aggregate fund level. Further, their findings have suggested –from morning to afternoon- a strong relationship between outflow and intraday volatility, whilst the interaction between inflow and intraday volatility becomes weaker.

Cao et al. (2008) have presented two fundamental channels for the purpose of explaining the stock market volatility-fund flow relationship. First, at the individual fund level, they have observed lumpy cash infusion (withdraw) into (out of) funds over short periods of time. The past performance is considered as the main incentive for such exogenous cash flow. With regards to negative feedback strategies followed by specified fund managers, these managers drive security prices –through their trades- towards their fundamental values. On the contrary, other fund managers might follow positive feedback strategies, they rely on past performance for the purpose of predicting future returns and push security prices away from their fundamental values through buying securities in up markets and selling in down markets. The extent to which flow-induced trades might depend on past return is important since negative (positive) feedback strategies decrease (increase) short-term volatility. As a result, managers' aggregate actions might be offsetting as they pursue diverse investment strategies.

Second, as noise traders and investor sentiment are essential factors in the overall market movement, noise traders might cause vast swings away from fundamentals. This result has been concluded by Lee et al. (1991). Because of mutual funds' investors are normally the least informed investors, it is rational to employ mutual fund flow as a proxy for such uninformed investor sentiment. Flow into/out of mutual funds might be related to market-wide returns and volatility with respect to the investor sentiment' importance as well as aggregate fund flow is considered as an appropriate proxy of this sentiment.

Lee et al. (2015) have investigated the dynamic relationships among aggregate equity fund flow, market volatility and market return in an international context. By employing monthly data covering the period between January 2000 and June 2011 and structural VAR approach, they have given evidence that the interactions among these three variables are most apparent in the U.S. amongst the sample's countries. In addition, they have showed that the contemporaneous impacts are most relevant to the interactions among these three variables. The aggregate equity fund flows are differently affected by both return and volatility shocks amongst the sample' countries. They have also demonstrated that Asian investors are less concerned with market return and market volatility than Western investors when buying and redeeming equity funds. In the Asian countries, the dynamic interactions between these three variables are less significant than in the Western countries. Finally, they have demonstrated the importance of contemporaneous impacts in the relationships amongst these three aforementioned variables as the hypothesis tests have revealed that the overall impacts presented in this study are largely attributable to the contemporaneous impacts.

4.3. Data

We have obtained daily data on S&P 500 stock index from Thomson Reuters Database in order to calculate both Rogers-Satchell volatility and Garman-Klass-Yang-Zhang volatility. The aim of employing two different measures of volatility is to capture the modifications in the volatility-flow linkage through applying various types of volatility.

Firstly, we have calculated the daily Rogers-Satchell (RS) volatility as follows:

$$Volatility_{RS} = LN\left(\frac{h_i}{c_i}\right) LN\left(\frac{h_i}{o_i}\right) + LN\left(\frac{l_i}{c_i}\right) LN\left(\frac{l_i}{o_i}\right),$$

Where h_i and l_i represent the high stock price and low stock price at time i respectively. In addition, c_i and o_i represent the close stock price and open close price at time i respectively.

Secondly, the daily Garman-Klass-Yang-Zhang (GKYZ) volatility has been calculated as follows:

$$Volatility_{GKYZ} = \left(LN(\frac{o_i}{c_{i-1}})\right)^2 + \frac{1}{2}\left(LN(\frac{h_i}{l_i})\right)^2 - (2LN(2) - 1)\left(LN(\frac{c_i}{o_i})\right)^2,$$

Where o_i and c_i represent the open stock price and close stock price at time i respectively. In addition, h_i and l_i represent the high stock price and low stock price at time i respectively and c_{i-1} represent the close stock price at time $i - 1$.

Finally, the aggregate mutual funds flow' data has been presented in details in the preceding chapter (chapter three).

4.4. Sub-Samples

The whole data set that is examined in this chapter is covering the period spanning from 3rd February 1998 to 20th March 2012 (see figure 4.1. for RS volatility and figure 4.2. for GKYZ volatility). This whole sample comprises of 3,538 daily observations.

We have run our statistical analysis on this whole sample through dividing it into three different ways. Two sub-samples (A and B) are obtained in the first case. The sub-sample (A) includes 2,369 observations and covers the period 3rd February 1998 to 25th July 2007, whereas the data set spanning from 26th July 2007 to 20th March 2012 is obtained in the sub-sample (B) with 1,169 daily observations.

In addition, the second case includes five ups-and-downs (UDs) sub-samples. The sub-sample (UD1) involves the period 3rd February 1998 to 1st September 2000. The second sub-sample (UD2) spans from 5th September 2000 to 7th October 2002. The period from 8th October 2002 until 19th July 2007 is included in the sub-sample (UD3). Whereas the fourth sub-sample (UD4) covers the period 20th July 2007 to 9th March 2009, the sub-sample (UD5) spans from 10th March 2009 until 20th March 2012.

Moreover, the third case consists of two cyclical (CYs) sub-samples. The sub-samples CYs cover the periods 3rd February 1998 until 7th October 2002 and 8th October 2002 to 9th March 2009 respectively. In other words, the sub-sample (CY1) involves the sub-samples (UD1) and (UD2), whereas the sub-samples (UD3) and (UD4) comprise the sub-sample (CY2).

We take into consideration three primary indicators when splitting the whole data set, which are the 2000 Dot-Com Bubble, the 2007 Financial Crisis as well as the 2009 European Sovereign Debt Crisis. The essential purpose of obtaining these nine sub-samples is examining the positive/negative return-flow interaction amongst several periods of time and realizing the possible changes that could happen to this linkage.

Figure.4.1. Market Return Volatility (RS)

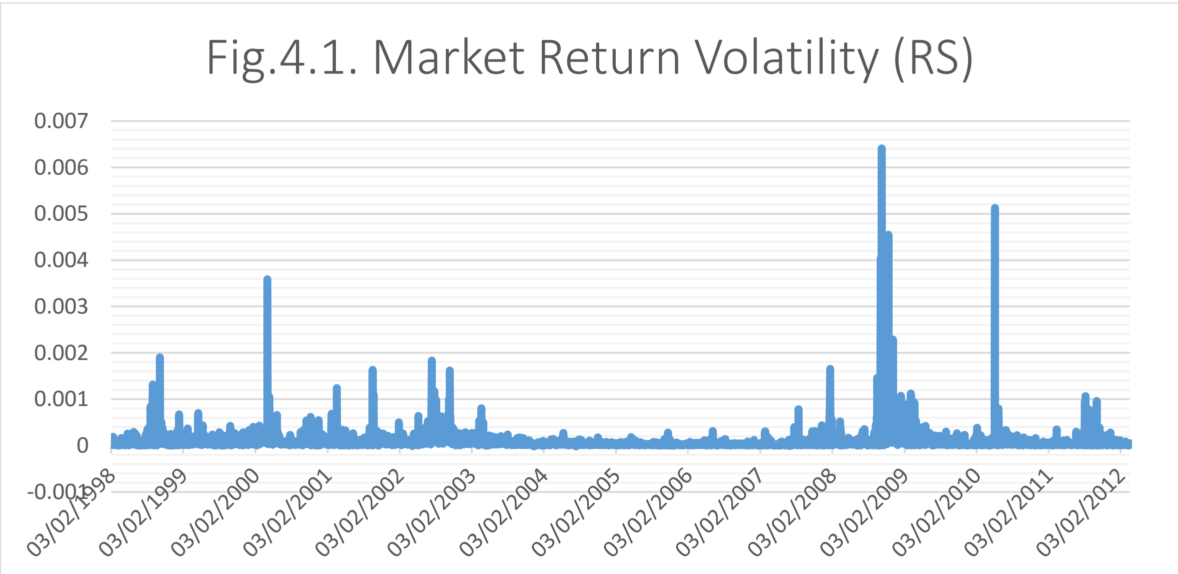
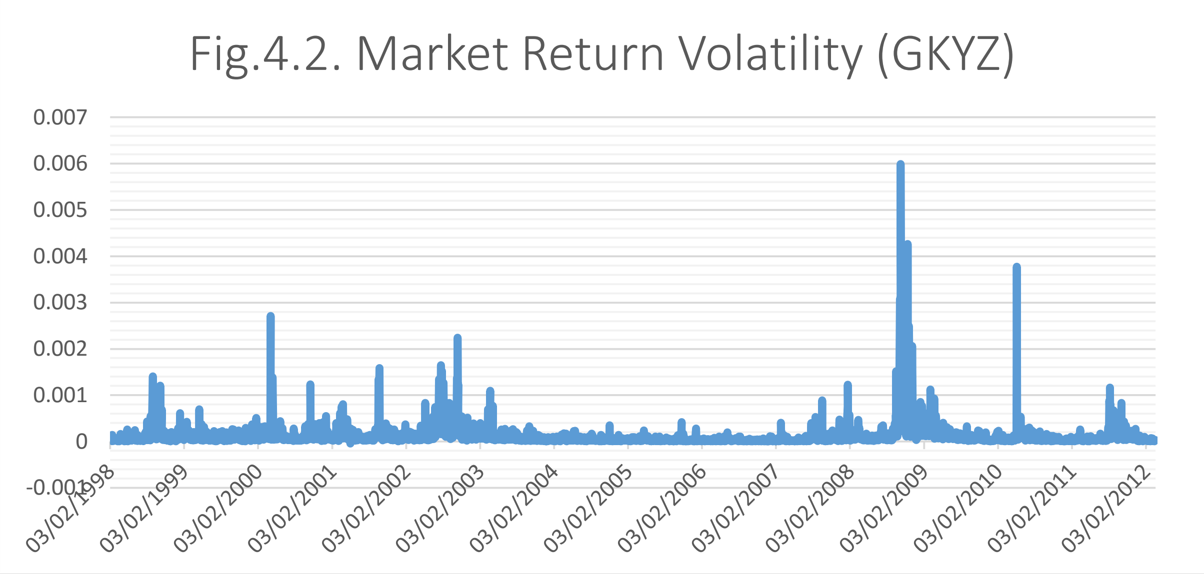


Figure.4.2. Market Return Volatility (GKYZ)



4.5. The Econometric Models

The estimates of the various formulations are obtained by quasi maximum likelihood estimation (QMLE) as it has been implemented by James Davidson (2007) in Time Series Modelling (TSM). We employ a range of starting values to be ensured that the estimation procedures have converged to a global maximum for the purpose of checking for the robustness of our estimates. In addition, the minimum value of the information criteria is considered when choosing the best fitting specification.

The VAR-GARCH (1,1) Models

Mean Equation

In order to capture the potential interactions between volatility and flow, stock market volatility (y_{vt}) and mutual funds flow (y_{ft}) follow a bivariate VAR model as follows:

$$\boldsymbol{\Phi}(L)\boldsymbol{y_t} = \boldsymbol{\mu} + \boldsymbol{\varepsilon_t}, \qquad 4.1$$

With $\boldsymbol{y_t} = \begin{bmatrix} y_{vt} \\ y_{ft} \end{bmatrix}$, $\boldsymbol{\mu} = \begin{bmatrix} \mu_{vt} \\ \mu_{ft} \end{bmatrix}$, $\boldsymbol{\varepsilon_t} = \begin{bmatrix} \varepsilon_{vt} \\ \varepsilon_{ft} \end{bmatrix}$ and $\boldsymbol{\Phi}(L) = \begin{bmatrix} (1 - \Phi_{vv}(L)) & -\Phi_{vf}(L) \\ -\Phi_{fv}(L) & (1 - \Phi_{ff}(L)) \end{bmatrix}$, where $\Phi_{ij}(L) = \sum_{l=1}^{l_{ij}} \phi_{ij}^{l} L^{l}$ and (L) denotes the lag operator, $i, j = v, f$.

The lag polynomials $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$ indicate respectively the response of stock market volatility and mutual funds flow to their own lags, whereas $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$ measure respectively the causality from flow to volatility and vice versa. In other words, the own effects are captured by $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$, whereas the cross effects are captured by $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$ polynomials for return and flow respectively.

With the bivariate VAR Model, we examine the case of volatility as a dependent variable with its' lagged values to investigate which of those lagged values have a significant impact on the volatility itself.

In particular, following equation (4.1), the lag polynomial $\Phi_{vv}(L)$ could be represented as follows:

$$\Phi_{vv}(L) = \sum_{l=1}^{l_{vv}} \phi_{vv}^{l} L^{l}. \qquad 4.2$$

In addition, by applying the bivariate VAR Model, the case of flow has been included as a dependent variable with its' own lagged values to show which of these lagged values have a significant effect on the flow itself.

Following equation (4.1), the lag polynomial $\Phi_{ff}(L)$ could be represented as follows:

$$\Phi_{ff}(L) = \sum_{l=1}^{l_{ff}} \phi_{ff}^{l} L^{l}. \qquad 4.3$$

The bi-directional causality between volatility and flow is captured by the lag polynomials $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$. The polynomial $\Phi_{vf}(L)$ in equation (4.1) represents the effect of flow on volatility in the mean equation, and could be written as follows:

$$\Phi_{vf}(L) = \sum_{l=1}^{l_{vf}} \phi_{vf}^{l} L^{l}. \qquad 4.4$$

The polynomial $\Phi_{fv}(L)$ in equation (4.1) captures the effect of volatility on flow in the mean equation, and could be written as follows:

$$\Phi_{fv}(L) = \sum_{l=1}^{l_{fv}} \phi_{fv}^{l} L^{l}. \qquad 4.5$$

Variance Equation

Moreover, the bivariate vector of innovations ε_t is conditionally normal with mean zero and variance-covariance matrix $\mathbf{H_t}$. That is $\varepsilon_t|\Omega_{t-1} \sim N\ (\mathbf{0}, \mathbf{H_t})$**:**

$$\mathbf{H_t} = \begin{bmatrix} h_{vt} & h_{vf,t} \\ h_{fv,t} & h_{ft} \end{bmatrix}, \quad 4.6$$

where $h_{it}, i = v, f$ denotes the conditional variance of stock market volatility and mutual funds flow respectively. $h_{vf,t}$ denotes the conditional covariance of the two variables.

