Design of Non-Fragile State Estimators for Discrete Time-Delayed Neural Networks with Parameter Uncertainties

Yajing Yu, Hongli Dong*, Zidong Wang, Weijian Ren and Fuad E. Alsaadi

Abstract

This paper is concerned with the problem of designing a non-fragile state estimator for a class of uncertain discrete-time neural networks with time-delays. The norm-bounded parameter uncertainties enter into all the system matrices, and the network output is of a general type that contains both linear and nonlinear parts. The additive variation of the estimator gain is taken into account that reflects the possible implementation error of the neuron state estimator. The aim of the addressed problem is to design a state estimator such that the estimation performance is non-fragile against the gain variations and also robust against the parameter uncertainties. Sufficient conditions are presented to guarantee the existence of the desired non-fragile state estimators by using the Lyapunov stability theory and the explicit expression of the desired estimators is given in terms of the solution to a linear matrix inequality. Finally, a numerical example is given to demonstrate the effectiveness of the proposed design approach.

Index Terms

Discrete-time neural networks; Time-delayed neural networks; State estimation; Non-fragile state estimator; Uncertain systems.

I. Introduction

The past two decades have witnessed a surge of interest on both theoretical investigations and algorithm developments of the recurrent neural networks (RNNs) due mainly to their remarkable ability to exhibit dynamic temporal behavior in order to extract/detect/approximate functional information from complicated or imprecise data. So far, the RNNs have come to play a more and more important role in a variety of application areas including pattern recognition, image processing, optimization calculation and so on. On

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the other hand, since early 90s, the time-delay has been recognized as a ubiquitous phenomenon that could cause undesired oscillations or even instability in both biological and man-made neural networks [2]. As an active research branch, the study on the dynamical behaviors of RNN with time-delays has recently attracted an ever-increasing interest from many communities including neural science, signal processing and control engineering. Accordingly, in the past years, a large number of results have been available in the literature on the dynamics analysis issues (e.g. stability, synchronization and estimation) for RNNs with various kinds of time-delays such as constant, time-varying, discrete, distributed or mixed delays, see [1], [6], [11], [13], [18], [22], [28] for some representative works. Very recently, in [21], the passivity analysis problem has been investigated for a class of switched neural networks subject to stochastic disturbances and time-varying delays by using the average dwell-time approach. In [33], the adaptive synchronization problem has been addressed for memristor-based neural networks with time-varying delays by virtue of the differential equation theory with discontinuous right-hand sides.

In many practical applications, the actual values of the neural states of a RNN are vitally important. For example, in optimization problems, the RNNs can be implemented physically in designated hardware such as application-specific integrated circuits where the optimization is carried out in a truly parallel and distributed manner. In this case, the neuron states are closely related to the equilibria as well as the decision-making solutions. However, because of the large scale of the RNN as well as the implementation cost, the neuron states are often not fully observable and only the network outputs are available that contain partial information about the network states. As such, accurate estimation of the neuron states through measured outputs becomes an essential prerequisite for successful accomplishment of certain tasks such as approximation and optimization by using RNNs. In [34], the problem of state estimation has been first proposed for neural networks with time-varying delays. Since then, such a problem has received considerable research attention for both continuous- and discrete-time neural networks, see e.g. [15], [17], [23], [25], [30], [34], [35].

As is well known, the parameter uncertainties are often unavoidable in real systems due to modeling inaccuracies and/or changes in the environment. In recent years, a great deal of effort has been devoted to the robustness analysis for uncertain systems [7], [12], [16], [29]. Despite the rich body of literature on the state estimation issues for RNN with parameter uncertainties, most results obtained so far have been based on the assumption that the desired state estimator can be realized precisely. Such an assumption is, however, not necessarily true in certain engineering practice. When implementing a state estimator digitally, the implementation errors are often inevitable due probably to analogue-to-digital conversion, rounding errors, finite precision or internal noise. As discussed in [19], a small or even tiny drift/fluctuation/error with the parameter implementation of the designed controller/estimator could lead to unexpected fragility of the closed-loop system as a whole. In other words, the parameters of the actually implemented controller/estimator might have slight deviations from their expected values, and therefore designed controller/estimator should have certain degree of tolerance or non-fragility against the possible parameter deviations. In the past decade, the problem of non-fragile control has gained much attention with respect to the implementation errors in controllers/estimators [24], [27], [32], [36], [37]. However, when it comes to the discrete-time RNNs with time-delays, the non-fragile state estimation problem has not been fully studied yet, not to mention the case where the uncertainties also enter into other network parameters. It is, therefore, the main purpose of this paper to shorten such a gap.

Motivated by the above discussion, we aim to design a non-fragile state estimator for a class of discrete-time neural networks with parameter uncertainties. A sufficient condition for the asymptotic stability of the error dynamics of the state estimation is obtained and the gain matrix of the state estimator is derived by solving a linear matrix inequality (LMI). A numerical example is presented to demonstrate the effectiveness of the theoretical results obtained. The main contribution of this paper lies mainly on the problem addressed and the model proposed, which is twofold as follows: 1) the non-fragile state estimation problem is put forward for discrete time-delay neural networks in the presence of parameter uncertainties in all network parameters; and 2) the network output is quite general that is subject to nonlinear disturbances.

