Event-Based H_∞ Filter Design for A Class of Nonlinear Time-Varying Systems with Fading Channels and Multiplicative Noises

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***Abstract*—In this paper, a general event-triggered framework is developed to deal with the finite-horizon H_∞ filtering problem for discrete time-varying systems with fading channels, randomly occurring nonlinearities and multiplicative noises. An event indicator variable is constructed and the corresponding event-triggered scheme is proposed. Such a scheme is based on the relative error with respect to the measurement signal in order to determine whether the measurement output should be transmitted to the filter or not. The fading channels are described by modified stochastic Rice fading models. Some uncorrelated random variables are introduced, respectively, to govern the phenomena of state-multiplicative noises, randomly occurring nonlinearities as well as fading measurements. The purpose of the addressed problem is to design a set of time-varying filter such that the influence from the exogenous disturbances onto the filtering errors is attenuated at the given level quantified by a H_∞-norm in the mean square sense. By utilizing stochastic analysis techniques, sufficient conditions are established to ensure that the dynamic system under consideration satisfies the H_∞ filtering performance constraint, and then a recursive linear matrix inequality (RLMI) approach is employed to design the desired filter gains. Simulation results demonstrate the effectiveness of the developed filter design scheme.**

***Index Terms*—Event-triggered mechanism; Finite-horizon filtering; Fading measurements; Multiplicative noise; Nonlinear time-varying systems.**

I. Introduction

For decades, filtering or state estimation techniques have been playing an important role in a variety of application areas such as target tracking, image processing, signal processing and control engineering, and a great number of important results have been reported in the literature, see, for example [1], [9], [13], [16], [17], [25], [30] and the references therein. Among the existing filtering methods, the H_∞ filtering approach is closely related to many robustness problems such as stabilization and sensitivity minimization of uncertain systems, and has therefore gained persistent research attention. For example, the H_∞ filtering problem has been investigated for a variety of complex dynamic systems including linear uncertain systems [3], Markovian jumping systems [20], fuzzy systems [18], time-delay systems [9], [10], stochastic systems [21] and nonlinear systems [24], etc. It is worth pointing out that, although fruitful results have been available for H_∞ filter design, most of them have been concerned with time-invariant systems only. On the other hand, virtually almost all models for real-time systems are time-varying over a finite-horizon and the corresponding filtering process could provide a better transient performance especially when the noise inputs are nonstationary [4], [24], [28]. As such, it makes more sense to consider filter design problems for time-varying systems over a finite-horizon.

Nonlinear control has been a mainstream of research topics due primarily to the fact that nonlinearity is a ubiquitous feature in a large class of practical systems and, if not properly coped with, the nonlinearity would inevitably degrade the system performance or even lead to the instability of the controlled systems. As discussed in [6], [7], [29], in today's popular networked systems such as the internet-based three-tank system for leakage fault diagnosis, the occurrence of nonlinearities is often of *random nature* resulting from sudden environment changes, intermittent transmission congestion, random failure and repairs of components, etc. Accordingly, the so-called randomly occurring nonlinearities (RONs) have started to gain some research interest and several initial results have been reported on the filtering problems subject to *additive noises*, see e.g. [4], [24]. Note that many plants may be modeled by systems with *multiplicative noises* and some characteristics of nonlinear systems can be closely approximated by models with multiplicative noises rather than by linearized models. In the context of nonlinear finite-horizon H_∞ filtering, the results on state-multiplicative noises have been very few, and this constitutes partial motivation for the present research on the H_∞ filtering issue for the time-varying stochastic systems with RONs, exogenous disturbance and state-multiplicative noises.

So far, most available filter algorithms have implicitly adopted the time-triggered strategy whose communication interval is designed *a priori* to reduce the complexity for analysis and design. Such a communication strategy, however, does not consider efficient usage of limited communication resources such as channel bandwidth or capacity in the network environment. To alleviate the unnecessary waste of computation and communication resources in a conventional time-triggered

This work was supported in part by the National Natural Science Foundation of China under Grants 61329301, 61333012, 61422301 and 61374127, the Engineering and Physical Sciences Research Council (EPSRC) of the U.K., the Royal Society of the U.K., and the Alexander von Humboldt Foundation of Germany.

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strategy, the event-triggered strategy has recently been proposed in [19] where the signal is transmitted only when certain conditions are satisfied. In comparison with conventional time-triggered communication, event-triggering allows a considerable reduction of the network resource occupancy while maintaining the guaranteed filtering performance. Clearly, when energy saving becomes a concern, the event-triggered strategy stands out as a competent candidate because of its capability of reducing the data communication frequency and network bandwidth usages. In the past few years, a growing number of results have been reported on the applications of event-based strategies to various engineering systems such as networked control systems [15], [19], sensor networks [26] and neural networks [22], etc. However, when it comes to the filtering or state estimation problems, the corresponding results have been relatively few, most of which have been concerned with the implementation problems rather than the system analysis and synthesis issues.