GARCH Models

In this study, we will examine four alternative GARCH processes. First, we will assume that h_{it} follow univariate GARCH (1,1) processes:

$$h_{it} = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1}, i = v, f, \qquad 4.7$$

and that $h_{vf,t} = 0$. Note that $\omega_i > 0$, $\alpha_i > 0$, and $\beta_i \geq 0$ in order for $h_{it} > 0$ for all t. Moreover, $\alpha_i + \beta_i < 1$ for the unconditional variance to exist.

The literature on GARCH (1,1) model is enormous (for instance, see Engle, 1982, Bollerslev, 1986, Bollerslev et al. 1992, Bollerslev et al. 1994, and the references therein). The ARCH model that introduced by Engle (1982) has explicitly recognized the difference between the conditional variance and the unconditional variance, allowing the conditional variance to change over time as a function of past errors.

Second, we will assume that $\mathbf{H_t}$ follows the bivariate constant conditional correlation (CCC) GARCH (1,1) model of Bollerslev et al. (1992). That is, h_{it} is given by equation (4.7),

and that $h_{vf,t}$ is given by:

$$h_{vf,t} = \rho\sqrt{h_{vt}}\sqrt{h_{ft}}\,, \quad 4.8$$

where ρ denotes the ccc.

We employ a bivariate GARCH (1,1) model to examine the dual linkage between stock market volatility and U.S. mutual funds flow. Bollerslev (1986) has developed the GARCH (1,1) model through allowing the conditional variance to depend on the past conditional variances. Whilst the autoregressive components capture the persistence in the conditional variance of volatility and flow, the past squared residual components capture the information shocks to volatility and flow.

The ARFIMA-FIGARCH (1,d,1) Models

Third, we will employ the bivariate ARFIMA-FIGARCH (1,d,1) process in order to capture the long-memory component in the mean equations as follows (see Granger and Joyeux, 1980, and Hosking, 1981, and the references therein):

$$(1-L)^{d_{mv}}(1-\Phi_{\boldsymbol{vv}}(L))\left[y_{vt} - \Phi_{vf}(L)y_{ft} - \mu_{vt}\right] = \varepsilon_{vt}\,,\ \ Volatility \quad 4.9$$

$$(1-\Phi_{ff}(L))y_{ft} = \mu_{ft} + \Phi_{fv}(L)y_{vt} + \varepsilon_{ft}, \quad Flow \quad 4.10$$

where the lag polynomials $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$ are represented by equations (4.2) and (4.3) respectively. Moreover, the lag polynomials $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$ are given by equations (4.4) and (4.5) respectively.

In addition, we will assume that h_{it} follow univariate FIGARCH (1,d,1) processes:

$$(1-\beta_i L)h_{it} = \omega_i + [(1-\beta_i L) - (1-c_i L)(1-L)^{d_{vi}}]\varepsilon_{i,t-1}^2, i = v, f, \quad 4.11$$

where $c_i = \alpha_i + \beta_i$ and d_{vi} is the long memory parameter. Note that if $d_{vi} = 0$, then the above FIGARCH (1,d,1) model reduces to the GARCH (1,1) model in equation (4.7). The sufficient conditions of Bollerslev and Mikkelsen (1996) for the positivity of the conditional variance of a FIGARCH (1,d,1) model: $\omega_i > 0$, $\beta_i - d_{vi} \leq c_i \leq \frac{2-d_{vi}}{3}$, and $d_{vi}\left(c_i - \frac{1-d_{vi}}{2}\right) \leq \beta_i(c_i - \beta_i + d_{vi})$ should be satisfied for both i (see also Conrad and Haag, 2006, Conrad, 2010, and Karanasos et al. 2016,). We also assume that $h_{vf,t} = 0$.

The FIGARCH model proposed by Baillie et al. (1996) has been proved to handle some typical data features in various empirical applications (see, for instance, Bollerslev and Mikkelsen, 1996, Beine and Laurent, 2003, Conrad and Karanasos, 2005, and Conrad and Haag, 2006,).

Furthermore, Giraitis et al. (2005) have presented an up-to-date overview of theoretical findings on ARFIMA-FIGARCH processes. Baillie (1996) and Henry and Zaffaroni (2003) have provided excellent surveys of major econometric analysis on long memory processes and their applications in finance and economics.

In the fourth case (bivariate ARFIMA-FIGARCH (1,d,1)), we will assume that the variance equation is given by equation (4.11) and $h_{vf,t}$ is given by equation (4.8).

Finally, Conrad and Haag (2006) have stated that the ARFIMA-FIGARCH model is not covariance stationary and conditions on the parameters should be imposed to ensure the non-negativity of the conditional variances.

4.6. Empirical Results

Bivariate VAR-CCC GARCH Model (RS Volatility)

Firstly, we should mention that the results with respect to the bivariate VAR with univariate GARCH (1,1) processes in the case of RS volatility are presented in details in Appendix (4.A). In this section, we will present the findings related to the bivariate VAR-CCC GARCH model in the mean equation (including the cases of own effects and cross effects respectively) as well as the results related to the variance equation (the GARCH coefficients).

Mean Equation

With the bivariate VAR model, we will examine the case of own effects. In addition, we will examine the case of cross effects (the effect of the lagged values of flow obtained in the mean equation of volatility and vice versa).

Own Effects

Table (4.1) reports the chosen lags for the own effects $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$. The $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$ columns report results for the volatility and flow equations respectively.

For example, the lag polynomial $\Phi_{vv}(L)$ for the sub-sample (B) might be written with regards to the equation (4.2) as follows (see Table 4.1):

$$\Phi_{vv}(L) = \left(\phi_{vv}^{1} L^{1} + \phi_{vv}^{3} L^{3}\right).$$

As another example, and by applying equation (4.3), the lag polynomial $\Phi_{ff}(L)$ for the sub-sample (UD3) might be written as follows (see Table 4.1):

$$\Phi_{ff}(L) = \left(\phi_{ff}^{2} L^{2} + \phi_{ff}^{3} L^{3}\right).$$

Table 4.1. Mean Equation: AR Lags (Own Effects)

Samples	**Eq. (4.2):** $\Phi_{vv}(L)$	**Eq. (4.3):** $\Phi_{ff}(L)$
Whole Sample	**2,3,7**	**2,3**
<u>Panel A:</u>		
A	**2,3,4**	**2,3,4,5**
B	**1,3**	**2,3,4**
<u>Panel B:</u>		
UD1	**1,3,4**	**1,2,3,6**
UD2	**1,2,6**	**1,3,4**
UD3	**1,2,3**	**2,3**
UD4	**1,2,5**	**6**
UD5	**1,2,3,4,5**	**2,3,4**
<u>Panel C:</u>		
CY1	**2**	**1,2,4,5,6**
CY2	**1,2,3,4,6**	**2,3,4,5,6**

Notes: This table reports significant lags for the own effects $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$.

Cross Effects (The Volatility-Flow Linkage)

Table (4.2) reports the chosen lags for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$. For instance and by following equation (4.4), the lag polynomial $\Phi_{\mathrm{vf}}(L)$ for the sub-sample (UD4) is as follows:

$$\Phi_{\mathrm{vf}}(L) = \phi_{vf}^{2} L^{2}.$$

Moreover, by applying equation (4.5), the lag polynomial for the sub-sample (UD4) could be written as follows (see Table 4.2):

$$\Phi_{\mathrm{fv}}(L) = \phi_{fv}^{1} L^{1}.$$

Table 4.2. Mean Equations: AR Lags (Cross Effects)

Samples	$\Phi_{vf}(L)$	$\Phi_{fv}(L)$
Whole Sample	**9**	**1**
Panel A:		
A	**4**	**3**
B	**4**	**1**
Panel B:		
UD1	**4**	**1**
UD2	**3**	**6**
UD3	**2**	**3**
UD4	**2**	**1**
UD5	**5**	**1**
Panel C:		
CY1	**4**	**6**
CY2	**5**	**3**

Notes: This table reports significant lags for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$.

As presented in Table (4.3), we notice a bidirectional mixed feedback between market return volatility and aggregate mutual fund flow in the whole sample, sub-samples (UD1) and (CY1). In particular, volatility affects flow negatively whereas the reverse impact is of the opposite sign. Moreover, we find a negative bi-directional causality between volatility and flow in the sub-samples (B), (UD2), (UD4) and (UD5). This finding is in line with the results presented by Cao et al. (2008). However, a positive bi-directional causality is noticed in the sub-samples (A), (UD3) and (CY2). That is, the evidence for the total sample suggests that the negative (positive) effect from volatility (flow) to flow (volatility) reflects the causal relation between flow and volatility in the sub-samples (CY1) and (UD1).

Our findings reveal that the positive impact of flow on volatility in the whole sample comes from the same impact in the sub-sample (A), which in turns comes from the sub-sample (CY1). In addition, the aforementioned effect in the sub-samples (CY1) and (CY2) comes from the same impact in the sub-samples (UD1) and (UD3) respectively. However, the negative effect of volatility on flow in the whole sample comes from the same effect in the sub-sample (B), which in turns comes from the sub-sample (CY1). This negative impact in the sub-sample (CY1) comes from the same impact in the sub-samples (UD1) and (UD2).

Whereas, the positive effect in the sub-sample (CY2) comes from the positive impact of volatility on flow in the sub-sample (UD3).

It is noteworthy that the positive effect of flow on volatility in the sub-sample (A) is converted to be negative in the sub-sample (B). Because the former sub-sample covers the periods among the 2007 Financial Crisis as well as the 2009 European Sovereign Debt Crisis, this negative impact might be explained by the relative role of mutual funds in both investment attitude and stock market volatility especially during the crises' period.

It is also interesting to highlight that the positive impact of flow on volatility in the sub-sample (UD1) is turned into a negative effect in the sub-sample (UD2), then it turns back to be positive in the sub-sample (UD3) and eventually it reverses to the negative sign in the sub-sample (UD4). This fluctuated impact could be explained by the rational investment sentiment which seems to cause market return volatility to respond positively to mutual fund flow in the stable market (outside the range of crises' period), as buying mutual funds has a high investment risk in the volatile stock market.

On the other hand, we notice that the negative impact of volatility on flow in the sub-sample (UD2) is converted to be a positive effect in the sub-sample (UD3), and consequently it reverts to the negative sign in the sub-sample (UD4). This phenomenon might be addressed by the risk attitude of the U.S. investors, as more than 80% of investors are institutional investors in the U.S. stock market. Likewise, institutional investors play a more important role than the individual investors in the U.S. stock market when we neglect foreign investors. This reveals that many U.S. institutional investors are likely to be risk-averse, they might not buy mutual funds when the unexpected volatility increases, causing a negative impact of market return volatility on mutual fund flow during the aforementioned crises.

Table 4.3. The Volatility-Flow Linkage: VAR-CCC GARCH (RS Volatility)

Samples	Effect of Flow on Volatility	Effect of Volatility on Flow
Whole Sample	**Positive**	**Negative**
Panel A:		
A	**Positive**	**Positive**
B	**Negative**	**Negative**
Panel B:		
UD1	**Positive**	**Negative**
UD2	**Negative**	**Negative**
UD3	**Positive**	**Positive**
UD4	**Negative**	**Negative**
UD5	**Negative**	**Negative**
Panel C:		
CY1	**Positive**	**Negative**
CY2	**Positive**	**Positive**

In addition, Table (4.4) presents the estimated coefficients for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$. The $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$ columns report results for the volatility and flow equations respectively.

Table 4.4. The Coefficients of The Volatility-Flow Link (RS Volatility)

Samples	$\Phi_{vf}(L)$	$\Phi_{fv}(L)$
Whole Sample	**0.07 (0.05)*****	**-0.22 (0.08)*****
Panel A:		
A	**0.01 (0.00)****	**0.36 (0.18)****
B	**-0.01 (0.00)*****	**-0.28 (0.09)*****
Panel B:		
UD1	**0.01 (0.00)****	**-0.84 (0.24)*****
UD2	**-0.02 (0.01)*****	**-0.54 (0.23)****
UD3	**0.00 (0.00)***	**0.52 (0.25)****
UD4	**-0.01 (0.00)*****	**-0.30 (0.22)***
UD5	**-0.01 (0.00)*****	**-0.32 (0.10)*****
Panel C:		
CY1	**0.01 (0.00)****	**-0.54 (0.15)*****
CY2	**0.01 (0.01)***	**0.23 (0.16)***

Notes: This table reports parameters' estimates for the Φ_{vf} and Φ_{fv} respectively.

***, **, and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Variance Equation

Table (4.5) reports estimates of ARCH, GARCH and CCC parameters. Following equation (4.7), we notice that the sum of the coefficients of the ARCH parameter (α) and the GARCH parameter (β) for the total sample and all the other sub-samples respectively is less than one. Additionally, the ARCH and GARCH coefficients are positive and significant in all cases.

With regards to equation (4.8), the conditional correlations between volatility and flow in the whole sample, sub-samples (A), (UD5) and (CY2) are very similar: (0.10, 0.10, 0.12 and 0.11). In the sub-samples (B), (UD1), (UD2), (UD4) and (CY1), the estimated values of ρ for volatility-flow (0.16, 0.25, 0.20, 0.25 and 0.21 respectively) are higher than the corresponding value for the whole sample which is (0.10). However, the lowest value of the constant conditional correlation between volatility and flow (0.05) is realized for sub-sample (UD3).