Notation: Throughout this paper, the superscript "T" represents the matrix transposition. $\mathbb{R}$ denotes the set of real numbers; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ denotes the set of all $n\times m$ real matrices. $\mathbb{N}^+$ stands for the sets of positive integers, I and 0 denote the identity matrix and zero matrix of appropriate dimensions, respectively. We use $X>0$ ($X<0$) to denote a positive-definite matrix (negative-definite matrix) X. $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. If A is a matrix, $\lambda_{\min}$ stands for the smallest eigenvalue of A. The notation $*$ always denotes the symmetric block in a symmetric matrix, and diag$\{\cdots\}$ stands for a block-diagonal matrix.

II. Problem formulation

In this paper, we consider a discrete-time neural network described by the following dynamical equations:

$$\begin{cases} x(k+1)=(C+\Delta C)x(k)+(A+\Delta A)f(x(k))+(B+\Delta B)f(x(k-d)) \\ x(k)=\phi(k), \quad k\in[-d,0) \end{cases} \tag{1}$$

where $x(k)=[x_1(k),x_2(k),\cdots,x_n(k)]^T\in\mathbb{R}^n$ is the neural state vector; $f(x(k))=[f_1(x_1(k)),f_2(x_2(k)),\cdots,f_n(x_n(k))]^T$ represents the nonlinear activation function with the initial condition $f(0)=0$; $C=\text{diag}\{c_1,c_2,\cdots,c_n\}$ is a positive diagonal matrix; $A=[a_{ij}]_{n\times n}$, $B=[b_{ij}]_{n\times n}$ are, respectively, the connection weight matrix and the delayed connection weight matrix; $d\geq 0$ denotes the discrete time-delay; $\phi(k)$ describes the initial condition. In addition, ΔA, ΔB and ΔC are time-varying parameter uncertainties that satisfy

$$\begin{bmatrix} \Delta A & \Delta B & \Delta C \end{bmatrix} = M_1F_1(k)\begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix} \tag{2}$$

where M_1, N_1, N_2, N_3 are known real-valued matrices with appropriate dimensions and $F_1(k)$ is an unknown matrix satisfying

$$F_1^T(k)F_1(k)\leq I, \quad \forall k\in\mathbb{N}^+ \tag{3}$$

Throughout this paper, we make the following assumption.

Assumption 1: For any α_1, $\alpha_2 \in \mathbb{R}$, $\alpha_1 \neq \alpha_2$, the activation function $f(\cdot)$ satisfies

$$\gamma_i^- \leq \frac{f_i(\alpha_1) - f_i(\alpha_2)}{\alpha_1 - \alpha_2} \leq \gamma_i^+, \quad (i = 1, 2, \cdots, n) \tag{4}$$

where γ_i^- and γ_i^+ are known constant scalars.

Remark 1: As shown in [22], the activation functions described in (1) are more general than the usual sigmoid functions and the commonly used Lipschitz conditions, where the constants γ_i^- and γ_i^+ are allowed to be positive, negative or zero. Therefore, such activation functions could be nonmonotonic and would induce less conservative results.

The outputs from the neural network (1) are of the following form:

$$y(k) = Dx(k) + Eg(k, x(k)) \tag{5}$$

where $y(k) = [y_1(k), y_2(k), \cdots, y_n(k)]^T$ represents the measurement output, D and E are known real-valued matrices with appropriate dimensions, and $g(k, x(k))$ is the neuron-dependent nonlinear disturbance that satisfies the following Lipschitz condition:

$$|g(k, \mu_1) - g(k, \mu_2)| \leq |G(\mu_1 - \mu_2)| \tag{6}$$

where G is a known constant matrix with appropriate dimension.

In order to estimate the state of the neural network (1) from available measurement output (5), we construct the following non-fragile state estimator:

$$\begin{cases} \hat{x}(k+1) = C\hat{x}(k) + Af(\hat{x}(k)) + Bf(\hat{x}(k-d)) + (K + \Delta K)\left[y(k) - D\hat{x}(k) - Eg(k, \hat{x}(k))\right] \\ \hat{x}(k) = \hat{\phi}(k), \quad k \in [-d, 0) \end{cases} \tag{7}$$

where $\hat{x}(k) \in \mathbb{R}^n$ is the state estimation, $\hat{\phi}(k)$ is the initial function of $\hat{x}(k)$, and K is the estimator gain parameter to be determined. ΔK quantifies the estimator gain variation in the following additive norm-bounded form:

$$\Delta K = M_2 F_2(k) N_4 \tag{8}$$

where M_2, N_4 are known real-valued matrices with appropriate dimensions and $F_2(k)$ is an unknown matrix satisfying

$$F_2^T(k) F_2(k) \leq I, \quad \forall k \in \mathbb{N}^+ \tag{9}$$

Letting the estimation error be $e(k) = x(k) - \hat{x}(k)$, the dynamics of the estimation error can be obtained from (1), (5) and (7) as follows:

$$\begin{aligned} e(k+1) =& (C - KD - \Delta KD)e(k) + \Delta Cx(k) + A\tilde{f}(e(k)) + \Delta Af(x(k)) + B\tilde{f}(e(k-d)) \\ &+ \Delta Bf(x(k-d)) - (K + \Delta K)E\tilde{g}(e(k)) \end{aligned} \tag{10}$$

where

$$\begin{aligned} \tilde{f}(e(k)) &:= f(x(k)) - f(\hat{x}(k)), \\ \tilde{f}(e(k-d)) &:= f(x(k-d)) - f(\hat{x}(k-d), \\ \tilde{g}(e(k)) &:= g(k, x(k)) - g(k, \hat{x}(k)). \end{aligned}$$

