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Two Novel Nonlinear Companding Schemes With Iterative Receiver to Reduce PAPR in Multi-Carrier Modulation Systems

Tao Jiang, Wenbing Yao, Peng Guo, Yonghua Song, and Daiming Qu

***Abstract*—Companding transform is an efficient and simple method to reduce the Peak-to-Average Power Ratio (PAPR) for Multi-Carrier Modulation (MCM) systems. But if the MCM signal is only simply operated by inverse companding transform at the receiver, the resultant spectrum may exhibit severe in-band and out-of-band radiation of the distortion components, and considerable peak regrowth by excessive channel noises etc. In order to prevent these problems from occurring, in this paper, two novel nonlinear companding schemes with a iterative receiver are proposed to reduce the PAPR. By transforming the amplitude or power of the original MCM signals into uniform distributed signals, the novel schemes can effectively reduce PAPR for different modulation formats and sub-carrier sizes. Despite moderate complexity increasing at the receiver, but it is especially suitable to be combined with iterative channel estimation. Computer simulation results show that the proposed schemes can offer good system performances without any bandwidth expansion.**

***Index Terms*—Iterative receiver, multi-carrier modulation (MCM), nonlinear companding, peak-to-average power ratio (PAPR).**

I. Introduction

MULTI-CARRIER MODULATION (MCM) has become popular technique in various high-speed wireless systems owing to the high spectrum efficiency and channel robustness and has been used in many wireless communication standards such as various Digital Audio Broadcasting (DAB), Terrestrial Digital Video Broadcasting (DVB-T) and Digital Subscriber Loop systems (xDSL) [1], [2]. These systems use Discrete Multi-Tone (DMT) over wire media and Orthogonal Frequency Division Multiplexing (OFDM) for wireless communication.

According to the central limit theorem, the superposition of many carriers in multi-tone signaling leads to a Gaussian-like density with a high Peak-to-Average Power Ratio (PAPR). So the amplitude distribution of MCM signals with random input data is approximately Gaussian for large number of carriers. Therefore the MCM signals will occasionally present very high peaks, namely the PAPR is very high, which require a high dynamic range of the Analog-to-Digital Converter (ADC) and analog front end in absence of any clipping or peak reduction techniques. This would result in inefficient amplifiers (with excessive power dissipation) and expensive transceivers [2].

Recently, many reduction PAPR have been proposed for MCM systems, such as clipping and filtering [3], window shaping [4], block coding [5], [6], Partial Transmit Sequence (PTS) [7], [8], and Selective Mapping (SLM) [9], [10], phase optimization [11], tone reservation and injection [12], [13], among which, nonlinear companding transform schemes are the most attractive ones due to good system performance and low complexity and no bandwidth expansion [14]–[16].

Companding transform is a type of nonlinear process that may lead to significant distortion and performance loss by companding noise. Companding noise can be defined that the noises are caused by the peak regrowth after D/A conversion to generate in-band distortion and out-band noise, by the excessive channel noises magnified after inverse companding transform etc. For out-of-band noise, it needs to be filtered and oversampled. For in-band distortion and channel noises magnified, they need to iterative estimation. Unlike Additive White Gaussian Noise (AWGN), companding noise is generated by a process known and that can be recreated at the receiver, and subsequently be removed. Based on this observation and the analysis of the companding process, two novel nonlinear companding schemes with iterative receiver are proposed for companded and filtered MCM signals.

In [17], author has paid attention to the spectral regrowth and proposed that it can be estimate by an analytical expression for spectrum on the basis of the Price's formula at the output of the smooth limiter and proposed it can be implemented by a filter in the frequency domain as a miltiplier bank. In order to restore the information at the receiver, the values of companding function parameters must be submitted to the receiver with the information, and it will expand systemic bandwidth. At the same time, we know the error function is a single-valued function, but its inversion function is no more and may not restore the information correctly for channels noise and companding noise. Moreover it is key and important that how to estimate and cancel the companding noise at the receiver. In this paper, we derive the accurate companding function based on the Gaussian distribution of MCM signals and correlative mathematic theory. In order to cancel the companding noise and channel noise, we apply an iterative receiver in the MCM system. Iterative receiver can also improve other system performances.