Table 4.5. Variance Equation: GARCH And CCC Coefficients (RS Volatility)

Samples	h_{1t}(Volatility)	h_{2t}(Flow)
Whole Sample		
α_i	**0.08 (0.05)***	**0.08 (0.03)*****
β_i	**0.72 (0.27)*****	**0.91 (0.03)*****
ρ	**0.10 (0.02)*****	-
Panel A: **Sub-Sample A**		
α_i	**0.10 (0.04)****	**0.12 (0.04)*****
β_i	**0.78 (0.05)*****	**0.87 (0.04)*****
ρ	**0.10 (0.02)*****	-
Sub-Sample B		
α_i	**0.09 (0.05)****	**0.10 (0.05)***
β_i	**0.85 (0.42)***	**0.89 (0.06)*****
ρ	**0.16 (0.05)*****	-

Panel B:		
Sub-Sample UD1		
α_i	**0.12 (0.07)****	**0.06 (0.02)***
β_i	**0.76 (0.23)*****	**0.92 (0.07)*****
ρ	**0.25 (0.06)*****	-
Sub-Sample UD2		
α_i	**0.08 (0.05)****	**0.09 (0.02)*****
β_i	**0.76 (0.13)*****	**0.90 (0.02)*****
ρ	**0.20 (0.06)*****	-
Sub-Sample UD3		
α_i	**0.15 (0.05)*****	**0.18 (0.09)***
β_i	**0.82 (0.05)*****	**0.70 (0.16)*****
ρ	**0.05 (0.02)***	-
Sub-Sample UD4		
α_i	**0.09 (0.03)*****	**0.13 (0.05)*****
β_i	**0.62 (0.12)*****	**0.86 (0.05)*****
ρ	**0.25 (0.08)***	-
Sub-Sample UD5		
α_i	**0.10 (0.05)****	**0.07 (0.14)***
β_i	**0.66 (0.20)***	**0.91 (0.19)*****
ρ	**0.12 (0.04)*****	-
Panel C:		
Sub-Sample CY1		
α_i	**0.16 (0.13)***	**0.07 (0.01)*****
β_i	**0.70 (0.07)*****	**0.92 (0.01)*****
ρ	**0.21 (0.04)*****	-
Sub-Sample CY2		
α_i	**0.12 (0.06)*****	**0.18 (0.06)*****
β_i	**0.83 (0.04)*****	**0.80 (0.05)*****
ρ	**0.11 (0.03)*****	-

Notes: This table reports parameters' estimates for the ARCH ($\boldsymbol{\alpha_i}$), GARCH ($\boldsymbol{\beta_i}$) and ccc ($\boldsymbol{\rho}$) coefficients.

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Bivariate VAR-CCC GARCH Model (GKYZ Volatility)

Firstly, we present the detailed findings related to the bivariate VAR with univariate GARCH (1,1) models in the case of GKYZ volatility in Appendix (4.B). In this section, we will present the results with respect to the bivariate VAR-CCC GARCH model in the mean equation (including the cases of own effects and cross effects respectively) and the findings related to the variance equation (the GARCH coefficients).

Mean Equation

With regards to the bivariate VAR model, we will present the case of own effects (the effect of the lagged values of volatility (flow) on the volatility (flow) in the mean equation). In addition, we will present the case of cross effects (the effect of the lagged values of volatility obtained in the mean equation of flow and vice versa).

Own Effects

Table (4.6) presents the chosen lags for the own effects $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$. The $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$ columns present results for the volatility and flow equations respectively.

Table 4.6. Mean Equation: AR Lags (Own Effects)

Samples	**Eq. (4.2):** $\Phi_{vv}(L)$	**Eq. (4.3):** $\Phi_{ff}(L)$
Whole sample	**2,3,4,6**	**2,3,4**
Panel A:		
A	**1,2,3,4,6**	**1,3,4,5**
B	**2,3,4**	**2,3,4**
Panel B:		
UD1	**1,2,6**	**1,2,3,6**
UD2	**2,3**	**1,3,4**
UD3	**2,3,4**	**2,3,4,5**
UD4	**5**	**6**
UD5	**1,2,3,4,5,6**	**2,3,4,6**
Panel C:		
CY1	**1,2,6**	**1,2,5,6**
CY2	**1,2,3,4,6**	**2,3,4,5,6**

Notes: This table reports significant lags for the own effects $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$. As an example, the equation (4.2) for the sub-sample (UD2) can be written as follows: $\Phi_{vv}(L) = (\phi_{vv}^2 L^2 + \phi_{vv}^3 L^3)$ and the equation (4.3) for the sub-sample (UD2) can be written as follows: $\Phi_{ff}(L) = (\phi_{ff}^1 L^1 + \phi_{ff}^3 L^3 + \phi_{ff}^4 L^4)$.

Cross Effects (The Volatility-Flow Linkage)

The significant lags for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$ are presented in Table (4.7).

Table 4.7. Mean Equation: AR Lags (Cross Effects)

Samples	$\Phi_{vf}(L)$	$\Phi_{fv}(L)$
Whole sample	**5**	**1**
Panel A:		
A	**6**	**3**
B	**4**	**1**
Panel B:		
UD1	**3**	**6**
UD2	**4**	**2**
UD3	**1**	**3**
UD4	**7**	**1**
UD5	**5**	**1**
Panel C:		
CY1	**3**	**1**
CY2	**4**	**3**

Notes: This table reports significant lags for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$. Following equation (4.4), $\Phi_{vf}(L)$ for the sub-sample (UD2) might be written as follows: $\Phi_{vf}(L) = \phi_{vf}^4 L^4$ and $\Phi_{fv}(L)$ for the sub-sample (UD5) could be represented with regards to the equation (4.5) as follows: $\Phi_{fv}(L) = \phi_{fv}^1 L^1$.

As we can see in Table (4.8), there is a negative bi-directional causality between stock market volatility and aggregate mutual fund flow in the whole sample, sub-samples (B), (UD2), (UD4), (UD5) and (CY1). This result is consistent with the finding obtained by Cao et al. (2008). In addition, there is a bidirectional mixed feedback between volatility and flow in the sub-sample (UD1). In particular, flow affects volatility positively whereas the reverse impact is of the opposite sign. A positive bi-directional causality is noticed in the sub-samples (A), (UD3) and (CY2). That is, the evidence for the whole sample suggests that the negative effect from volatility (flow) to flow (volatility) reflects the negative interaction between volatility and flow in the sub-samples (B) and (CY1).

Summarizing, the negative impact of flow on volatility in the whole sample comes from the same impact in the sub-sample (B). This aforesaid negative effect in the sub-sample (B) is consistent with the same impact in the sub-samples (UD4) and (UD5). Moreover, this negative effect in the sub-sample (CY1) comes from sub-sample (UD2), whereas the positive impact in the sub-sample (CY2) comes from the relevant sign in the sub-sample (UD3).

It is important to mention that two exceptional cases are captured in comparison between these findings and our previous findings in the case of RS volatility which are the effect of flow on volatility in the whole sample as well as the sub-sample (CY1). Whilst this impact is noticed with a negative sign in this case, a positive effect is realized in the case of RS volatility.

Table 4.8. The Volatility-Flow Linkage: VAR-CCC GARCH (GKYZ Volatility)

Samples	Effect of Flow on Volatility	Effect of Volatility on Flow
Whole Sample	**Negative**	**Negative**
Panel A:		
A	**Positive**	**Positive**
B	**Negative**	**Negative**
Panel B:		
UD1	**Positive**	**Negative**
UD2	**Negative**	**Negative**
UD3	**Positive**	**Positive**
UD4	**Negative**	**Negative**
UD5	**Negative**	**Negative**
Panel C:		
CY1	**Negative**	**Negative**
CY2	**Positive**	**Positive**

Moreover, the estimated coefficients for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$ are presented in Table (4.9). The $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$ columns report findings for the volatility and flow equations respectively.

Table 4.9. The Coefficients of The Volatility-Flow Link (GKYZ Volatility)

Samples	$\Phi_{vf}(L)$	$\Phi_{fv}(L)$
Whole Sample	**-0.01 (0.00)***	**-0.17 (0.10)***
Panel A:		
A	**0.01 (0.00)*****	**0.28 (0.15)***
B	**-0.02 (0.00)*****	**-0.37 (0.15)****
Panel B:		
UD1	**0.01 (0.00)****	**-0.77 (0.29)*****
UD2	**-0.01 (0.01)****	**-0.46 (0.24)***
UD3	**0.01 (0.00)***	**0.44 (0.20)****
UD4	**-0.02 (0.00)*****	**-0.30 (0.24)***
UD5	**-0.00 (0.00)*****	**-0.45 (0.13)*****
Panel C:		
CY1	**-0.01 (0.00)***	**-0.83 (0.21)*****
CY2	**0.02 (0.01)****	**0.17 (0.14)***

Notes: This table reports parameters' estimates for the Φ_{vf} and Φ_{fv} respectively.

***, **, and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Variance Equation

The estimates of ARCH, GARCH and CCC parameters are presented in Table (4.10). We notice that the ARCH and GARCH coefficients are positive and significant in all various cases. In addition, we note by following equation (4.7) that the sum of the coefficients of the ARCH parameter (α) and the GARCH parameter (β) for the whole sample and all various sub-samples respectively is less than one.

Following equation (4.8), the constant conditional correlations between volatility and flow in the whole sample, sub-samples (B), (UD3), (UD4) and (CY2) are very identical: (0.08, 0.09 and 0.07). In the sub-samples (A), (UD1), (UD2), (UD5) and (CY1), the estimated values of ρ for volatility-flow (0.13, 0.20, 0.21, 0.15 and 0.23 respectively) are higher than the corresponding value for the whole sample which is (0.08).

Table 4.10. Variance Equation: GARCH And CCC Coefficients (GKYZ Volatility)

Samples	h_{1t}(Volatility)	h_{2t}(Flow)
Whole Sample		
α_i	**0.11 (0.07)***	**0.08 (0.03)*****
β_i	**0.72 (0.16)*****	**0.91 (0.03)*****
ρ	**0.08 (0.03)*****	-
Panel A:		
Sub-Sample A		
α_i	**0.07 (0.03)*****	**0.12 (0.04)*****
β_i	**0.72 (0.06)*****	**0.87 (0.03)*****
ρ	**0.13 (0.03)*****	-
Sub-Sample B		
α_i	**0.11 (0.08)***	**0.09 (0.05)****
β_i	**0.83 (0.34)****	**0.89 (0.05)*****
ρ	**0.09 (0.05)***	-

Panel B:		
Sub-Sample UD1		
α_i	**0.19 (0.07)****	**0.06 (0.03)***
β_i	**0.73 (0.33)****	**0.90 (0.10)*****
ρ	**0.20 (0.05)*****	-
Sub-Sample UD2		
α_i	**0.10 (0.06)***	**0.08 (0.01)*****
β_i	**0.71 (0.13)*****	**0.91 (0.02)*****
ρ	**0.21 (0.07)*****	-
Sub-Sample UD3		
α_i	**0.10 (0.08)***	**0.11 (0.07)***
β_i	**0.81 (0.10)*****	**0.75 (0.18)****
ρ	**0.08 (0.04)****	-
Sub-Sample UD4		
α_i	**0.07 (0.03)*****	**0.13 (0.05)*****
β_i	**0.70 (0.18)*****	**0.86 (0.05)*****
ρ	**0.08 (0.11)***	-
Sub-Sample UD5		
α_i	**0.08 (0.08)***	**0.07 (0.06)****
β_i	**0.73 (0.30)****	**0.91 (0.10)*****
ρ	**0.15 (0.04)*****	-
Panel C:		
Sub-Sample CY1		
α_i	**0.07 (0.05)****	**0.07 (0.01)*****
β_i	**0.72 (0.23)*****	**0.92 (0.01)*****
ρ	**0.23 (0.04)*****	-
Sub-Sample CY2		
α_i	**0.11 (0.06)*****	**0.11 (0.04)*****
β_i	**0.86 (0.04)*****	**0.84 (0.07)*****
ρ	**0.07 (0.05)***	-

Notes: This table reports parameters' estimates for the ARCH (α_i), GARCH (β_i) and ccc (ρ) coefficients.

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Bivariate CCC ARFIMA-FIGARCH (1,d,1) Model (RS Volatility)

First of all, we should mention that the results with respect to the univariate ARFIMA-FIGARCH (1,d,1) process in the case of RS volatility are presented in details in Appendix (4.C). In this section, we will present the findings related to the mean equation (including the cases of own effects and cross effects respectively), the fractional mean parameters and the variance equation (GARCH coefficients and CCC parameters).

Mean Equation

We will examine the case of own effects (the effect of the lagged values of volatility (flow) on the volatility (flow) in the mean equation) in this sub-section. Moreover, we will examine the case of cross effects (the effect of the lagged values of flow obtained in the mean equation of volatility and vice versa).

Own Effects

Table (4.11) reports the chosen lags for the own effects $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$. The $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$ columns report results for the volatility and flow equations respectively.

Table 4.11. Mean Equations: AR Lags (Own Effects)

Samples	Eq. (4.2): $\Phi_{vv}(L)$	Eq. (4.3): $\Phi_{ff}(L)$
Whole Sample	**2**	**2,3,4**
Panel A:		
A	**1,5**	**2,3,4,5**
B	**1,3**	**1,2,3,4,5**
Panel B:		
UD1	**2,3**	**1,2,3,4**
UD2	**6**	**1,3,4**
UD3	**1**	**1,2,3**
UD4	**3**	**2**
UD5	**1,4**	**2,3,4**
Panel C:		
CY1	**1,4,5**	**1,4,5**
CY2	**1,3**	**2,3,4**

Notes: This table reports significant lags for the own effects $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$. As an example, the equation (4.2) for the sub-sample (A) can be written as follows: $\Phi_{vv}(L) = \phi_{vv}^{1}L^{1} + \phi_{vv}^{5}L^{5}$, and the equation (4.3) for the sub-sample (UD5) can be written as follows: $\Phi_{ff}(L) = \phi_{ff}^{2}L^{2} + \phi_{ff}^{3}L^{3} + \phi_{ff}^{4}L^{4}$.