From (4) and (6), we have that

$$\gamma_i^- \leq \frac{\tilde{f}_i(e(k))}{e(k)} \leq \gamma_i^+ \tag{11}$$

$$|\tilde{g}(e(k))| \leq |Ge(k)| \tag{12}$$

Set

$$\begin{aligned}
\eta(k) &= \begin{bmatrix} x^T(k) & e^T(k) \end{bmatrix}^T, \\
\varphi(\eta(k)) &= \begin{bmatrix} f^T(x(k)) & \tilde{f}^T(e(k)) \end{bmatrix}^T, \\
\psi(\eta(k)) &= \begin{bmatrix} g^T(k, x(k)) & \tilde{g}^T(e(k)) \end{bmatrix}^T.
\end{aligned}$$

Considering (1) and (10), we obtain the following augmented system:

$$\eta(k+1) = \tilde{C}\eta(k) + \tilde{A}\varphi(\eta(k)) + \tilde{B}\varphi(\eta(k-d)) - \tilde{E}\psi(\eta(k)) \tag{13}$$

where

$$\tilde{C} = \bar{C} + \Delta\tilde{C}, \quad \tilde{A} = \bar{A} + \Delta\tilde{A}, \quad \tilde{B} = \bar{B} + \Delta\tilde{B}, \quad \tilde{E} = \bar{E} + \Delta\tilde{E},$$

$$\bar{C} = \mathrm{diag}\{C, C - KD\}, \quad \bar{A} = \mathrm{diag}\{A, A\}, \quad \bar{B} = \mathrm{diag}\{B, B\}, \quad \bar{E} = \mathrm{diag}\{0, KE\},$$

$$\Delta\tilde{C} = \begin{bmatrix} \Delta C & 0 \\ \Delta C & -\Delta KD \end{bmatrix}, \quad \Delta\tilde{A} = \begin{bmatrix} \Delta A & 0 \\ \Delta A & 0 \end{bmatrix}, \quad \Delta\tilde{B} = \begin{bmatrix} \Delta B & 0 \\ \Delta B & 0 \end{bmatrix}, \quad \Delta\tilde{E} = \begin{bmatrix} 0 & 0 \\ 0 & \Delta KE \end{bmatrix}.$$

Before proceeding further, we introduce the following definition.

Definition 1: The augmented system (13) is said to be asymptotically stable if, for any solution $\eta(k)$ of it, the following holds:

$$\lim_{k\to\infty} |\eta(k)|^2 = 0.$$

The objective of this paper is to design appropriate estimator parameter K for the state estimator (7) such that, in the presence of admissible gain variations as well as parameter uncertainties, the augmented system (13) is asymptotically stable, which implies that the error dynamics $e(k)$ tends to zero asymptotically.

III. Main Results

In this section, the stability is analyzed for the augmented system (13). A sufficient condition is given to guarantee that the augmented system (13) is asymptotically stable and then the explicit expression of the desired estimator gain is proposed in terms of the solution to certain matrix inequality derived according to the obtained condition.

Before stating our main results, we introduce the following lemmas.

Lemma 1: [3] (Schur Complement) Given constant matrices $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$ where $\mathcal{S}_1 = \mathcal{S}_1^{\mathrm{T}}$ and $0 < \mathcal{S}_2 = \mathcal{S}_2^{\mathrm{T}}$, then $\mathcal{S}_1 + \mathcal{S}_3^{\mathrm{T}}\mathcal{S}_2^{-1}\mathcal{S}_3 < 0$ if and only if

$$\begin{bmatrix} \mathcal{S}_1 & \mathcal{S}_3^{\mathrm{T}} \\ \mathcal{S}_3 & -\mathcal{S}_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\mathcal{S}_2 & \mathcal{S}_3 \\ \mathcal{S}_3^{\mathrm{T}} & \mathcal{S}_1 \end{bmatrix} < 0.$$

Lemma 2: [3] (S-procedure) Let $\mathcal{U}, \mathcal{V}(t), \mathcal{W}$ and $\mathcal{Q}$ be real matrices of appropriate dimensions with $\mathcal{Q}$ satisfying $\mathcal{Q} = \mathcal{Q}^T$. Then, for all $\mathcal{V}(t)\mathcal{V}^T(t) \leq I$,

$$\mathcal{Q} + \mathcal{U}\mathcal{V}(t)\mathcal{W} + \mathcal{W}^T\mathcal{V}^T(t)\mathcal{U}^T < 0$$

holds if and only if there exists a positive scalar μ such that

$$\mathcal{Q} + \mu^{-1}\mathcal{U}\mathcal{U}^T + \mu\mathcal{W}^T\mathcal{W} < 0.$$