The rest of this paper is organized as follows. In Section II, a typical companded MCM with iterative receiver system is given, and how to estimate and cancel the companding noise in MCM systems according to the iterative processing are illuminated. Then, two novel nonlinear companding schemes are proposed and analyzed in Section III. In Section IV, the performances of

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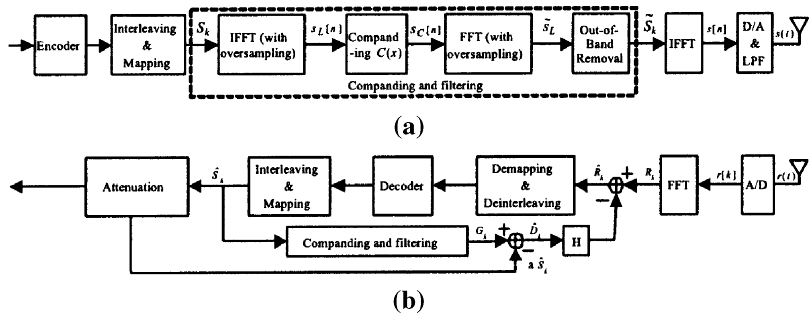

Fig. 1. Block diagram of companded MCM system with iterative receiver.

the proposed nonlinear companding schemes with the given iterative receiver over AWGN and fading channels are studied and compared with the μ-law companding scheme through computer simulations, followed by conclusions in Section V.

II. Theoretical Analysis of Companded MCM System With Iterative Receiver

Fig. 1 shows the block diagram of a typical companded MCM system with iterative receiver for PAPR reduction. The incoming bit stream is packed into x bits per symbol to form a complex number S_k, where x is determined by the QAM or PSK signal constellation and let N denote the number of sub-carriers used for parallel information transmission so that S_k $(0 \leq k \leq N-1)$ can be considered as the k^{th} complex modulated symbol in a block of N information symbols. For a real sequence output at the Inverse Fast discrete Fourier Transform (IFFT), the complex input to the IFFT has Hermitian symmetry and is considered as follows [18]

$$S_{N-k} = S_k^* \tag{1}$$

where $k = 0, \cdots, (N/2) - 1$ and $*$ denotes the complex conjugate. In practice, the carriers at DC and Nyquist frequency are not used as usual, which means when N is even and $S_k = a_k + j \cdot b_k$, the $S_0 = a_0 = b_0 = 0$ and $S_{(N/2)} = 0$ [19].

An oversampled MCM signal can be obtained by padding $\{S_k\}_{k=0}^{N-1}$ with $(L-1)N$ zeros and taking the IFFT, where L is the oversampling factor. The discrete time MCM signals with L oversampling over one symbol interval $s_L[n]$ is then expressed

$$s_L[n] = \frac{1}{\sqrt{N}} \sum_{k=1}^{\frac{N}{2}-1} \left(a_k cos \frac{2\pi kn}{LN} - b_k sin \frac{2\pi kn}{LN} \right) \tag{2}$$

where $n = 0, 1, \cdots, LN - 1$.

From the central limit theorem it follows that for large values of N (e.g. $N \geq 64$), $s_L[n]$ become Gaussian distributed with the Probability Density Function (PDF) as follows

$$p_{s_L[n]}(s) = \frac{1}{\sqrt{2\pi}\sigma} exp \left\{ -\frac{s^2}{2\sigma^2} \right\} \tag{3}$$

where σ^2 is the variance of the MCM signals. So the signals $s_L[n]$ has distribution with the Cumulative Distribution Function (CDF) as following

$$F_{s_L[n]}(s) = \frac{1}{2} \left(1 + erf \left(\frac{s}{\sqrt{2}\sigma} \right) \right) \tag{4}$$

where $erf(\cdot)$ is the error function with

$$erf(x) = \int_0^x \frac{2}{\sqrt{\pi}} e^{-t^2} dt \tag{5}$$

The PAPR of MCM signals in one symbol period is then defined as

$$PAPR(s_L[n]) = 10 \log_{10} \frac{Max \left\{ |s_L[n]|^2 \right\}}{P_{av}} \tag{6}$$

where P_{av} is the average power of the MCM signals.

How to perform the filtering digitally on the MCM symbol at the transmitter, [20] has described in detailed. To reduce peak power regrowth and distortion, the time domain signal is usually oversampled by $L > 2$. Following oversampling, the amplitude or power of the time domain signal samples are companded by a nonlinear transform function with the signals phase unchanged. Then companded signal $s_C(n)$ $(0 \leq n \leq LN - 1)$ is given by

$$s_C[n] = C(s_L[n]) \tag{7}$$

where $C(\cdot)$ is the companding function.