Cross Effects (The Volatility-Flow Linkage)

Table (4.12) below presents the chosen lags for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$.

Table 4.12. Mean Equations: AR Lags (Cross Effects)

Samples	$\Phi_{vf}(L)$	$\Phi_{fv}(L)$
Whole Sample	**4**	**1**
Panel A:		
A	**2**	**3**
B	**1**	**5**
Panel B:		
UD1	**4**	**1**
UD2	**3**	**6**
UD3	**2**	**3**
UD4	**2**	**3**
UD5	**2**	**1**
Panel C:		
CY1	**4**	**4**
CY2	**2**	**3**

Notes: This table reports significant lags for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$. Following equation (4.4), $\Phi_{vf}(L)$ for the sub-sample (UD5) might be written as follows: $\Phi_{vf}(L) = \phi_{vf}^{2}L^{2}$, and $\Phi_{fv}(L)$ for the sub-sample (UD4) could be represented with regards to the equation (4.5) as follows: $\Phi_{fv}(L) = \phi_{fv}^{3}L^{3}$.

Following the presented in Table (4.13), there is a bidirectional mixed feedback between market return volatility and aggregate mutual fund flow in the whole sample, sub-samples

(UD1) and (CY1). In particular, flow affects volatility positively whereas the reverse impact is of the opposite sign. In addition, a positive bi-directional causality is noticed in the sub-samples (A), (UD3) and (CY2). This result is in sharp contrast with the finding presented by Cao et al. (2008). However, we notice a negative bi-directional causality between volatility and flow in the sub-samples (B), (UD2), (UD4) and (UD5). That is, the evidence for the total sample suggests that the positive (negative) impact from flow (volatility) to volatility (flow) reflects the causal relation between volatility and flow in the sub-samples (UD1) and (CY1). In brief, the negative effect of volatility on flow in the whole sample comes from the same effect in the sub-samples (B) and (CY1). This negative impact in the sub-sample (B) and (CY1) is consistent with the same impact in the sub-samples (UD4 and UD5) and sub-samples (UD1 and UD2) respectively. These mentioned results are consistent with the findings related to the bivariate VAR-GARCH (1,1) model with respect to the case of RS volatility. Additionally, in comparison between the bivariate VAR-CCC GARCH model and bivariate CCC ARFIMA-FIGARCH process in the case of RS volatility, we notice that, qualitatively, the results are identical with regards to the volatility-flow relationship.

Table 4.13. The Volatility-Flow Linkage: CCC ARFIMA-FIGARCH (RS Volatility)

Samples	**Effect of Flow on Volatility**	**Effect of Volatility on Flow**
Whole Sample	**Positive**	**Negative**
Panel A:		
A	**Positive**	**Positive**
B	**Negative**	**Negative**
Panel B:		
UD1	**Positive**	**Negative**
UD2	**Negative**	**Negative**
UD3	**Positive**	**Positive**
UD4	**Negative**	**Negative**
UD5	**Negative**	**Negative**
Panel C:		
CY1	**Positive**	**Negative**
CY2	**Positive**	**Positive**

Table (4.14) reports the estimated coefficients for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$. The $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$ columns report results for the volatility and flow equations respectively.

Table 4.14. The Coefficients of The Volatility-Flow Link (RS Volatility)

Samples	$\Phi_{vf}(L)$	$\Phi_{fv}(L)$
Whole Sample	**0.01 (0.00)***	**-0.21 (0.08)****
Panel A:		
A	**0.01 (0.00)*****	**0.27 (0.19)*****
B	**-0.02 (0.01)*****	**-0.24 (0.13)*****
Panel B:		
UD1	**0.00 (0.00)****	**-0.88 (0.24)***
UD2	**-0.01 (0.01)*****	**-0.78 (0.30)***
UD3	**0.00 (0.00)*****	**0.44 (0.28)*****
UD4	**-0.01 (0.01)*****	**-0.18 (0.14)*****
UD5	**-0.01 (0.01)*****	**-0.18 (0.06)***
Panel C:		
CY1	**0.01 (0.01)*****	**-0.33 (0.18)*****
CY2	**0.00 (0.00)*****	**0.36 (0.18)****

Notes: This table reports parameters' estimates for the Φ_{vf} and Φ_{fv} respectively.

***, **, and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Fractional Mean Parameters

Following equation (4.9), estimates of the volatility's fractional mean parameters are presented in Table (4.15). In the whole sample and sub-sample (CY1), volatility generated very similar fractional mean parameters: (0.43 and 0.45). In the sub-samples (UD2, UD3, UD5 and CY2), the long memory mean parameters are lower than the corresponding value for the whole sample: (0.30, 0.31, 0.23 and 0.38). However, in the sub-samples (A), (B), (UD1) and (UD4), the estimated values of d_{mv} are higher than the corresponding value for the whole sample: (0.60, 0.49, 0.59 and 0.55).

Table 4.15. Mean Equation: Fractional Parameters (RS Volatility)

Samples	Volatility d_{mv}
Whole Sample	**0.43 (0.06)*** **
<u>Panel A:</u>	
A	**0.60 (0.07)*** **
B	**0.49 (0.09)*** **
<u>Panel B:</u>	
UD1	**0.59 (0.45)***
UD2	**0.30 (0.08)*** **
UD3	**0.31 (0.03)*** **
UD4	**0.55 (0.08)*** **
UD5	**0.23 (0.04)*** **
<u>Panel C:</u>	
CY1	**0.45 (0.22)*** **
CY2	**0.38 (0.05)*** **

Notes: this table reports parameters' estimates for the GARCH Long Memory in the mean equation for volatility.

*** and * Stand for significance at the 1% and 10% significant levels respectively.

Variance Equation (FIGARCH Specifications)

Following equation (4.11), the analysing dynamic adjustments of the conditional variances of both volatility and flow can be seen in Table (4.16). Table (4.16) presents estimates of the ARCH and the GARCH parameters.

We note that the sum of the coefficients of the ARCH parameter (α) and the GARCH parameter (β) for the total sample and all various sub-samples respectively is less than one. Additionally, all the ARCH and GARCH coefficients are positive and significant in all various sub-samples. In other words, the GARCH coefficients in all cases have satisfied the sufficient and necessary conditions for the non-negativity of the conditional variances (see, for instance, Conrad and Haag, 2006,).

With respect to the equation (4.8), the constant conditional correlations between volatility and flow in the whole sample, sub-samples (A), (B) and (UD5) are very similar: (0.09 and 0.10). In the sub-samples (UD1), (UD2), (UD4), (CY1) and (CY2), the estimated values of ρ for volatility-flow (0.25, 0.23, 0.21, 0.14 and 0.12 respectively) are higher than the corresponding value for the whole sample which is (0.08). In addition, this value in the sub-sample (UD3) is the lowest value among all aforesaid sub-samples which is (0.06).

Table 4.16. Variance equations: GARCH And CCC Coefficients (RS Volatility)

Samples	h_{1t}(Volatility)	h_{2t}(Flow)
Total sample		
α_i	**0.06 (0.07)*** **	**0.07 (0.01)***
β_i	**0.36 (0.08)***	**0.29 (0.02)***
ρ	**0.09 (0.01)*** **	-
Panel A: **Sample A**		
α_i	**0.09 (0.04)***	**0.08 (0.04)***
β_i	**0.34 (0.30)*** **	**0.23 (0.06)***
ρ	**0.10 (0.02)*** **	-
Sample B		
α_i	**0.13 (0.04)***	**0.07 (0.03)***
β_i	**0.27 (0.16)*** **	**0.28 (0.07)***
ρ	**0.10 (0.04)*** **	-

<u>Panel B:</u>		
Sample UD1		
α_i	**0.10 (0.09)***	**0.06 (0.05)***
β_i	**0.32 (0.10)***	**0.27 (0.15)***
ρ	**0.25 (0.06)***	-
Sample UD2		
α_i	**0.09 (0.04)***	**0.10 (0.07)***
β_i	**0.34 (0.08)***	**0.24 (0.09)***
ρ	**0.23 (0.06)***	-
Sample UD3		
α_i	**0.07 (0.05)***	**0.15 (0.14)***
β_i	**0.25 (0.22)***	**0.39 (0.27)***
ρ	**0.06 (0.03)****	-
Sample UD4		
α_i	**0.11 (0.16)***	**0.07 (0.04)***
β_i	**0.40 (0.15)***	**0.29 (0.06)***
ρ	**0.21 (0.07)***	-
Sample UD5		
α_i	**0.08 (0.09)***	**0.09 (0.09)***
β_i	**0.24 (0.13)***	**0.35 (0.08)***
ρ	**0.09 (0.04)****	-
<u>Panel C:</u>		
Sample CY1		
α_i	**0.09 (0.05)***	**0.10 (0.03)***
β_i	**0.36 (0.09)***	**0.35 (0.06)***
ρ	**0.14 (0.03)***	-
Sample CY2		
α_i	**0.08 (0.06)***	**0.08 (0.10)***
β_i	**0.32 (0.22)***	**0.32 (0.09)***
ρ	**0.12 (0.03)***	-

Notes: This table reports parameters' estimates for the ARCH ($\boldsymbol{\alpha_i}$), GARCH ($\boldsymbol{\beta_i}$) and ccc ($\boldsymbol{\rho}$) coefficients.

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

As we can see in Table (4.17), the estimated of d_{vv} and d_{vf} have governed the long-run dynamics of the conditional heteroscedasticity. The estimation of bivariate ARFIMA-FIGARCH (1,d,1) processes for all volatility and flow's samples have realized estimated values of d_{vv} and d_{vf} that are significantly different from zero or one. In other words, the conditional variances of the two variables have been characterized by the FIGARCH behavior.

In the whole sample, we can notice that the fractional variance parameter which has been generated by volatility is: (0.24). In addition, we find that the fractional differencing parameters in the sub-samples (A), (B), (UD2), (UD3), (UD5), (CY1) and (CY2) are: (0.35, 0.30, 0.29, 0.29, 0.31, 0.33 and 0.34) which are higher than the corresponding value in the whole sample. Moreover, the values of this coefficient for sub-samples (UD1) and (UD4) are: (0.19 and 0.17) which are lower than the corresponding value in the whole sample (0.24). However, although these estimated values are relatively small, they are significantly different from zero.

Moreover, flow has generated very similar fractional differencing parameters for the whole sample, sub-samples (UD5) and (CY2): (0.24, 0.20 and 0.22). We notice that the values of this coefficient in the sub-samples (A), (B), (UD1 up to UD4) and (CY1) are: (0.19, 0.13, 0.09, 0.18, 0.15, 0.08 and 0.19) which are lower than the corresponding value of the fractional variance parameter in the whole sample (0.24). However, although these estimated values are relatively small, they are remarkably different from zero.

Table 4.17. Variance Equation: Fractional Parameters (RS Volatility)

Samples	Volatility d_{vv}	Flow d_{vf}
Whole Sample	**0.24 (0.03)****	**0.24 (0.06)***
Panel A:		
A	**0.35 (0.08)***	**0.19 (0.02)****
B	**0.30 (0.04)****	**0.13 (0.06)***
Panel B:		
UD1	**0.19 (0.01)****	**0.09 (0.04)****
UD2	**0.29 (0.09)***	**0.18 (0.09)***
UD3	**0.29 (0.08)***	**0.15 (0.09)***
UD4	**0.17 (0.07)***	**0.08 (0.09)***
UD5	**0.31 (0.08)***	**0.20 (0.06)***
Panel C:		
CY1	**0.33 (0.08)***	**0.19 (0.04)****
CY2	**0.34 (0.09)***	**0.22 (0.07)***

Notes: this table reports parameters' estimates for the GARCH Long Memory in the variance equation for volatility and flow respectively.

** and * Stand for significance at the 5% and 10% significant levels respectively.

Bivariate CCC ARFIMA-FIGARCH (1,d,1) Model (GKYZ Volatility)

Firstly, we present the detailed findings that related to the univariate ARFIMA-FIGARCH (1,d,1) model in the case of GKYZ volatility in Appendix (4.D). In this section, we will present the results with respect to the mean equation (including the cases of own effects and cross effects respectively), the fractional mean parameters and the variance equation (GARCH coefficients and CCC parameters).

Mean Equation

We will assess the case of own effects (the effect of the lagged values of volatility (flow) on the volatility (flow) in the mean equation) in this sub-section. Additionally, we will examine the case of cross effects (the effect of the lagged values of volatility obtained in the mean equation of flow and vice versa).

Own Effects

The chosen lags for the own effects $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$ are presented in Table (4.18). The $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$ columns report findings for the volatility and flow equations.

Table 4.18. Mean Equations: AR Lags (Own Effects)

Samples	**Eq. (4.2):** $\Phi_{vv}(L)$	**Eq. (4.3):** $\Phi_{ff}(L)$
Whole Sample	**1**	**2,3,4**
Panel A:		
A	**1**	**2,3**
B	**1,5,6**	**2,3,4,5,6**
Panel B:		
UD1	**6,7**	**1,2,6**
UD2	**2,3**	**1,2**
UD3	**1,4**	**2,3,4,5,6,7**
UD4	**1,3**	**5,6**
UD5	**1,2**	**2,3**
Panel C:		
CY1	**2,3**	**1,2**
CY2	**1**	**2,3,4,5,6,7**

Notes: This table reports significant lags for the own effects $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$. As an example, the equation (4.2) for the sub-sample (UD1) can be written as follows: $\Phi_{vv}(L) = \phi_{vv}^{6}L^{6} + \phi_{vv}^{7}L^{7}$, and the equation (4.3) for the sub-sample (UD1) can be written as follows: $\Phi_{ff}(L) = \phi_{ff}^{1}L^{1} + \phi_{ff}^{2}L^{2} + \phi_{ff}^{6}L^{6}$.