Lemma 3: [22] Suppose that $\mathcal{B} = \mathrm{diag}\{\beta_1, \beta_2, \cdots, \beta_n\}$ is a positive-semidefinite diagonal matrix. Let $x = [x_1, x_2, \cdots, x_n]^T \in \mathbb{R}^n$ and $\mathcal{H}(x) = [h_1(x_1), h_2(x_2), \cdots, h_n(x_n)]^T$ be a continuous nonlinear function satisfying

$$l_i^- \leq \frac{h_i(m)}{m} \leq l_i^+, \quad m \neq 0, \quad m \in \mathbb{R}, \quad i = 1, 2, \cdots, n$$

with l_i^- and l_i^+ being constant scalars. Then

$$\begin{bmatrix} x \\ \mathcal{H}(x) \end{bmatrix}^T \begin{bmatrix} \mathcal{B}L_1 & -\mathcal{B}L_2 \\ -\mathcal{B}L_2 & \mathcal{B} \end{bmatrix} \begin{bmatrix} x \\ \mathcal{H}(x) \end{bmatrix} \leq 0$$

or

$$x^T\mathcal{B}L_1x - 2x^T\mathcal{B}L_2\mathcal{H}(x) + \mathcal{H}^T(x)\mathcal{B}\mathcal{H}(x) \leq 0$$

where $L_1 = \mathrm{diag}\{l_1^+l_1^-, l_2^+l_2^-, \cdots, l_n^+l_n^-\}$ and $L_2 = \mathrm{diag}\{\frac{l_1^+ + l_1^-}{2}, \frac{l_2^+ + l_2^-}{2}, \cdots, \frac{l_n^+ + l_n^-}{2}\}$.

Let us now consider the stability analysis problem for the augmented system (13).

Theorem 1: Consider the neural network model (1) and suppose that the estimator parameter K is given. The augmented system (13) is asymptotically stable if there exist a positive constant ε, symmetric positive definite matrices $P > 0$, $Q > 0$, $R > 0$ and four sets of diagonal matrices $F > 0$, $H > 0$, $X > 0$, $Y > 0$ satisfying the following inequality

$$\tilde{\Gamma} = \begin{bmatrix} \Omega_1 & * & * & * & * \\ 0 & \Omega_2 & * & * & * \\ \Omega_3 & 0 & \Omega_4 & * & * \\ \tilde{B}^TP\tilde{C} & -\Lambda_{22} & \tilde{B}^TP\tilde{A} & \Omega_5 & * \\ -\tilde{E}^TP\tilde{C} & 0 & -\tilde{E}^TP\tilde{A} & -\tilde{E}^TP\tilde{B} & \Omega_6 \end{bmatrix} < 0 \tag{14}$$

where

$$\bar{I} = \begin{bmatrix} 0 & I \end{bmatrix}, \quad \bar{G} = \begin{bmatrix} 0 & G \end{bmatrix}, \quad \Lambda_{11} = \begin{bmatrix} F\Lambda_1 & 0 \\ 0 & X\Lambda_1 \end{bmatrix}, \quad \Lambda_{21} = \begin{bmatrix} -F\Lambda_2 & 0 \\ 0 & -X\Lambda_2 \end{bmatrix},$$

$$\Lambda_{12} = \begin{bmatrix} H\Lambda_1 & 0 \\ 0 & Y\Lambda_1 \end{bmatrix}, \quad \Lambda_{22} = \begin{bmatrix} -H\Lambda_2 & 0 \\ 0 & -Y\Lambda_2 \end{bmatrix}, \quad \bar{F} = \begin{bmatrix} F & 0 \\ 0 & X \end{bmatrix}, \quad \bar{H} = \begin{bmatrix} H & 0 \\ 0 & Y \end{bmatrix},$$

$$\Gamma_1 = \tilde{C}^TP\tilde{C} - P + Q, \quad \Gamma_2 = \tilde{A}^TP\tilde{A} + R, \quad \Gamma_3 = \tilde{B}^TP\tilde{B} - R,$$

$$\Omega_1 = \Gamma_1 - \Lambda_{11} + \varepsilon\bar{G}^T\bar{G}, \quad \Omega_2 = -Q - \Lambda_{12}, \quad \Omega_3 = \tilde{A}^TP\tilde{C} - \Lambda_{21},$$

$$\Omega_4 = \Gamma_2 - \bar{F}, \quad \Omega_5 = \Gamma_3 - \bar{H}, \quad \Omega_6 = \tilde{E}^TP\tilde{E} - \varepsilon\bar{I}^T\bar{I},$$

$$\Lambda_1 = \text{diag}\{\gamma_1^+\gamma_1^-, \gamma_2^+\gamma_2^-, \cdots, \gamma_n^+\gamma_n^-\}, \quad \Lambda_2 = \text{diag}\left\{\frac{\gamma_1^+ + \gamma_1^-}{2}, \frac{\gamma_2^+ + \gamma_2^-}{2}, \cdots, \frac{\gamma_n^+ + \gamma_n^-}{2}\right\}.$$

Proof: Choose the following Lyapunov-Krasovskii functional for system (13):

$$V(x(k),k) = \eta^T(k)P\eta(k) + \sum_{i=k-d}^{k-1} \eta^T(i)Q\eta(i) + \sum_{i=k-d}^{k-1} \varphi^T(\eta(i))R\varphi^T(\eta(i)).$$

For $P > 0$, $Q > 0$ and $R > 0$, we have

$$\begin{aligned}
\Delta V(k) =& V(x(k+1),k+1) - V(x(k),k) \\
=& \eta^T(k+1)P\eta(k+1) + \sum_{i=k-d+1}^{k} \eta^T(i)Q\eta(i) + \sum_{i=k-d+1}^{k} \varphi^T(\eta(i))R\varphi(\eta(i)) \\
& - \eta^T(k)P\eta(k) - \sum_{i=k-d}^{k-1} \eta^T(i)Q\eta(i) - \sum_{i=k-d}^{k-1} \varphi^T(\eta(i))R\varphi(\eta(i)) \\
=& [\eta^T(k)\tilde{C}^T + \varphi^T(\eta(k))\tilde{A}^T + \varphi^T(\eta(k-d))\tilde{B}^T - \psi^T(\eta(k))\tilde{E}^T]P[\tilde{C}\eta(k) \\
& + \tilde{A}\varphi(\eta(k)) + \tilde{B}\varphi(\eta(k-d)) - \tilde{E}\psi(\eta(k))] - \eta^T(k)P\eta(k) + \eta^T(k)Q\eta(k) \\
& - \eta^T(k-d)Q\eta(k-d) + \varphi^T(\eta(k))R\varphi(\eta(k)) - \varphi^T(\eta(k-d))R\varphi(\eta(k-d)) \\
=& \xi^T(k)\Gamma\xi(k)
\end{aligned}$$