Then the MCM signals in frequency domain can be obtained by taking $\{s_C[n]\}_{n=0}^{LN-1}$ LN-point FFT. After the out-of band removed, N-point IFFT, Low-Pass Filter (LPF) and D/A converter, the continuous-time MCM signals $s(t)$ can be obtained.

According to [21], the extension of the Bussgang theorem to complex or real Gaussian inputs can give the separateness of a nonlinear output as the sum of a useful attenuated input replica and an uncorrelated nonlinear distortion noise [20], [22], [23]. So the companded MCM signal $\{s_C[n]\}_{n=0}^{LN-1}$ can be modeled as the aggregate of an attenuated signal component and companding noise $\{d_n\}_{n=0}^{LN-1}$

$$s_C[n] = \alpha \cdot s_L[n] + d_n, \quad n = 0, \cdots, LN - 1 \tag{8}$$

where α is attenuation coefficient, which is a time invariant for stationary input processes. Although the MCM signal $s_L[n]$ is not stationary, it has been shown in [22], [24] that an MCM signal guarantees α to be time invariant.

α has been given in [24]

$$\alpha = \frac{E\{s_C[n] s_L^*[n]\}}{E\{s_L[n] s_L^*[n]\}} \tag{9}$$

Consequently, we can apply this theory to reconstruct the useful information, namely to estimate the $s_L[n]$. According to [23], an alternative signal reconstruction approach, which

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attempts to restore the companded signal to it noncompanded form, can be applied to MCM system with a iterative receiver.

To remove the out-of-band components resulting from companding, the time domain samples (8) are converted back to the frequency domain by taking FFT. Using (8), seeing the Fig. 1, the terms $\hat{S}_k$ can be expressed

$$\hat{S}_k = \alpha \cdot S_k + D_k, \quad k = 0, \cdots, LN-1 \tag{10}$$

where $\{S_k\}_{k=0}^{LN-1}$ and $\{D_k\}_{k=0}^{LN-1}$ are respectively, the FFT of $\{s_L[n]\}_{n=0}^{LN-1}$ and $\{d_n\}_{n=0}^{LN-1}$ in (8). In particular, $\{D_k\}_{k=0}^{LN-1}$ is the sequence representing the companding noise in the frequency domain. Out-of-band components are removed by processing only the in-band-components through an N-point IFFT, please see the Fig. 1. For simplicity, no guard interval is considered because it has no bearing on the analysis in this paper.

Assuming perfect synchronization and following FFT, the signal at the receiver is

$$R_k = H_k(\alpha S_k + D_k + W_k) \tag{11}$$

where H_k is the complex channel gain of the k-th subcarrier assumed to be perfectly known, and W_k is AWGN channel.

The main idea of companding noise cancellation scheme is to iterative process at the receiver using detected symbols, then estimate and cancel the frequency domain companding noise caused by it. The iterative receiver is described as following with reference to Fig. 1.

Step 1) Channel observations $\{R_k\}_{k=0}^{N-1}$ are decoded and detected. Let decisions of the transmitted sequence be denoted $\{\hat{S}_k\}_{k=0}^{N-1}$;

Step 2) The sequence $\{\hat{S}_k\}_{k=0}^{N-1}$ is then processed through two branches. One branch regenerates the attenuated frequency domain samples of the noncompanded signals $\{\alpha\hat{S}_k\}_{k=0}^{N-1}$. The other branch regenerates the companded signals at the receiver by passing $\{\hat{S}_k\}_{k=0}^{N-1}$ through the same companding and filtering process as at the transmitter. Denote regenerated companded samples by $\{\hat{G}_k\}_{k=0}^{N-1}$. Similar to (10), these companded signals can be represented as the sum of an attenuated noncompanded signal $\alpha\hat{S}_k$ and the companding noise $\hat{D}_k$

$$G_k = \alpha\hat{S}_k + \hat{D}_k, \quad k = 0, \cdots, N-1 \tag{12}$$

Since G_k and $\hat{S}_k$ are observable and α can be computed from (9), the companding noise can be estimated according to

$$\hat{D}_k = G_k - \alpha\hat{S}_k, \quad k = 0, \cdots, N-1 \tag{13}$$

Step 3) The estimated clipping noise terms $\hat{D}_k$ are subtracted from the current channel observation to obtain the refined channel observation for the next iteration

$$\hat{R}_k = R_k - H_k\hat{D}_k, \quad k = 0, \cdots, N-1 \tag{14}$$

Step 4) When the estimation of the companding noise components $\{\hat{D}_k\}_{k=0}^{N-1}$ is found to become increasingly accurate and the receiver performance is improved, all iteration processing are over. Otherwise, go back to step 1) and replace $\{R_k\}_{k=0}^{N-1}$ with $\{\hat{R}_k\}_{k=0}^{N-1}$.