Cross Effects (The Volatility-Flow Linkage)

The chosen lags for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$ are showed in Table (4.19).

Table 4.19. Mean Equations: AR Lags (Cross Effects)

Samples	$\Phi_{vf}(L)$	$\Phi_{fv}(L)$
Whole Sample	**4**	**1**
Panel A:		
A	**2**	**3**
B	**1**	**1**
Panel B:		
UD1	**3**	**6**
UD2	**1**	**3**
UD3	**4**	**6**
UD4	**4**	**6**
UD5	**1**	**1**
Panel C:		
CY1	**1**	**1**
CY2	**7**	**7**

Notes: This table reports significant lags for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$. Following equation (4.4), $\Phi_{vf}(L)$ for the sub-sample (A) might be written as follows: $\Phi_{vf}(L) = \phi_{vf}^{2} L^{2}$, and $\Phi_{fv}(L)$ for the sub-sample (UD2) could be represented with regards to the equation (4.5) as follows: $\Phi_{fv}(L) = \phi_{fv}^{3} L^{3}$.

As presented in Table (4.20), there is a negative bi-directional causality between stock market volatility and aggregate mutual fund flow in the whole sample, sub-samples (B), (UD2), (UD4), (UD5) and (CY1). However, we find a positive bi-directional causality in the sub-samples (A), (UD3) and (CY2). This finding is on the contrary of the result obtained by Cao et al. (2008).

In addition, there is a bidirectional mixed feedback between volatility and flow in the sub-sample (UD1). In particular, flow affects volatility positively whereas the reverse impact is of the opposite sign. That is, the evidence for the total sample suggests that the negative effect from flow (volatility) to volatility (flow) reflects the negative interaction between volatility and flow in the sub-samples (B) and (CY1). In brief, the negative impact of volatility on flow in the whole sample comes from the same impact in the sub-samples (B) and (CY1). This negative impact in the sub-sample (CY1) is consistent with the same impact in the sub-samples (UD1) and (UD2). However, the positive effect of volatility on flow in sub-samples (A) and (CY2) comes from the same effect in the sub-sample (UD3). These aforementioned findings are in line with the results related to the both bivariate VAR-GARCH and bivariate VAR-CCC GARCH processes in the case of GKYZ volatility.

Table 4.20. The Volatility-Flow Linkage: CCC ARFIMA-FIGARCH (GKYZ Volatility)

Samples	Effect of Flow on Volatility	Effect of Volatility on Flow
Whole Sample	**Negative**	**Negative**
Panel A:		
A	**Positive**	**Positive**
B	**Negative**	**Negative**
Panel B:		
UD1	**Positive**	**Negative**
UD2	**Negative**	**Negative**
UD3	**Positive**	**Positive**
UD4	**Negative**	**Negative**
UD5	**Negative**	**Negative**
Panel C:		
CY1	**Negative**	**Negative**
CY2	**Positive**	**Positive**

Moreover, the estimated coefficients for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$ are presented in Table (4.21). The $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$ columns report findings for the volatility and flow equations respectively.

Table 4.21. The Coefficients of The Volatility-Flow Link (GKYZ Volatility)

Samples	$\Phi_{vf}(L)$	$\Phi_{fv}(L)$
Whole Sample	**-0.01 (0.00)***	**-0.10 (0.09)****
Panel A:		
A	**0.00 (0.00)***	**0.05 (0.14)****
B	**-0.01 (0.00)***	**-0.22 (0.14)***
Panel B:		
UD1	**0.02 (0.00)*****	**-0.78 (0.28)*****
UD2	**-0.02 (0.01)****	**-0.92 (0.23)*****
UD3	**0.01 (0.00)****	**0.38 (0.19)****
UD4	**-0.01 (0.00)*****	**-0.16 (0.20)***
UD5	**-0.01 (0.00)***	**-0.34 (0.10)*****
Panel C:		
CY1	**-0.01 (0.00)***	**-0.73 (0.20)*****
CY2	**0.02 (0.01)***	**0.30 (0.17)***

Notes: This table reports parameters' estimates for the Φ_{vf} and Φ_{fv} respectively.

***, **, and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Fractional Mean Parameters

The estimates of the volatility's fractional mean parameters are presented in Table (4.22). We notice that the fractional mean parameters in the whole sample is: (0.67). In the sub-samples (A), (B), (UD1 up to UD3), (UD5) and (CY1, CY2), the long memory mean parameters are: (0.51, 0.41, 0.27, 0.06, 0.30, 0.23, 0.37 and 0.35) which are lower than the corresponding value for the whole sample. However, in the sub-sample (UD4), the estimated values of d_{mv} is the highest value for fractional mean parameter amongst the aforementioned sub-samples: (0.82).

Table 4.22. Mean Equation: Fractional Parameters (GKYZ Volatility)

Samples	Volatility d_{mv}
Whole Sample	**0.67 (0.15)*** **
<u>Panel A:</u>	
A	**0.51 (0.07)*** **
B	**0.41 (0.05)*** **
<u>Panel B:</u>	
UD1	**0.27 (0.08)*** **
UD2	**0.06 (0.08)* **
UD3	**0.30 (0.04)*** **
UD4	**0.82 (0.08)*** **
UD5	**0.23 (0.03)*** **
<u>Panel C:</u>	
CY1	**0.37 (0.09)* **
CY2	**0.35 (0.05)*** **

Notes: this table reports parameters' estimates for the GARCH Long Memory in the mean equation for volatility.

*** and * Stand for significance at the 1% and 10% significant levels respectively.

Variance Equation (FIGARCH Specifications)

Following equation (4.11), the analysing dynamic adjustments of the conditional variances of both volatility and flow can be seen in Table (4.23). We note that the sum of the coefficients of the ARCH parameter (α) and the GARCH parameter (β) for the total sample and all sub-samples respectively is less than one. Additionally, all the ARCH and the GARCH coefficients are positive and significant in all various sub-samples.

With respect to the equation (4.8), the constant conditional correlations between volatility and flow in the whole sample and sub-sample (UD3) as well as sub-samples (A), (B) and (UD5) are identical which are: (0.09) and (0.12) respectively. In the sub-sample (UD1), the estimated values of ρ for volatility-flow is: (0.25) which is the highest value of the constant conditional correlation between volatility and flow. However, this value in the sub-sample (CY2) is the lowest value among all aforesaid sub-samples which is (0.08).

Table 4.23. Variance equations: GARCH And CCC Coefficients (GKYZ Volatility)

Samples	h_{1t}(Volatility)	h_{2t}(Flow)
Whole Sample		
α_i	**0.08 (0.03)*** **	**0.13 (0.03)*** **
β_i	**0.25 (0.18)***	**0.35 (0.07)*** **
ρ	**0.09 (0.03)*** **	-
Panel A: **Sub-Sample A**		
α_i	**0.08 (0.03)*** **	**0.06 (0.05)*** **
β_i	**0.30 (0.28)***	**0.24 (0.06)*** **
ρ	**0.12 (0.02)*** **	-
Sub-Sample B		
α_i	**0.12 (0.11)***	**0.08 (0.03)*** **
β_i	**0.30 (0.13)** **	**0.36 (0.03)*** **
ρ	**0.12 (0.05)** **	-
Panel B: **Sub-Sample UD1**		
α_i	**0.13 (0.05)** **	**0.06 (0.04)***
β_i	**0.25 (0.08)*** **	**0.30 (0.11)*** **
ρ	**0.25 (0.05)*** **	-
Sub-Sample UD2		
α_i	**0.08 (0.02)*** **	**0.08 (0.05)*** **
β_i	**0.24 (0.18)***	**0.21 (0.04)*** **
ρ	**0.23 (0.07)*** **	-
Sub-Sample UD3		
α_i	**0.07 (0.04)*** **	**0.07 (0.05)*** **
β_i	**0.28 (0.07)*** **	**0.22 (0.13)***
ρ	**0.09 (0.04)** **	-
Sub-Sample UD4		
α_i	**0.11 (0.06)** **	**0.07 (0.06)*** **
β_i	**0.38 (0.18)** **	**0.19 (0.04)*** **
ρ	**0.11 (0.09)** **	-
Sub-Sample UD5		
α_i	**0.09 (0.05)*** **	**0.09 (0.04)***
β_i	**0.22 (0.28)***	**0.33 (0.10)*** **
ρ	**0.12 (0.05)** **	-

Panel C:		
Sub-Sample CY1		
α_i	**0.14 (0.05)*** **	**0.08 (0.05)*** **
β_i	**0.29 (0.16)***	**0.33 (0.07)*** **
ρ	**0.22 (0.04)*** **	-
Sub-Sample CY2		
α_i	**0.09 (0.06)***	**0.09 (0.12)***
β_i	**0.38 (0.13)*** **	**0.33 (0.13)*** **
ρ	**0.08 (0.06)** **	-

Notes: This table reports parameters' estimates for the ARCH (α_i), GARCH (β_i) and ccc (ρ) coefficients.

***, ** and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

As presented in Table (4.24), the estimation of bivariate ARFIMA-FIGARCH (1,d,1) models for all volatility and flow's samples have realized estimated values of d_{vv} and d_{vf} that are significantly different from zero or one.

We can see in the sub-sample (CY1)/whole sample that the fractional variance parameters for volatility/flow are the highest values amongst all various sub-samples: (0.88 and 0.62). Even though, the values of the fractional differencing parameters for all other sub-samples are lower than the corresponding values in the sub-sample (CY1)/whole sample for volatility and flow respectively, they are significantly different from zero. As a result, the findings in both the case of volatility and flow are asymmetric.

Table 4.24. Variance Equation: Fractional Parameters (GKYZ Volatility)

Samples	Volatility d_{vv}	Flow d_{vf}
Whole Sample	**0.80 (0.07)***	**0.62 (0.09)***
Panel A:		
A	**0.76 (0.09)***	**0.33 (0.08)***
B	**0.42 (0.06)***	**0.39 (0.08)***
Panel B:		
UD1	**0.32 (0.07)***	**0.22 (0.03)****
UD2	**0.34 (0.07)***	**0.27 (0.09)***
UD3	**0.42 (0.08)***	**0.40 (0.07)***
UD4	**0.34 (0.06)***	**0.22 (0.07)***
UD5	**0.69 (0.08)***	**0.30 (0.06)***
Panel C:		
CY1	**0.88 (0.08)***	**0.23 (0.08)***
CY2	**0.67 (0.08)***	**0.41 (0.08)***

Notes: this table reports parameters' estimates for the GARCH Long Memory in the variance equation for volatility and flow respectively.

** and * Stand for significance at the 5% and 10% significant levels respectively.

4.7. Conclusion

In this chapter, we have examined the dynamic causalities between market return volatility and aggregate mutual fund flow. The variables under consideration were inextricably linked. We have detected different key behavioural features which were observed across the various univariate and bivariate specifications. We have taken into consideration the 2000 Dot-Com Bubble, the 2007 Financial Crisis as well as the 2009 European Sovereign Debt Crisis and presented how these changes have affected the relationships among these two variables.

Our contribution in this paper has been considered as follows: employing two different measurements of market return volatility (RS and GKYZ volatilities), obtaining a long span of daily data (from February 3rd 1998 to March 20th 2012), dividing the whole data set into three different cases with nine sub-samples, and applying the bivariate VAR model with univariate GARCH and bivariate CCC GARCH models in addition to the univariate ARFIMA-FIGARCH and bivariate CCC ARFIMA-FIGARCH processes in order to capture all the changeable results.

By employing the case of RS volatility with respect to the various models of GARCH and ARFIMA-FIGARCH, we have detected a bidirectional mixed feedback between market return volatility and aggregate mutual fund flow in the whole sample, sub-samples (UD1) and (CY1). Moreover, we have observed a negative bi-directional causality between volatility and flow in the sub-samples (B), (UD2), (UD4) and (UD5). However, a positive bi-directional causality has been noticed in the sub-samples (A), (UD3) and (CY2).

On the other hand, the following findings have been observed by employing the various cases of GARCH and ARFIMA-FIGARCH processes in order to assess the volatility-flow interaction in the case of GKYZ volatility. We have presented a negative bi-directional causality between stock market volatility and aggregate mutual fund flow in the whole sample, sub-samples (B), (UD2), (UD4), (UD5) and (CY1). Additionally, a bidirectional mixed feedback between volatility and flow has been observed in the sub-sample (UD1). Furthermore, a positive bi-directional causality has been found in the sub-samples (A), (UD3) and (CY2).

As a result, we have detected one exceptional issue in comparison between the results observed by employing RS volatility and GKYZ volatility. Whereas we have observed a bidirectional mixed feedback between volatility and flow in the whole sample and sub-sample (CY1), this relation has converted to be negative in the case of GKYZ volatility with respect to the aforesaid samples. Finally, most of the bidirectional effects have been found to be quite robust to the dynamics of the different GARCH models employed in this chapter.

Appendix 4

Appendix 4.A

VAR-GARCH (1,1) Model (RS Volatility)

In this section, we will present the results related to the bivariate VAR-GARCH model in the mean equation (including the cases of own effects and cross effects respectively) as well as the findings related to the variance equation (the GARCH coefficients).

Mean Equation

With the bivariate VAR model, we will examine the case of own effects (the impact of the lagged values of volatility (flow) on the volatility (flow) as dependent variables). In addition, we will examine the case of cross effects (the impact of the lagged values of flow obtained in the mean equation of volatility and vice versa).

Own Effects

Table (A.4.1.) reports the chosen lags for the own effects $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$. The $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$ columns report results for the volatility and flow equations respectively.

Volatility as a Dependent Variable

By examining the effect of the first seven lagged values of volatility on the volatility itself, the results –with respect to the obtained various samples- have been reported as Eq. (4.2) in Table (A.4.1.).