where

$$\xi(k) := [\eta^T(k) \quad \eta^T(k-d) \quad \varphi^T(\eta(k)) \quad \varphi^T(\eta(k-d)) \quad \psi^T(\eta(k))]^T,$$

$$\Gamma = \begin{bmatrix} \Gamma_1 & * & * & * & * \\ 0 & -Q & * & * & * \\ \tilde{A}^T P\tilde{C} & 0 & \Gamma_2 & * & * \\ \tilde{B}^T P\tilde{C} & 0 & \tilde{B}^T P\tilde{A} & \Gamma_3 & * \\ -\tilde{E}^T P\tilde{C} & 0 & -\tilde{E}^T P\tilde{A} & -\tilde{E}^T P\tilde{B} & \tilde{E}^T P\tilde{E} \end{bmatrix}.$$

Moreover, it follows from Assumption 1 and Lemma 3 that

$$\begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix}^T \begin{bmatrix} F\Lambda_1 & -F\Lambda_2 \\ -F\Lambda_2 & F \end{bmatrix} \begin{bmatrix} x(k) \\ f(x(k)) \end{bmatrix} \leq 0 \tag{15}$$

$$\begin{bmatrix} e(k) \\ \tilde{f}(e(k)) \end{bmatrix}^T \begin{bmatrix} X\Lambda_1 & -X\Lambda_2 \\ -X\Lambda_2 & X \end{bmatrix} \begin{bmatrix} e(k) \\ \tilde{f}(e(k)) \end{bmatrix} \leq 0 \tag{16}$$

$$\begin{bmatrix} x(k-d) \\ f(x(k-d)) \end{bmatrix}^T \begin{bmatrix} H\Lambda_1 & -H\Lambda_2 \\ -H\Lambda_2 & H \end{bmatrix} \begin{bmatrix} x(k-d) \\ f(x(k-d)) \end{bmatrix} \leq 0 \tag{17}$$

$$\begin{bmatrix} e(k-d) \\ \tilde{f}(e(k-d)) \end{bmatrix}^T \begin{bmatrix} Y\Lambda_1 & -Y\Lambda_2 \\ -Y\Lambda_2 & Y \end{bmatrix} \begin{bmatrix} e(k-d) \\ \tilde{f}(e(k-d)) \end{bmatrix} \leq 0 \tag{18}$$

From (15)-(18), we have immediately that

$$\begin{bmatrix} \eta(k) \\ \varphi(\eta(k)) \end{bmatrix}^T \begin{bmatrix} \Lambda_{11} & \Lambda_{21} \\ \Lambda_{21} & \bar{F} \end{bmatrix} \begin{bmatrix} \eta(k) \\ \varphi(\eta(k)) \end{bmatrix} \leq 0 \tag{19}$$

$$\begin{bmatrix} \eta(k-d) \\ \varphi(\eta(k-d)) \end{bmatrix}^T \begin{bmatrix} \Lambda_{12} & \Lambda_{22} \\ \Lambda_{22} & \bar{H} \end{bmatrix} \begin{bmatrix} \eta(k-d) \\ \varphi(\eta(k-d)) \end{bmatrix} \leq 0 \tag{20}$$

Furthermore, it follows from (6) that

$$\begin{aligned} \psi^T(\eta(k))\bar{I}^T\bar{I}\psi(\eta(k)) = [\bar{I}\psi(\eta(k))]^T[\bar{I}\psi(\eta(k))] &= \tilde{g}^T(e(k))\tilde{g}(e(k)) \\ &= |g(x(k)) - g(\hat{x}(k))|^2 \leq |Ge(k)|^2 = e^T(k)G^TGe(k) \\ &= \eta^T(k)\bar{G}^T\bar{G}\eta(k). \end{aligned}$$

Then, for a positive scalar ε, one can obtain

$$0 \leq \varepsilon[\eta^T(k)\bar{G}^T\bar{G}\eta(k) - \psi^T(\eta(k))\bar{I}^T\bar{I}\psi(\eta(k))] \tag{21}$$

From (19)-(20), it is not difficult to derive that

$$\begin{aligned} \Delta V(k) = \xi^T(k)\Gamma\xi(k) \leq & \xi^T(k)\Gamma\xi(k) - \begin{bmatrix} \eta(k) \\ \varphi(\eta(k)) \end{bmatrix}^T \begin{bmatrix} \Lambda_{11} & \Lambda_{21} \\ \Lambda_{21} & \bar{F} \end{bmatrix} \begin{bmatrix} \eta(k) \\ \varphi(\eta(k)) \end{bmatrix} \\ & - \begin{bmatrix} \eta(k-d) \\ \varphi(\eta(k-d)) \end{bmatrix}^T \begin{bmatrix} \Lambda_{12} & \Lambda_{22} \\ \Lambda_{22} & \bar{H} \end{bmatrix} \begin{bmatrix} \eta(k-d) \\ \varphi(\eta(k-d)) \end{bmatrix} \\ & + \varepsilon[\eta^T(k)\bar{G}^T\bar{G}\eta(k) - \psi^T(\eta(k))\bar{I}^T\bar{I}\psi(\eta(k))] \\ = & \xi^T(k)\tilde{\Gamma}\xi(k) \end{aligned}$$