Obviously, each iteration requires a pair of IFFT/FFT operations. In the numerical results provided in [23], it is shown that no more than two of the iterations are required, implying that the iterative receiver incurs only a moderate increase in the complexity.

III. Two Novel Nonlinear Companding Schemes

In this section, we propose two new nonlinear companding techniques, that can effectively reduce the PAPR of transmitted (companded) MCM signals by transforming the statistics of the amplitudes or power of these signals into uniform distribution. The new schemes also have the advantage of maintaining a constant average power level in the nonlinear companding operation. The strict linearity requirements on HPA can then be partially relieved.

Let us denote X and Y as random variables (i.i.d) representing the amplitudes of the inputs and outputs signals of the companding function $C_1(\cdot)$ with the CDF's (Cumulative Distribution Functions) marked $F_X(x)$ and $F_Y(y)$, respectively. Since Y is to be desired the uniform distribution in the interval $[0, h_1]$ $(h_1 > 0)$, the CDF's of Y is given by

$$F_Y(y) = \frac{y}{2h_1} + \frac{1}{2} \quad (0 \le y \le h_1) \tag{15}$$

We know that the $F_X(x)$ and $F_Y(y)$ are strictly monotone increasing functions so as to have corresponding inverse functions. At the same time, the companding function $C_1(\cdot)$ is also restricted to be a strictly monotone increasing function and has its inverse function. When these conditions are satisfied, we can educe these conclusions as following

$$\begin{aligned} F_X(x) &= Prob\{X \le x\} \\ &= Prob\{C_1(X) \le C_1(x)\} \\ &= F_Y(C_1(x)) \end{aligned} \tag{16}$$

So, the companding function $C_1(\cdot)$ is given by the following identity

$$C_1(x) = F_Y^{-1}[F_X(x)] \tag{17}$$

Substituting (4) and (15) into (17) shows that

$$C_1(x) = h_1 \cdot erf\left(\frac{x}{\sqrt{2}\sigma}\right) \quad (0 \le x \le 1) \tag{18}$$

The positive constant h_1 determines the average power of the output signals. In order to keep the input and output signals at the same average power level, we let

$$h_1 = \sqrt{\frac{E\left[|s_L[n]|^2\right]}{E\left[\left|erf\left(\frac{s_L[n]}{\sqrt{2}\sigma}\right)\right|^2\right]}} \tag{19}$$

Similarly, when we want to transform the power of MCM signals into uniform distribution, another companding function can be obtained as following

$$C_2(x) = sgn(x) \cdot \sqrt{h_2 \cdot erf\left(\frac{|x|}{\sqrt{2}\sigma}\right)} \tag{20}$$

where $sgn(\cdot)$ is the sign function.

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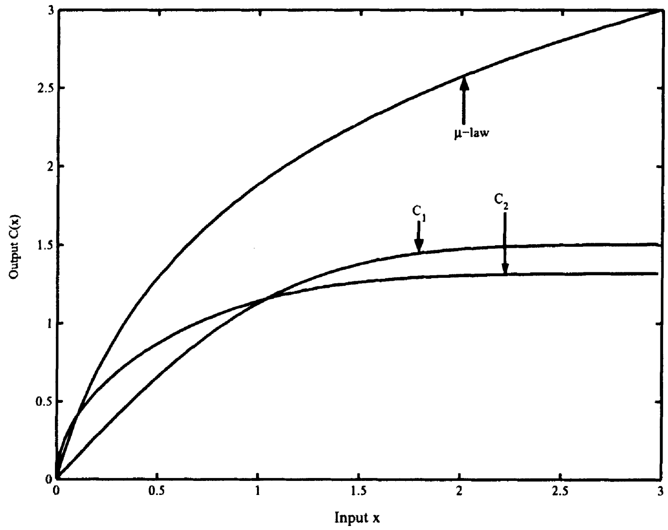

Fig. 2. The nonlinear companding functions $C(\cdot)$ transforming the MCM signals into uniform distributions.