As an example, the equation (4.2) for the sub-sample (B) can be written as follows:

$$\Phi_{vv}(L) = \phi_{vv}^{2} L^{2}.$$

Table A.4.1. Mean Equations: AR Lags (Own Effects)

Samples	Eq. (4.2): $\Phi_{vv}(L)$	Eq. (4.3): $\Phi_{ff}(L)$
Whole Sample	**2,3,7**	**2,3**
Panel A:		
A	**2,3**	**2,3**
B	**2**	**2,3,4**
Panel B:		
UD1	**1,3**	**1,2,3,6**
UD2	**1,2**	**1,2**
UD3	**1,2**	**1,2**
UD4	**1,2**	**2,6**
UD5	**1,3,4**	**2,3**
Panel C:		
CY1	**2,6**	**1,2**
CY2	**1,2,3**	**1,2,3**

Notes: This table reports significant lags for the own effects $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$.

Flow as a Dependent Variable

By examining the effect of the first six lagged values of flow on the flow itself, the results – with respect to the previous mentioned samples- have been reported as Eq.(4.3) in Table (A.4.1.).

As an example, the equation (4.3) for the sub-sample (B) can be written as follows:

$$\Phi_{ff}(L) = \phi_{ff}^{2} L^{2} + \phi_{ff}^{3} L^{3} + \phi_{ff}^{4} L^{4}.$$

Cross Effects (The Volatility-Flow Linkage)

Table (A.4.2.) reports the chosen lags for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$. Following equation (4.4), $\Phi_{vf}(L)$ for the sub-sample (UD1) might be written as follows (Table A.4.2.):

$$\Phi_{vf}(L) = \phi_{vf}^{4} L^{4}.$$

For example, $\Phi_{fv}(L)$ for the whole sample could be represented with regards to the equation (4.5) as follows (Table A.4.2.):

$$\Phi_{fv}(L) = \phi_{fv}^{1} L^{1}.$$

Table A.4.2. Mean Equations: AR Lags (Cross Effects)

Samples	$\Phi_{vf}(L)$	$\Phi_{fv}(L)$
Whole Sample	**3**	**1**
Panel A:		
A	**6**	**7**
B	**2**	**1**
Panel B:		
UD1	**4**	**1**
UD2	**3**	**2**
UD3	**2**	**3**
UD4	**1**	**8**
UD5	**4**	**1**
Panel C:		
CY1	**4**	**1**
CY2	**7**	**4**

Notes: This table reports significant lags for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$.

The results presented in Table (A.4.3.) are identical to the results obtained by employing the bivariate VAR-CCC GARCH (1,1) model in the case of RS volatility. In brief, the positive effect of flow on volatility in the whole sample comes from the same effect in the sub-samples (A), (UD1) and (UD3). This aforementioned positive impact in the sub-sample (A) comes from sub-samples (UD1) and (UD3). Finally, this impact in the sub-samples (CY1) and (CY2) comes from sub-samples (UD1) and (UD3) respectively.

Table A.4.3. The Volatility-Flow Linkage: VAR-GARCH (RS Volatility)

Samples	Effect of Flow on Volatility	Effect of Volatility on Flow
Whole Sample	**Positive**	**Negative**
Panel A:		
A	**Positive**	**Positive**
B	**Negative**	**Negative**
Panel B:		
UD1	**Positive**	**Negative**
UD2	**Negative**	**Negative**
UD3	**Positive**	**Positive**
UD4	**Negative**	**Negative**
UD5	**Negative**	**Negative**
Panel C:		
CY1	**Positive**	**Negative**
CY2	**Positive**	**Positive**

Table (A.4.4.) reports the estimated coefficients for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$. The $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$ columns report results for the volatility and flow equations respectively.

Table A.4.4. The Coefficients of The Volatility-Flow Link (RS Volatility)

Samples	$\Phi_{vf}(L)$	$\Phi_{fv}(L)$
Whole Sample	**0.01 (0.00)***	**-0.22 (0.08)***
Panel A:		
A	**0.01 (0.01)**	**0.35 (0.15)**
B	**-0.01 (0.00)***	**-0.30 (0.10)***
Panel B:		
UD1	**0.01 (0.00)***	**-0.92 (0.25)***
UD2	**-0.01 (0.01)**	**-0.79 (0.28)***
UD3	**0.00 (0.00)***	**0.48 (0.24)**
UD4	**-0.01 (0.01)***	**-0.16 (0.14)**
UD5	**-0.01 (0.00)***	**-0.31 (0.11)***
Panel C:		
CY1	**0.01 (0.00)***	**-0.71 (0.20)***
CY2	**0.01 (0.01)**	**0.17 (0.16)***

Notes: This table reports parameters' estimates for the Φ_{vf} and Φ_{fv} respectively.

***, **, and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Variance Equation

With regards to equation (4.7), the analysing dynamic adjustments of the conditional variances for both volatility and flow can be seen in Table (A.4.5.). Table (A.4.5.) reports estimates of the ARCH and the GARCH parameters.

We can note that the sum of the coefficients of the ARCH parameter (α) and the GARCH parameter (β) for the total sample and all the other sub-samples respectively is less than one. Additionally, all the ARCH and GARCH coefficients are positive and significant in all various sub-samples.

Table A.4.5. Variance Equations: GARCH Coefficients (RS Volatility)

Samples	h_{1t}(Volatility)	h_{2t}(Flow)
Whole Sample		
α_i	**0.39 (0.45)***	**0.08 (0.03)*****
β_i	**0.43 (0.12)*****	**0.91 (0.03)*****
Panel A: **Sub-Sample A**		
α_i	**0.23 (0.09)****	**0.11 (0.03)*****
β_i	**0.76 (0.05)*****	**0.88 (0.03)*****
Sub-Sample B		
α_i	**0.07 (0.06)***	**0.10 (0.05)***
β_i	**0.92 (0.32)****	**0.89 (0.06)*****
Panel B: **Sub-Sample UD1**		
α_i	**0.10 (0.10)****	**0.05 (0.04)****
β_i	**0.59 (0.07)*****	**0.34 (0.17)****
Sub-Sample UD2		
α_i	**0.19 (0.13)***	**0.11 (0.02)*****
β_i	**0.74 (0.07)*****	**0.88 (0.02)*****
Sub-Sample UD3		
α_i	**0.14 (0.04)*****	**0.18 (0.08)****
β_i	**0.83 (0.04)*****	**0.72 (0.12)*****
Sub-Sample UD4		
α_i	**0.27 (0.16)****	**0.14 (0.06)****
β_i	**0.42 (0.09)*****	**0.85 (0.05)*****
Sub-Sample UD5		
α_i	**0.12 (0.07)***	**0.01 (0.02)***
β_i	**0.76 (0.25)****	**0.98 (0.02)*****
Panel C: **Sub-Sample CY1**		
α_i	**0.13 (0.09)***	**0.08 (0.01)*****
β_i	**0.64 (0.10)*****	**0.91 (0.01)*****
Sub-Sample CY2		
α_i	**0.14 (0.06)****	**0.15 (0.05)*****
β_i	**0.82 (0.06)*****	**0.83 (0.05)*****

Notes: this table reports parameters' estimates for the ARCH ($\boldsymbol{\alpha_i}$) and GARCH ($\boldsymbol{\beta_i}$) coefficients.

***, ** and * Stand for significance at the 1%, 5% and 10% significant levels.

The numbers in parentheses are standard errors.

Appendix 4.B

VAR-GARCH (1,1) Model (GKYZ Volatility)

In this section, we will present the results with respect to the bivariate VAR-GARCH model in the mean equation (including the cases of own effects and cross effects respectively) as well as the findings related to the variance equation (the GARCH coefficients).

Mean Equation

With the bivariate VAR model, we will assess the case of own effects (the effect of the lagged values of volatility (flow) on the volatility (flow) as dependent variables). In addition, we will examine the case of cross effects (the effect of the lagged values of flow obtained in the mean equation of volatility and vice versa).

Own Effects

Table (B.4.1.) reports the chosen lags for the own effects $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$. The $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$ columns report results for the volatility and flow equations respectively.

Volatility as a Dependent Variable

By examining the impact of the first five lagged values of volatility on the volatility itself, the results –with respect to the obtained various samples- have been reported as Eq. (4.2) in Table (B.4.1.).

Table B.4.1. Mean Equations: AR Lags (Own effects)

Samples	**Eq. (4.2):** $\Phi_{vv}(L)$	**Eq. (4.3):** $\Phi_{ff}(L)$
Whole Sample	**2,3,4**	**2,3**
<u>Panel A:</u>		
A	**1,2,3**	**2,3**
B	**2,3,4**	**2,3**
<u>Panel B:</u>		
UD1	**1,2,3**	**1,2,3**
UD2	**1,2,5**	**1,3**
UD3	**1,2,3**	**1,2,3**
UD4	**1,4**	**2,6**
UD5	**2,3**	**2,3,4,6**
<u>Panel C:</u>		
CY1	**1,2,3**	**1,2**
CY2	**1,2,3**	**1,2,3**

Notes: This table reports significant lags for the own effects $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$. As an example, the equation (4.2) for the sub-sample (UD4) can be written as follows: $\Phi_{vv}(L) = \phi_{vv}^{1}L^{1} + \phi_{vv}^{4}L^{4}$, and the equation (4.3) for the sub-sample (UD1) can be written as follows: $\Phi_{ff}(L) = \phi_{ff}^{1}L^{1} + \phi_{ff}^{2}L^{2} + \phi_{ff}^{3}L^{3}$.

Flow as a Dependent Variable

By examining the impact of the first six lagged values of flow on the flow itself, the results – with respect to the previous mentioned samples- have been reported as Eq.(4.3) in Table (B.4.1.).

Cross Effects (The Volatility-Flow Linkage)

Table (B.4.2.) reports the chosen lags for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$.

Table B.4.2. Mean Equations: AR Lags (Cross Effects)

Samples	$\Phi_{vf}(L)$	$\Phi_{fv}(L)$
Whole Sample	**7**	**1**
<u>Panel A:</u>		
A	**6**	**7**
B	**4**	**1**
<u>Panel B:</u>		
UD1	**8**	**1**
UD2	**1**	**1**
UD3	**1**	**1**
UD4	**2**	**5**
UD5	**1**	**1**
<u>Panel C:</u>		
CY1	**4**	**1**
CY2	**4**	**8**

Notes: This table reports significant lags for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$. Following equation (4.4), $\Phi_{vf}(L)$ for the sub-sample (B) might be written as follows: $\Phi_{vf}(L) = \phi_{vf}^{4}L^{4}$, and $\Phi_{fv}(L)$ for the sub-sample (B) could be represented with regards to the equation (4.5) as follows: $\Phi_{fv}(L) = \phi_{fv}^{1}L^{1}$.

The findings presented in Table (B.4.3.) are identical to the findings obtained by employing the bivariate VAR-CCC GARCH (1,1) model in the case of GKYZ volatility. Summarizing, the negative effect of flow on volatility in the whole sample comes from the same effect in the sub-samples (B), (CY1). This aforementioned negative impact in the sub-sample (B) is consistent with the same impact in the sub-samples (UD4) and (UD5). Finally, this impact in the sub-sample (CY1) comes from sub-sample (UD2).

Table B.4.3. The Volatility-Flow Linkage: VAR-GARCH (GKYZ Volatility)

Samples	Effect of Flow on Volatility	Effect of Volatility on Flow
Whole Sample	**Negative**	**Negative**
<u>Panel A:</u>		
A	**Positive**	**Positive**
B	**Negative**	**Negative**
<u>Panel B:</u>		
UD1	**Positive**	**Negative**
UD2	**Negative**	**Negative**
UD3	**Positive**	**Positive**
UD4	**Negative**	**Negative**
UD5	**Negative**	**Negative**
<u>Panel C:</u>		
CY1	**Negative**	**Negative**
CY2	**Positive**	**Positive**

Table (B.4.4.) reports the estimated coefficients for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$. The $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$ columns report results for the volatility and flow equations respectively.

Table B.4.4. The Coefficients of The Volatility-Flow Link (GKYZ Volatility)

Samples	$\Phi_{vf}(L)$	$\Phi_{fv}(L)$
Whole Sample	**-0.02 (0.00)*** **	**-0.22 (0.10)** **
<u>Panel A:</u>		
A	**0.01 (0.00)*** **	**0.41 (0.13)*** **
B	**-0.02 (0.00)*** **	**-0.41 (0.16)** **
<u>Panel B:</u>		
UD1	**0.01 (0.00)*** **	**-1.14 (0.34)*** **
UD2	**-0.02 (0.01)** **	**-0.38 (0.23)* **
UD3	**0.01 (0.00)** **	**0.38 (0.22)* **
UD4	**-0.02 (0.01)*** **	**-0.17 (0.12)* **
UD5	**-0.01 (0.00)* **	**-0.45 (0.15)*** **
<u>Panel C:</u>		
CY1	**-0.01 (0.00)** **	**-0.84 (0.22)*** **
CY2	**0.02 (0.01)* **	**0.17 (0.13)** **

Notes: This table reports parameters' estimates for the Φ_{vf} and Φ_{fv} respectively.

***, **, and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Variance Equation

With respect to equation (4.7), the analysing dynamic adjustments of the conditional variances for both volatility and flow can be seen in Table (B.4.5.). We can see that the sum of the coefficients of the ARCH parameter (α) and the GARCH parameter (β) for the total sample and all various sub-samples respectively is less than one. Moreover, all the ARCH and GARCH coefficients are positive and significant in all various sub-samples.