In terms of inequality (14), we have

$$\Delta V(k) \leq -\lambda_{\min}(-\tilde{\Gamma})|\xi(k)|^2 < 0 \tag{22}$$

which implies

$$\Delta V(k) \leq -\lambda_{\min}(-\tilde{\Gamma})|\xi(k)|^2 \tag{23}$$

Given a positive integer m, the recursive sum of both sides of (23) from 0 to m leads to

$$V(x(m+1), m+1) - V(x(0), 0) \leq -\lambda_{\min}(-\tilde{\Gamma})\sum_{k=0}^{m}|\xi(k)|^2 \tag{24}$$

which results in

$$\sum_{k=0}^{m}|\xi(k)|^2 \leq \frac{1}{\lambda_{\min}(-\tilde{\Gamma})}V(x(0), 0) \tag{25}$$

Letting $m \to \infty$, we know that the series $\sum_{k=0}^{m}|\xi(k)|^2$ is convergent, which means

$$\lim_{k\to\infty}|\eta(k)|^2 = 0.$$

The proof is now complete. ■

Having conducted the estimating performance analysis in Theorem 1, we are now in a position to deal with the problem of designing estimator and the main results are given in the following theorem.

Theorem 2: There exists a non-fragile state estimator of the type (7) with ΔK satisfying (8) such that the augmented system (14) is asymptotically stable if there exist positive constants ε and ϵ, symmetric positive definite matrices $P = \mathrm{diag}\{P_1, P_2\} > 0$, $Q > 0$ and $R > 0$, four sets of diagonal matrices $F > 0$,

$H > 0$, $X > 0$, $Y > 0$ and a matrix Z with appropriate dimensions such that the following linear matrix inequality (LMI)

$$\begin{bmatrix} \Theta_1 & * & * & * & * & * & * \\ 0 & \Omega_2 & * & * & * & * & * \\ -\Lambda_{21} & 0 & \Theta_2 & * & * & * & * \\ 0 & -\Lambda_{22} & 0 & \Theta_3 & * & * & * \\ 0 & 0 & 0 & 0 & \Theta_4 & * & * \\ \Theta_5 & 0 & P\bar{A} & P\bar{B} & -Z_2 & -P & * \\ 0 & 0 & 0 & 0 & 0 & \mathcal{D}_1^T & -\epsilon I \end{bmatrix} < 0 \quad (26)$$

holds, where

$$\tilde{M}_1 = \begin{bmatrix} M_1 & 0 \\ M_1 & M_2 \end{bmatrix}, \quad \bar{M}_1 = \begin{bmatrix} M_1 & 0 \\ M_1 & 0 \end{bmatrix}, \quad \bar{M}_2 = \begin{bmatrix} 0 & 0 \\ 0 & M_2 \end{bmatrix},$$

$$Z_1 = \begin{bmatrix} 0 & 0 \\ 0 & ZD \end{bmatrix}, \quad Z_2 = \begin{bmatrix} 0 & 0 \\ 0 & ZE \end{bmatrix}, \quad \hat{C} = \mathrm{diag}\{C, C\},$$

$$\bar{N}_1 = \mathrm{diag}\{N_1, N_1\}, \quad \bar{N}_2 = \mathrm{diag}\{N_2, N_2\}, \quad \bar{N}_3 = \mathrm{diag}\{N_3, -N_4 D\}, \quad \bar{N}_4 = \mathrm{diag}\{N_4, N_4 E\},$$

$$\mathcal{D}_1 = \begin{bmatrix} P\tilde{M}_1 & 0 & P\bar{M}_1 & P\bar{M}_1 & P\bar{M}_2 & 0 \end{bmatrix}, \quad \Theta_1 = -P + Q - \Lambda_{11} + \varepsilon \bar{G}^T \bar{G} + \epsilon \bar{N}_3^T \bar{N}_3,$$

$$\Theta_2 = R - \bar{F} + \epsilon \bar{N}_1^T \bar{N}_1, \Theta_3 = -R - \bar{H} + \epsilon \bar{N}_2^T \bar{N}_2, \Theta_4 = -\varepsilon \bar{I}^T \bar{I} + \epsilon \bar{N}_4^T \bar{N}_4, \Theta_5 = P\hat{C} - Z_1.$$

and other parameters are defined as those in Theorem 1. In this case, the estimator gain K of the desired non-fragile state estimator (7) can be characterized by

$$K = P_2^{-1} Z \quad (27)$$

Proof: First, we denote

$$\Xi_1 = -P + Q - \Lambda_{11} + \varepsilon \bar{G}^T \bar{G}, \quad \Xi_2 = R - \bar{F}, \quad \Xi_3 = -R - \bar{H}$$

$$\Sigma_1 = \begin{bmatrix} \Xi_1 & * & * & * & * \\ 0 & \Omega_2 & * & * & * \\ -\Lambda_{21} & 0 & \Xi_2 & * & * \\ 0 & -\Lambda_{22} & 0 & \Xi_3 & * \\ 0 & 0 & 0 & 0 & -\varepsilon \bar{I}^T \bar{I} \end{bmatrix}, \quad \Sigma_2 = \begin{bmatrix} P\tilde{C} & 0 & P\tilde{A} & P\tilde{B} & -P\tilde{E} \end{bmatrix}$$

and then $\tilde{\Gamma}$ in (14) can be rewritten as

$$\tilde{\Gamma} = \Sigma_1 + \Sigma_2^T P^{-1} \Sigma_2 \leq 0$$

which, according to Lemma 1, is equivalent to

$$\begin{bmatrix} \Sigma_1 & \Sigma_2^{\mathrm{T}} \\ \Sigma_2 & -P \end{bmatrix} < 0 \quad (28)$$