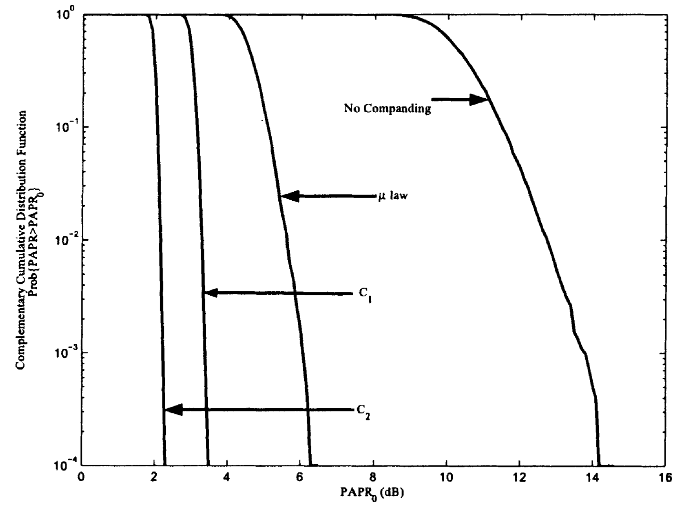

Fig. 3. The CCDF's of original MCM signals and companded signals ($\mathrm{N} = 256$, $\mathrm{L} = 4$, 16 QAM).

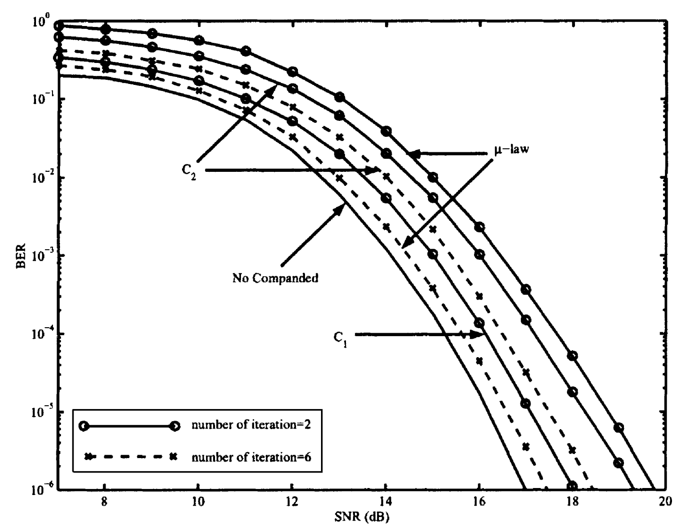

Fig. 4. BER performance of the proposed nonlinear companding with iterative receiver over AWGN channel.

In order to keep the average power of the output signals the same with that of the input signals, the constant h_2 should be given to

$$h_2 = \frac{E\left[\left(|s_L[n]|\right)^2\right]}{E\left[erf\left(\frac{x_L[n]}{\sqrt{2}\sigma}\right)\right]} \tag{21}$$

Fig. 2 shows the nonlinear companding function $C_1(x)$ and $C_2(x)$, from that we can see the proposed schemes can compress large input signals and expand small signals simultaneously. While the μ-law companding scheme can only enlarge small signals and does not change the signal peaks, which leads to a higher average power level of output signals [14]–[16].

We know, from Section II, how to calculate the value of α is a key problem for the proposed nonlinear companding schemes to apply the iterative receiver. According to (9), for the proposed two novel companding schemes, the α is given as following

$$\alpha = \frac{1}{\sigma^2} \int_{-\infty}^{\infty} x \cdot C(x) \cdot p(x) dx \tag{22}$$

When $\sigma^2 = 1$, respectively substituting the (3), (18) and (20) into (22) show that

$$\alpha = \begin{cases} \frac{h_1}{\pi}, & for\ C_1(x) \\ \frac{h_2 \cdot \sqrt{h_2}}{\pi}, & for\ C_2(x) \end{cases} \tag{23}$$

IV. Numerical Results and Discussions

To verify the performance of the two novel schemes in the reduction of PAPR and the system performances, numerical simulation results are presented for MCM signals over AWGN and fading channels, with that the number of subcarriers $N = 256$ and the oversampling factor $L = 4$ and the modulation is 16-QAM. Transmitted MCM signals are companded by two novel nonlinear companding transform and filtered respectively.

Fig. 3 shows respectively the Complementary Cumulative Distribution Functions (CCDF) of PAPR for 10^4 random original MCM symbols generated, μ-law companded signals, and the companded signals with $C_1(\cdot)$, $C_2(\cdot)$. When $CCDF = 10^{-4}$, the PAPR are 14.3 dB, 6.4 dB, 3.6 dB and 2.4 dB for the original MCM signals, μ-law companded signals, C_1 and C_2 companded signals respectively. Obviously, the signals companded by the nonlinear functions with $C_1(\cdot)$, $C_2(\cdot)$ can reduce the PAPR greater than that of μ-law companding function.