Table B.4.5. Variance Equations: GARCH Coefficients (GKYZ Volatility)

Samples	h_{1t}(Volatility)	h_{2t}(Flow)
Whole Sample		
α_i	**0.24 (0.11)****	**0.08 (0.03)*****
β_i	**0.57 (0.08)*****	**0.91 (0.03)*****
Panel A: **Sub-Sample A**		
α_i	**0.22 (0.09)****	**0.11 (0.03)*****
β_i	**0.76 (0.06)*****	**0.88 (0.03)*****
Sub-Sample B		
α_i	**0.11 (0.07)***	**0.10 (0.05)***
β_i	**0.88 (0.30)****	**0.89 (0.06)*****
Panel B: **Sub-Sample UD1**		
α_i	**0.09 (0.06)***	**0.02 (0.02)***
β_i	**0.48 (0.11)*****	**0.86 (0.10)*****
Sub-Sample UD2		
α_i	**0.08 (0.07)***	**0.10 (0.02)*****
β_i	**0.41 (0.19)****	**0.89 (0.02)*****
Sub-Sample UD3		
α_i	**0.14 (0.08)***	**0.19 (0.12)****
β_i	**0.82 (0.08)*****	**0.69 (0.19)*****
Sub-Sample UD4		
α_i	**0.06 (0.02)*****	**0.14 (0.06)****
β_i	**0.53 (0.04)*****	**0.85 (0.06)*****
Sub-Sample UD5		
α_i	**0.10 (0.10)***	**0.01 (0.01)****
β_i	**0.67 (0.27)****	**0.98 (0.02)*****
Panel C: **Sub-Sample CY1**		
α_i	**0.15 (0.11)****	**0.08 (0.01)*****
β_i	**0.62 (0.14)*****	**0.91 (0.01)*****
Sub-Sample CY2		
α_i	**0.12 (0.04)*****	**0.15 (0.05)*****
β_i	**0.87 (0.03)*****	**0.83 (0.05)*****

Notes: this table reports parameters' estimates for the ARCH ($\boldsymbol{\alpha_i}$) and GARCH ($\boldsymbol{\beta_i}$) coefficients.

***, ** and * Stand for significance at the 1%, 5% and 10% significant levels.

The numbers in parentheses are standard errors.

Appendix 4.C

ARFIMA-FIGARCH (1,d,1) Model (RS Volatility)

In this section, we will present the results with respect to the mean equation (including the cases of own effects and cross effects respectively) as well as the results related to the variance equation.

Mean Equation

In this sub-section, we will examine the case of own effects (the impact of the lagged values of volatility (flow) on the volatility (flow) as dependent variables). In addition, we will examine the case of cross effects (the impact of the lagged values of flow obtained in the mean equation of volatility and vice versa).

Own Effects

We take into consideration two fundamental points when choosing our models which are information criteria and significance of the coefficients. Table (C.4.1.) reports the chosen lags for the own effects $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$. The $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$ columns report results for the return and flow equations respectively.

Volatility as a Dependent Variable

By examining the effect of the first six lagged values of volatility on the volatility itself, the results –with respect to the obtained various samples- have been reported as Eq. (4.2) in Table (C.4.1.).

Table C.4.1. Mean Equations: AR Lags (Own Effects)

Samples	Eq. (4.2): $\Phi_{vv}(L)$	Eq. (4.3): $\Phi_{ff}(L)$
Whole Sample	**2,3**	**2,3**
Panel A:		
A	**1**	**2,3**
B	**1**	**2**
Panel B:		
UD1	**1,3**	**1,2,3**
UD2	**6**	**1,3**
UD3	**1**	**1,3**
UD4	**3**	**2,6**
UD5	**1**	**2,3**
Panel C:		
CY1	**2**	**1,2,4**
CY2	**1,3**	**1,2,3**

Notes: This table reports significant lags for of the own effects $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$. As an example, the lag polynomial $\Phi_{vv}(L)$ for the whole sample might be written as follows: $\Phi_{vv}(L) = \phi_{vv}^{2}L^{2} + \phi_{vv}^{3}L^{3}$. For instance, the lag polynomial $\Phi_{ff}(L)$ for the sub-sample (B) could be written as follows: $\Phi_{ff}(L) = \phi_{ff}^{2}L^{2}$.

Flow as a Dependent Variable

By examining the effect of the first six lagged values of flow on the flow itself, the results – with respect to the previous mentioned samples- have been reported as Eq.(4.3) in table (C.4.1.).

Cross Effects (The Volatility-Flow Linkage)

Table (C.4.2.) reports the chosen lags for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$ respectively.

Table C.4.2. Mean Equations: AR Lags (Cross Effects)

Samples	$\Phi_{vf}(L)$	$\Phi_{fv}(L)$
Whole Sample	**4**	**1**
Panel A:		
A	**2**	**7**
B	**7**	**1**
Panel B:		
UD1	**3**	**1**
UD2	**8**	**2**
UD3	**2**	**2**
UD4	**2**	**5**
UD5	**4**	**1**
Panel C:		
CY1	**6**	**1**
CY2	**8**	**3**

Notes: This table reports significant lags for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$. Following equation (4.4), $\Phi_{vf}(L)$ for the sub-sample (CY1) might be written as follows: $\Phi_{\mathrm{vf}}(L) = \phi_{vf}^{6} L^{6}$. Moreover, by applying equation (4.5), $\Phi_{fv}(L)$ for the sub-sample (B) could be represented as follows: $\Phi_{\mathrm{fv}}(L) = \phi_{fv}^{1} L^{1}$.

The results presented in Table (C.4.3.) are identical to the results obtained by employing the bivariate CCC ARFIMA-FIGARCH (1,d,1) model in the case of RS volatility. In brief, the negative impact of volatility on flow in the whole sample comes from the same impact in the sub-samples (B) and (CY1). This negative effect in the sub-sample (B) and (CY1) is consistent with the same effect in the sub-samples (UD4 and UD5) and sub-samples (UD1 and UD2) respectively.

Table C.4.3. The Volatility-Flow Linkage: ARFIMA-FIGARCH (RS Volatility)

Samples	Effect of Flow on Volatility	Effect of Volatility on Flow
Whole Sample	**Positive**	**Negative**
Panel A:		
A	**Positive**	**Positive**
B	**Negative**	**Negative**
Panel B:		
UD1	**Positive**	**Negative**
UD2	**Negative**	**Negative**
UD3	**Positive**	**Positive**
UD4	**Negative**	**Negative**
UD5	**Negative**	**Negative**
Panel C:		
CY1	**Positive**	**Negative**
CY2	**Positive**	**Positive**

Table (C.4.4.) reports the estimated coefficients for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$. The $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$ columns report results for the volatility and flow equations respectively.

Table C.4.4. The Coefficients of The Volatility-Flow Link (RS Volatility)

Samples	$\Phi_{vf}(L)$	$\Phi_{fv}(L)$
Whole Sample	**0.01 (0.00)****	**-0.19 (0.08)****
Panel A:		
A	**0.01 (0.00)***	**0.35 (0.15)****
B	**-0.01 (0.01)***	**-0.24 (0.10)****
Panel B:		
UD1	**0.01 (0.00)***	**-0.93 (0.26)*****
UD2	**-0.01 (0.01)***	**-0.54 (0.29)***
UD3	**0.00 (0.00)***	**0.49 (0.23)****
UD4	**-0.01 (0.01)***	**-0.19 (0.11)***
UD5	**-0.01 (0.01)***	**-0.28 (0.09)*****
Panel C:		
CY1	**0.01 (0.01)***	**-0.65 (0.18)*****
CY2	**0.01 (0.00)****	**0.22 (0.12)***

Notes: This table reports parameters' estimates for the Φ_{vf} and Φ_{fv} respectively.

***, **, and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Fractional Mean Parameters

Estimates of the volatility's fractional mean parameters are presented in Table (C.4.5.). In the whole sample and sub-sample (CY1), volatility generated very similar fractional mean parameters: (0.45 and 0.44). In the sub-samples (A), (B), (UD1) and (UD4), the estimated values of d_{mv} are higher than the corresponding value for the whole sample: (0.57, 0.56, 0.66 and 0.52). However, in the sub-samples (UD2, UD3, UD5 and CY2), the long memory mean parameters are lower than the corresponding value for the whole sample: (0.31, 0.32, 0.26 and 0.37).

Table C.4.5. Mean Equation: Fractional Parameters (RS Volatility)

Samples	Volatility d_{mv}
Whole Sample	**0.45 (0.06)*** **
Panel A:	
A	**0.57 (0.07)*** **
B	**0.56 (0.13)*** **
Panel B:	
UD1	**0.66 (0.33)*** **
UD2	**0.31 (0.08)*** **
UD3	**0.32 (0.03)*** **
UD4	**0.52 (0.07)*** **
UD5	**0.26 (0.05)*** **
Panel C:	
CY1	**0.44 (0.19)** **
CY2	**0.37 (0.05)*** **

Notes: this table reports parameters' estimates for the GARCH Long Memory in the mean equation for volatility.

*** and ** Stand for significance at the 1% and 5% significant levels respectively.

Variance Equation (FIGARCH Specifications)

Following equation (4.11), the analysing dynamic adjustments of the conditional variances of both volatility and flow can be seen in Table (C.4.6.). Table (C.4.6.) reports estimates of the ARCH and the GARCH parameters.

We note that the sum of the coefficients of the ARCH parameter (α) and the GARCH parameter (β) for the total sample and all various sub-samples respectively is less than one. Additionally, all the ARCH and GARCH coefficients are positive and significant in all various sub-samples. In other words, the GARCH coefficients in all cases have satisfied the sufficient and necessary conditions for the non-negativity of the conditional variances (see, for instance, Conrad and Haag, 2006,).

Table C.4.6. Variance Equations: GARCH Coefficients (RS Volatility)

Samples	h_{1t}(Volatility)	h_{2t}(Flow)
Whole Sample		
α_i	**0.07 (0.07)***	**0.02 (0.02)****
β_i	**0.71 (0.23)*****	**0.97 (0.02)*****
Panel A:		
Sub-Sample A		
α_i	**0.12 (0.02)*****	**0.13 (0.17)***
β_i	**0.49 (0.22)****	**0.86 (0.17)*****
Sub-Sample B		
α_i	**0.09 (0.08)****	**0.08 (0.04)***
β_i	**0.62 (0.10)***	**0.56 (0.08)*****
Panel B:		
Sub-Sample UD1		
α_i	**0.17 (0.03)*****	**0.05 (0.11)****
β_i	**0.75 (0.01)*****	**0.56 (0.22)****
Sub-Sample UD2		
α_i	**0.11 (0.10)****	**0.14 (0.03)*****
β_i	**0.85 (0.08)*****	**0.73 (0.06)*****
Sub-Sample UD3		
α_i	**0.15 (0.06)****	**0.18 (0.14)***
β_i	**0.73 (0.07)*****	**0.36 (0.19)***
Sub-Sample UD4		
α_i	**0.07 (0.10)***	**0.10 (0.04)*****
β_i	**0.87 (0.05)*****	**0.83 (0.07)*****
Sub-Sample UD5		
α_i	**0.11 (0.04)*****	**0.05 (0.01)*****
β_i	**0.87 (0.11)*****	**0.88 (0.01)*****
Panel C:		
Sub-Sample CY1		
α_i	**0.16 (0.13)***	**0.07 (0.00)*****
β_i	**0.56 (0.09)*****	**0.84 (0.04)*****
Sub-Sample CY2		
α_i	**0.19 (0.17)***	**0.09 (0.10)***
β_i	**0.63 (0.25)****	**0.73 (0.12)*****

Notes: this table reports parameters' estimates for the ARCH (α_i) and GARCH (β_i) coefficients.

***, ** and * Stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

As presented in Table (C.4.7.), the estimated of d_{vv} and d_{vf} have governed the long-run dynamics of the conditional heteroscedasticity. The estimation of univariate ARFIMA-FIGARCH (1,d,1) processes for all volatility and flow's samples have realized estimated values of d_{vv} and d_{vf} that are significantly different from zero or one. In other words, the conditional variances of the two variables have been characterized by the FIGARCH behavior.

In the sub-samples (UD2), (UD5) and (CY1) of volatility, we can notice that volatility has generated similar fractional variance parameters: (0.29). In addition, we find symmetric fractional differencing parameters in the sub-samples (B) and (UD3): (0.27). Moreover, the value of this coefficient for sub-sample (UD1) is the lowest value of the fractional variance parameter (0.17). However, although this estimated value is relatively small, it is significantly different from zero. Nevertheless, we notice that the highest value of the fractional differencing parameter is found in the sub-sample (CY2).

Moreover, flow has generated very similar fractional differencing parameters for the whole sample, sub-samples (A), (B), (UD2), (UD3), (UD5) and (CY1): (0.13, 0.15, 0.14, 0.17, 0.15, 0.18 and 0.18). We also notice that the highest value of the fractional differencing parameter for flow is found in the sub-sample (CY2). However, the value of this coefficient for sub-sample (UD1) is the lowest value of the fractional differencing parameter (0.17). However, although this estimated value is relatively small, it is remarkably different from zero. It is noteworthy that the previous two findings are the same in the case of volatility.

Table C.4.7. Variance Equation: Fractional Parameters (RS Volatility)

Samples	Volatility d_{vv}	Flow d_{vf}
Whole Sample	**0.24 (0.02)****	**0.13 (0.06)***
Panel A:		
A	**0.28 (0.08)***	**0.15 (0.09)***
B	**0.27 (0.08)***	**0.14 (0.03)****
Panel B:		
UD1	**0.17 (0.02)****	**0.07 (0.05)***
UD2	**0.29 (0.02)****	**0.17 (0.02)****
UD3	**0.27 (0.06)***	**0.15 (0.02)****
UD4	**0.18 (0.02)****	**0.09 (0.06)***
UD5	**0.29 (0.03)****	**0.18 (0.09)***
Panel C:		
CY1	**0.29 (0.09)***	**0.18 (0.09)***
CY2	**0.32 (0.07)***	**0.20 (0.03)****

Notes: this table reports parameters' estimates for the GARCH Long Memory in the variance equation for volatility and flow respectively.