Replacing $\tilde{C}$, $\tilde{A}$, $\tilde{B}$ and $\tilde{E}$ with $\bar{C}+\Delta\tilde{C}$, $\bar{A}+\Delta\tilde{A}$, $\bar{B}+\Delta\tilde{B}$ and $\bar{E}+\Delta\tilde{E}$ in (28), respectively, one obtains

$$\begin{bmatrix} \Xi_1 & * & * & * & * & * \\ 0 & \Omega_2 & * & * & * & * \\ -\Lambda_{21} & 0 & \Xi_2 & * & * & * \\ 0 & -\Lambda_{22} & 0 & \Xi_3 & * & * \\ 0 & 0 & 0 & 0 & -\varepsilon\bar{I}^T\bar{I} & * \\ P\bar{C} & 0 & P\bar{A} & P\bar{B} & -P\bar{E} & -P \end{bmatrix} + \mathcal{D}\mathcal{F}(k)\mathcal{N} + \mathcal{N}^T\mathcal{F}^T(k)\mathcal{D}^T \leq 0. \tag{29}$$

where

$$\mathcal{D} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \mathcal{D}_1^T \end{bmatrix}^T, \quad \mathcal{N} = \mathrm{diag}\left\{ \bar{N}_3 \quad 0 \quad \bar{N}_1 \quad \bar{N}_2 \quad -\bar{N}_4 \quad 0 \right\},$$
$$\mathcal{F}(k) = \mathrm{diag}\left\{ \mathrm{diag}\left\{F_1(k), F_2(k)\right\}, 0, \mathrm{diag}\left\{F_1(k), F_1(k)\right\}, \mathrm{diag}\left\{F_1(k), F_1(k)\right\}, \mathrm{diag}\left\{F_2(k), F_2(k)\right\}, 0 \right\}.$$

Employing Lemmas 1-2 again and applying the change of variable such that $K = P_2^{-1}Z$, we can see that $\tilde{\Gamma} = \Sigma_1 + \Sigma_2^T P^{-1}\Sigma_2 \leq 0$ is guaranteed by the LMI (26). Therefore, the system (13) is globally asymptotically stable and the proof is then completed. ■

Remark 2: It is worth noting that the non-fragile state estimation problem is introduced for the discrete-time RNNs with time-delays because the implementation of the designed controller/estimator is often inaccurate due to a variety of reasons such as analog-digital and digital-analog conversion, finite word length, finite resolution measuring instruments, programming errors, and roundoff errors in numerical computations. In Theorems 1-2, sufficient conditions for the existence and the derivation of the desired state estimator are provided, respectively. It is observed that all the network parameters, the sector-bounds of the activation function, the bounds of the Lipschitz-type nonlinearities in the network output as well as the bounds of the parameter uncertainties are all reflected in the main results. The obtained state estimator is capable of tolerating the admissible gain variations that might occur in the physical implementation. Conditions in Theorem 2 can be readily solved by utilizing the well-developed interior-point methods embedded in the Matlab software.

IV. An illustrative example

In this section, we present a simulation example to illustrate the effectiveness of the developed theoretical results.

Consider an uncertain delayed neural network (1) with the following parameters:

$$C=\begin{bmatrix}0.2&0&0\\0&0.3&0\\0&0&0.15\end{bmatrix}, A=\begin{bmatrix}-0.1&0.6&0.1\\0.7&0.2&0.2\\0.1&-0.2&0.3\end{bmatrix}, B=\begin{bmatrix}0.2&-0.2&0.1\\0.1&0.2&0.1\\0.4&-0.1&0.3\end{bmatrix}, M_1=\begin{bmatrix}0.3\\0.2\\-0.1\end{bmatrix},$$

$$M_2=\begin{bmatrix}0.2&0&0\\0&0.2&0\\0&0&0.2\end{bmatrix}, N_1=\begin{bmatrix}0.1&0.2&0.1\end{bmatrix}, N_2=\begin{bmatrix}0.1&-0.2&0.1\end{bmatrix},$$

$$N_3=\begin{bmatrix}0.1&0.2&-0.1\end{bmatrix}, N_4=\begin{bmatrix}0.3&0&0\\0&0.3&0\\0&0&0.3\end{bmatrix}, d=5, F_1(k)=\sin(0.6k), F_2(k)=\sin(0.6k).$$

The parameters of the network measurement (5) are given as

$$D=I,\quad E=1,\quad g(k,x(k))=0.4\cos(x(k)),\quad G=0.4I.$$

Take the activation functions as follows:

$$f_1(s)=-\tanh(0.4s),\quad f_2(s)=0.2\tanh(s),\quad f_3(s)=\tanh(0.6s).$$

It can be calculated that $\Lambda_1=0$, $\Lambda_2=\mathrm{diag}\{0.2,0.1,0.3\}$. By solving the LMI (26), the state estimator gain matrix can be obtained as follows:

$$K=\begin{bmatrix}0.1775&0.1924&0.0065\\0.1034&0.3321&-0.0152\\0.0173&-0.0427&0.1831\end{bmatrix}.$$