Fig. 4 depicts the performance of Bit-Error-Ratio (BER) versus Signal-to-Noise Ratio (SNR) of actual signals companded by different companding schemes over the AWGN channel, with comparisons to that of the siganls without any companding scheme and to the iterative receiver with companding noise cancellating and signal reconstruction as discussed in Section II and Section III. From Fig. 4, it is observed that for $C_1(\cdot)$ companding scheme, it is only about 1.01 dB loss compared to the case without any companding scheme applied is obtained after only two iteration at $BER = 10^{-6}$. But for μ-law companding scheme, it also can obtain an improvement of about 0.52 dB relative to $C_1(\cdot)$, but it needs 6

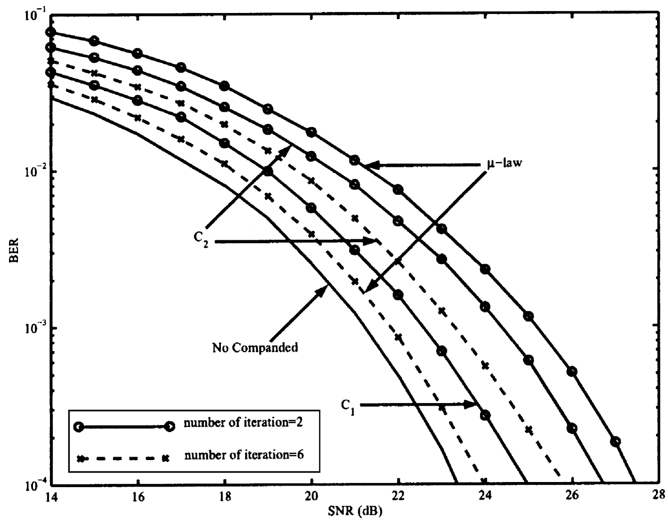

Fig. 5. BER performance of the proposed nonlinear companding with iterative receiver over Rayleigh fading channel.

iterations to converge. When the number of iteration equal to 2, it is about 19.72 dB for the μ-law companding scheme, but it doesn't cause to converge. For $C_2(\cdot)$ companding scheme, it converges to 6 iteration and it is about 1.42 dB loss compared to that of MCM signals without any companding scheme applied.

Fig. 5 shows the BER performance of the proposed nonlinear companding schemes with iterative receiver over a Rayleigh fading channel with an exponentially decaying power delay profile, with normalized delay spread equal to 2 [23]. It can be observed that after two iteration, $C_1(\cdot)$ converges and the BER performance of the reconstructed MCM signals can be restored to about 1.62 dB of the noncompanded case at $BER = 10^{-4}$. For $C_2(\cdot)$ companding scheme, it converges to 6 iteration and the reconstructed signal performs worse by about 3.39 dB compared to that of MCM signals without any companding scheme applied. For μ-law companding scheme, it has better BER performance than that of $C_1(\cdot)$ and $C_2(\cdot)$, its signal reconstruction performance better by about 1.01 dB than that of $C_1(\cdot)$, but it need much time to converge.

These simulations demonstrate that the proposed nonlinear companding schemes with iterative receiver can remove of the companding noise so as to significantly restore the system performance. Only considering the convergence speed, the $C_1(\cdot)$ is the best scheme; but μ-law has the best BER performance and $C_2(\cdot)$ has the best PAPR reduction performance. When all comes to all, $C_1(\cdot)$ is the best optimal scheme for reduction PAPR in MCM systems when it is be combined with the algorithms for iterative channel estimation.

V. Conclusion

In this letter, we propose two nonlinear companding transform schemes to efficiently reduce the PAPR of the transmitted signals in MCM systems. Two novel and effective nonlinear companding transform techniques with iterative receiver are proposed and evaluated to mitigate the PAPR for the MCM systems in this paper. By transforming the amplitude or power of the original MCM signals into uniform distributed signals, the novel schemes can effectively reduce PAPR for different modulation formats and sub-carrier sizes. Despite moderate complexity increasing at the receiver, but it is especially suitable to be combined with iterative channel estimation. Simulation results have shown that proposed nonlinear companding schemes could offer better system performance than that of μ-law companding scheme, although $C_2(\cdot)$ scheme has a little worse BER performance.

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