** and * Stand for significance at the 5% and 10% significant levels respectively.

Appendix 4.D

ARFIMA-FIGARCH (1,d,1) Model (GKYZ Volatility)

In this section, we will show the findings related to the mean equation (including the cases of own effects and cross effects respectively) as well as the results related to the variance equation.

Mean Equation

In this sub-section, we will examine the case of own effects (the effect of the lagged values of volatility (flow) on the volatility (flow) as dependent variables). In addition, we will examine the case of cross effects (the effect of the lagged values of flow obtained in the mean equation of volatility and vice versa).

Own Effects

Table (D.4.1.) reports the chosen lags for the own effects $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$. The $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$ columns report results for the return and flow equations respectively.

Volatility as a Dependent Variable

By examining the effect of the first ten lagged values of volatility on the volatility itself, the results –with respect to the obtained various samples- have been reported as Eq. (4.2) in Table (D.4.1.).

Table D.4.1. Mean Equations: AR Lags (Own Effects)

Samples	Eq. (4.2): $\Phi_{vv}(L)$	Eq. (4.3): $\Phi_{ff}(L)$
Whole Sample	**1**	**2,3**
Panel A:		
A	**1**	**2,3**
B	**1**	**2**
Panel B:		
UD1	**1**	**1,2,3,4**
UD2	**10**	**1,3**
UD3	**1,4**	**2,3,4,5**
UD4	**3**	**2,6**
UD5	**1**	**2,3**
Panel C:		
CY1	**2**	**1,2**
CY2	**1**	**1,2,3**

Notes: This table reports significant lags for the own effects $\Phi_{vv}(L)$ and $\Phi_{ff}(L)$. As an example, the lag polynomial $\Phi_{vv}(L)$ for the sub-sample (UD3) might be written as follows: $\Phi_{vv}(L) = \phi_{vv}^{1}L^{1} + \phi_{vv}^{4}L^{4}$. For instance, the lag polynomial $\Phi_{ff}(L)$ for the sub-sample (B) could be written as follows: $\Phi_{ff}(L) = \phi_{ff}^{2}L^{2}$.

Flow as a Dependent Variable

By examining the effect of the first six lagged values of flow on the flow itself, the results –with respect to the previous mentioned samples- have been reported as Eq.(4.3) in table (D.4.1.).

Cross Effects (The Volatility-Flow Linkage)

Table (D.4.2.) reports the chosen lags for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$ respectively.

Table D.4.2. Mean Equations: AR Lags (Cross Effects)

Samples	$\Phi_{vf}(L)$	$\Phi_{fv}(L)$
Whole Sample	**4**	**1**
Panel A:		
A	**8**	**7**
B	**9**	**1**
Panel B:		
UD1	**10**	**1**
UD2	**1**	**1**
UD3	**4**	**1**
UD4	**2**	**4**
UD5	**1**	**1**
Panel C:		
CY1	**2**	**1**
CY2	**9**	**8**

Notes: This table reports significant lags for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$. Following equation (4.4), $\Phi_{vf}(L)$ for the sub-sample (CY1) might be written as follows: $\Phi_{\text{vf}}(L) = \phi_{vf}^{2}L^{2}$. Moreover, by applying equation (4.5), $\Phi_{fv}(L)$ for the sub-sample (B) could be represented as follows: $\Phi_{\text{fv}}(L) = \phi_{fv}^{1}L^{1}$.

The findings presented in Table (D.4.3.) are identical to the findings obtained by employing the bivariate CCC ARFIMA-FIGARCH (1,d,1) model in the case of GKYZ volatility. In brief, the negative impact of volatility on flow in the whole sample comes from the same impact in the sub-samples (B) and (CY1). This negative impact in the sub-sample (CY1) is consistent with the same impact in the sub-samples (UD1) and (UD2). However, the positive effect of volatility on flow in sub-samples (A) and (CY2) comes from the same effect in the sub-sample (UD3).

Table D.4.3. The Volatility-Flow Linkage: ARFIMA-FIGARCH (GKYZ Volatility)

Samples	Effect of Flow on Volatility	Effect of Volatility on Flow
Whole Sample	**Negative**	**Negative**
Panel A:		
A	**Positive**	**Positive**
B	**Negative**	**Negative**
Panel B:		
UD1	**Positive**	**Negative**
UD2	**Negative**	**Negative**
UD3	**Positive**	**Positive**
UD4	**Negative**	**Negative**
UD5	**Negative**	**Negative**
Panel C:		
CY1	**Negative**	**Negative**
CY2	**Positive**	**Positive**

Table (D.4.4.) reports the estimated coefficients for the cross effects $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$. The $\Phi_{vf}(L)$ and $\Phi_{fv}(L)$ columns report results for the volatility and flow equations respectively.

Table D.4.4. The Coefficients of The Volatility-Flow Link (GKYZ Volatility)

Samples	$\Phi_{vf}(L)$	$\Phi_{fv}(L)$
Whole Sample	**-0.01 (0.00)****	**-0.17 (0.09)***
Panel A:		
A	**0.01 (0.00)*****	**0.40 (0.12)*****
B	**-0.01 (0.01)***	**-0.35 (0.17)****
Panel B:		
UD1	**0.01 (0.00)****	**-1.22 (0.36)*****
UD2	**-0.03 (0.01)*****	**-0.42 (0.24)***
UD3	**0.01 (0.01)***	**0.51 (0.20)****
UD4	**-0.02 (0.01)****	**-0.15 (0.17)***
UD5	**-0.01 (0.00)****	**-0.48 (0.18)*****
Panel C:		
CY1	**-0.01 (0.00)***	**-0.83 (0.21)*****
CY2	**0.02 (0.01)***	**0.15 (0.13)****

Notes: This table reports parameters' estimates for the Φ_{vf} and Φ_{fv} respectively.

***, **, and * stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

Fractional Mean Parameters

Table (D.4.5.) presents the estimates of the volatility's fractional mean parameters. In the whole sample and sub-sample (B), volatility generated very similar fractional mean parameters: (0.57 and 0.58). In the sub-samples (UD1 up to UD5) and (CY1, CY2), the long memory mean parameters are lower than the corresponding value for the whole sample: (0.49, 0.47, 0.29, 0.48, 0.17, 0.46 and 0.34). However, in the sub-sample (A), the estimated values of d_{mv} is higher than the corresponding value for the whole sample: (0.66).

Table D.4.5. Mean Equation: Fractional Parameters (GKYZ Volatility)

Samples	Volatility d_{mv}
Whole Sample	**0.57 (0.11)*** **
Panel A:	
A	**0.66 (0.18)*** **
B	**0.58 (0.19)** **
Panel B:	
UD1	**0.49 (0.12)*** **
UD2	**0.47 (0.13)*** **
UD3	**0.29 (0.04)*** **
UD4	**0.48 (0.09)*** **
UD5	**0.17 (0.03)*** **
Panel C:	
CY1	**0.46 (0.14)*** **
CY2	**0.34 (0.05)*** **

Notes: this table reports parameters' estimates for the GARCH Long Memory in the mean equation for volatility.

*** and ** Stand for significance at the 1% and 5% significant levels respectively.

Variance Equation (FIGARCH Specifications)

Following equation (4.11), the analysing dynamic adjustments of the conditional variances of both volatility and flow can be seen in Table (D.4.6.). We note that the sum of the coefficients of the ARCH parameter (α) and the GARCH parameter (β) for the total sample and all various sub-samples respectively is less than one. Additionally, all the ARCH and GARCH coefficients are positive and significant in all various sub-samples.

Table D.4.6. Variance Equations: GARCH Coefficients (GKYZ Volatility)

Samples	h_{1t}(Volatility)	h_{2t}(Flow)
Whole Sample		
α_i	**0.22 (0.07)*** **	**0.17 (0.05)*** **
β_i	**0.68 (0.10)*** **	**0.76 (0.06)*** **
Panel A:		
Sub-Sample A		
α_i	**0.05 (0.04)***	**0.11 (0.04)*** **
β_i	**0.90 (0.05)*** **	**0.75 (0.06)*** **
Sub-Sample B		
α_i	**0.09 (0.05)***	**0.08 (0.05)***
β_i	**0.27 (0.18)***	**0.26 (0.06)*** **
Panel B:		
Sub-Sample UD1		
α_i	**0.11 (0.05)** **	**0.07 (0.09)** **
β_i	**0.35 (0.12)*** **	**0.70 (0.17)*** **
Sub-Sample UD2		
α_i	**0.05 (0.03)***	**0.05 (0.01)*** **
β_i	**0.76 (0.13)*** **	**0.82 (0.04)*** **
Sub-Sample UD3		
α_i	**0.19 (0.02)*** **	**0.20 (0.13)***
β_i	**0.65 (0.09)*** **	**0.45 (0.31)** **
Sub-Sample UD4		
α_i	**0.13 (0.09)***	**0.07 (0.03)*** **
β_i	**0.41 (0.17)** **	**0.72 (0.05)*** **
Sub-Sample UD5		
α_i	**0.15 (0.05)*** **	**0.05 (0.02)*** **
β_i	**0.33 (0.60)** **	**0.89 (0.01)*** **
Panel C:		
Sub-Sample CY1		
α_i	**0.13 (0.09)** **	**0.15 (0.03)*** **
β_i	**0.73 (0.16)*** **	**0.83 (0.04)*** **
Sub-Sample CY2		
α_i	**0.19 (0.16)***	**0.10 (0.11)** **
β_i	**0.39 (0.20)***	**0.73 (0.12)*** **

Notes: this table reports parameters' estimates for the ARCH ($\boldsymbol{\alpha_i}$) and GARCH ($\boldsymbol{\beta_i}$) coefficients.

***, ** and * Stand for significance at the 1%, 5% and 10% significant levels respectively.

The numbers in parentheses are standard errors.

As we notice in Table (D.4.7.), the estimation of univariate ARFIMA-FIGARCH (1,d,1) processes for all volatility and flow's samples have realized estimated values of d_{vv} and d_{vf} that are significantly different from zero or one.

We can see in the whole sample that the fractional variance parameters for volatility and flow are the highest values amongst all various sub-samples: (0.76 and 0.64). Even though, the values of the fractional differencing parameters for all sub-samples are lower than the corresponding value in the whole sample, they are significantly different from zero. As a result, the findings in the case of both volatility and flow are symmetric.

Table D.4.7. Variance Equation: Fractional Parameters (GKYZ Volatility)

Samples	Volatility d_{vv}	Flow d_{vf}
Whole Sample	**0.76 (0.06)***	**0.64 (0.06)***
Panel A:		
A	**0.42 (0.02)****	**0.24 (0.03)****
B	**0.33 (0.03)****	**0.34 (0.04)****
Panel B:		
UD1	**0.18 (0.03)****	**0.11 (0.09)***
UD2	**0.27 (0.06)***	**0.17 (0.09)***
UD3	**0.23 (0.08)***	**0.15 (0.02)****
UD4	**0.24 (0.04)****	**0.16 (0.06)***
UD5	**0.31 (0.09)***	**0.18 (0.07)***
Panel C:		
CY1	**0.21 (0.06)***	**0.12 (0.08)***
CY2	**0.49 (0.09)***	**0.37 (0.03)****

Notes: this table reports parameters' estimates for the GARCH Long Memory in the variance equation for volatility and flow respectively.

** and * Stand for significance at the 5% and 10% significant levels respectively.

Chapter Five

Concluding Remarks

In this thesis, we have considered issues in the field of trading volume, market return volatility, aggregate mutual fund flow and stock market return.

Chapter 2 has simultaneously investigated the dynamics and interactions of the volume-volatility link. We have been able to highlight different keys of behavioral features which were presented across the various univariate and bivariate formulations. We have considered several changes according to different chosen samples and discussed how these changes would affect the linkages amongst these two variables.

In particular, we have taken into account the case of structural breaks and employed different specifications of the univariate and bivariate GARCH processes in order to obtain all the changeable results.

We have employed a long span of daily data (1990-2012 for S&P 500, 1992-2012 for Dow Jones respectively) with four sub-sample periods. As a result, we have observed a mixed bidirectional feedback between volume and volatility (volatility affects volume positively whereas the reverse impact is of its opposite sign) in a variety of these selected sub-samples, while a negative (or positive) bidirectional linkage has been detected in the other sub-samples (volume has a negative or positive impact on volatility and vice versa).

In chapter 3, we have examined the dynamic interactions between stock market return and U.S. aggregate mutual fund flow. We have taken into consideration the 2000 Dot-Com Bubble, the 2007 Financial Crisis as well as the 2009 European Sovereign Debt Crisis, and discussed how these changes have affected the relationship among the variables mentioned previously.

We have obtained a long span of daily data (from February 3rd 1998 to March 20th 2012), divided the whole data set into three different cases with nine sub-samples and applied the bivariate VAR model with four different GARCH processes for the purpose of capturing all the changeable results.

We have observed a bidirectional mixed feedback between return and flow for the majority of the samples obtained. In particular, the lagged values of flow have negatively affected return whereas the lagged values of return have a positive impact on flow. Nevertheless, we have detected a positive bi-directional causality between flow and return with respect to some sub-periods of up-/down-market movement.

Chapter 4 has studied the dynamic causalities between market return volatility and aggregate mutual fund flow in the U.S. market. With similarity to the sub-samples obtained in chapter 3, we have additionally employed two different measurements of market return volatility.

We have observed a negative bi-directional causality between volatility and flow in most cases of up-/down- market movements. This means that volatility has a negative impact on flow (particularly flow into mutual funds) and vice versa. However, a positive bi-directional causality has been noticed in some sub-samples of cyclical behavior. In other words, flow (specifically flow out of mutual funds) has a positive effect on volatility and vice versa.

In addition, we have presented a bidirectional mixed feedback between flow and volatility in the rest of the estimations. More specifically, volatility affects flow negatively whereas the reverse impact is of its opposite sign.

Last but not least, most of the bidirectional effects have been found to be quite robust to the dynamics of the different GARCH models employed in this thesis.

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