The simulation results are shown in Figs. 1–4, where the true states $x_1(k)$, $x_2(k)$, $x_3(k)$ and their estimates $\hat{x}_1(k)$, $\hat{x}_2(k)$, $\hat{x}_3(k)$ are depicted, respectively, in Fig. 1, Fig. 2 and Fig. 3 with the initial condition

$$x(0)=\begin{bmatrix}-0.12&0.28&-0.52\end{bmatrix}^T,\quad \hat{x}(0)=\begin{bmatrix}0.67&-1.14&0.2\end{bmatrix}^T.$$

Fig. 4 shows the dynamical evolution of the estimate error $e_1(k)$, $e_2(k)$, $e_3(k)$. The simulation results verify the effectiveness of the developed algorithm for designing the non-fragile state estimator for discrete-time neural networks with parameter uncertainties.

V. Conclusions

In this paper, the non-fragile state estimation problem has been studied for a class of uncertain discrete-time neural networks with time-delay. By employing the Lyapunov stability theory and the matrix analysis technique, a sufficient condition has been established to ensure that the dynamics of the estimation error achieves the asymptotic stability for all admissible parameter uncertainties as well as gain variations. The explicit expression of the gain matrix of the desired non-fragile estimator has been characterized by means of the feasibility to a linear matrix inequality. Finally, an example has been given to illustrate the

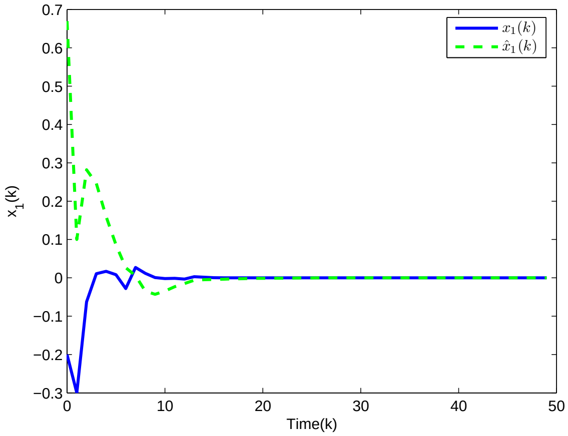

Fig. 1: The true states of $x_1(k)$ and its estimate.

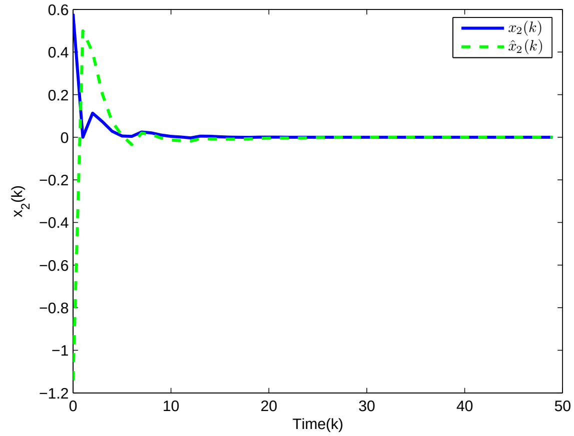

Fig. 2: The true states of $x_2(k)$ and its estimate.

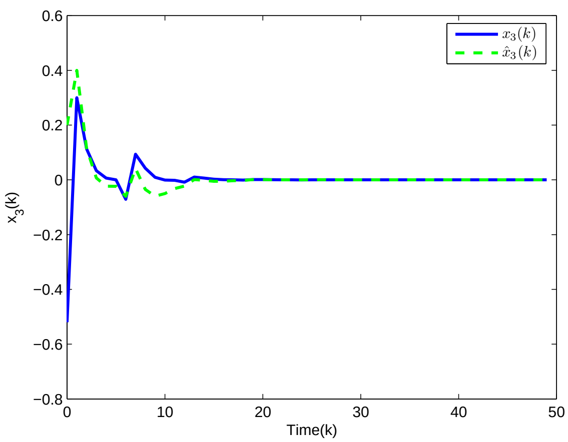

Fig. 3: The true states of $x_3(k)$ and its estimate.

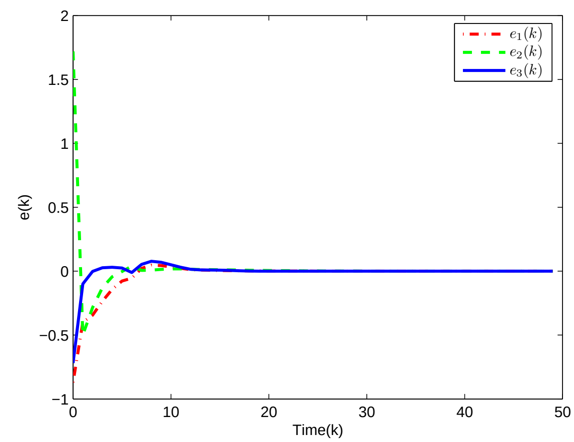

Fig. 4: The estimation errors.

usefulness of the developed state estimation approach. The results in this paper could be further extended to the non-fragile state estimation problems for discrete neural networks with more complicated network-induced phenomena such as fading measurements [4], [5], [10], [20], [26], missing measurements [8], sensor delays [9], randomly occurring faults [14] and mixed time-delays [31].

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