On another active research front, due to the rapid development of network technologies, network-induced phenomena such as packet dropouts [21], [28], communication delays [9] and signal quantization [12] have been well studied for filtering and control problems of networked systems. However, the network-induced channel fading problem has received little attention despite its practical significance in wireless mobile communications. Generally speaking, the main causes for fading effects are the multi-path propagation and the shadowing from obstacles, which are widely regarded as a kind of channel unreliability described by a random process reflecting the random changes of amplitude and phase of the transmitted signal. If not dealt with adequately, the phenomenon of network-induced channel fading would inevitably deteriorate the filtering performance of systems under investigation. To date, some pioneering work has appeared in the literature concerning networked control systems with fading channels, see [5] and the references therein. Nevertheless, the corresponding event-triggered filtering problem for time-varying systems with fading measurements has not yet been fully investigated, not to mention the case when the combined influences from both the RONs and the state-multiplicative noises are also involved. It is, therefore, the main purpose of this paper to shorten such a gap by addressing the event-based finite-horizon filtering problem for nonlinear time-varying systems with fading channels and multiplicative noises.

Motivated by the above discussions, in this paper, we aim to provide a systematic approach to the understanding, analysis and design of the event-based filters for time-varying systems with fading channels, RONs and multiplicative noise. The event-triggered scheme is based on the relative error with respect to the measurement signal and the fading channels are described by modified stochastic Rice fading models. Several uncorrelated random variables are introduced to cater for the random occurrences of the state-multiplicative noises, RONs and fading measurements. Some sufficient conditions are established, via intensive stochastic analysis, to guarantee the existence of the desired filter gains, and then such finite-horizon filter gains are obtained by solving sets of recursive matrix inequalities. A simulation example is finally presented to illustrate the effectiveness of the proposed design scheme. *The main contributions of this paper are highlighted as follows:*

1) *An event indicator variable is introduced to reflect the event-triggered information in the filter analysis with the hope of decreasing the data transmission frequency and also reduce conservatism in the filter design.*
2) *The event-triggered filter algorithm is proposed for discrete time-varying nonlinear stochastic systems with fading channels, randomly occurring nonlinearities and multiplicative noise. The system model addressed is quite comprehensive, hence reflecting reality more closely.*
3) *The developed finite-horizon filter design algorithm is recursive and is thus suitable for online applications.*

The rest of this paper is organized as follows: In Section II, the discrete time-varying nonlinear stochastic system with fading channels, randomly occurring nonlinearities and multiplicative noise is introduced and the problem under consideration is formulated. In Section III, the design problem of the event-based finite-horizon filtering problem is solved and a simulation example is given in Section IV to demonstrate the main results obtained. Finally, we conclude the paper in Section V.

Notation. The notation used here is standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are real symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). M^T represents the transpose of the matrix M. $\mathbf{0}_n$ (or simply $\mathbf{0}$) represents n-dimensional zero matrix. The n-dimensional identity matrix is denoted as I_n or simply I, if no confusion is caused. diag$\{\cdots\}$ stands for a block-diagonal matrix. $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|\, y\}$ will, respectively, denote expectation of the stochastic variable x and expectation of x conditional on y. $\mathrm{Prob}\{\cdot\}$ means the occurrence probability of the event "$\cdot$". In symmetric block matrices, "$*$" is used as an ellipsis for terms induced by symmetry. The symbol $\otimes$ denotes the Kronecker product. $\mathbf{1}_n = [1, 1, \ldots, 1]^T \in \mathbb{R}^n$. Matrices, if they are not explicitly specified, are assumed to have compatible dimensions.

II. Problem Formulation

Consider a discrete time-varying nonlinear stochastic system described by the following state-space model:

$$\begin{cases} x(k+1) = \big(A(k) + \displaystyle\sum_{i=1}^{r} w_i(k)A_i(k)\big)x(k) + \alpha(k)g(k, x(k)) \\ \qquad\qquad + D_1(k)v(k) \\ \quad \tilde{y}(k) = C(k)x(k) + D_2(k)v(k) \\ \quad z(k) = L(k)x(k) \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ represents the state vector; $\tilde{y}(k) \in \mathbb{R}^{n_y}$ is the process output; $z(k) \in \mathbb{R}^{n_z}$ is the signal to be estimated; $w_i(k) \in \mathbb{R}$ $(i = 1, 2, ..., r)$ with $w_i(k) \sim \mathcal{N}(0, 1)$; $v(k) \in \mathbb{R}^{n_v}$ is a deterministic disturbance noise that belongs to $l_2([0,N]$ where $l_2[0,N]$ denotes the space of square-summable sequences; $A(k)$, $A_i(k)$, $C(k)$, $D_1(k)$, $D_2(k)$ and $L(k)$ are known, real, time-varying matrices with appropriate dimensions.

The nonlinear vector-valued function $g:[0,N]\times\mathbb{R}^{n_x}\to\mathbb{R}^{n_x}$ is continuous, and satisfies $g(k,0)=0$ and the following sector-bounded condition:

$$\begin{aligned}&\big[g(k,x)-g(k,y)-\Phi(k)(x-y)\big]^T\big[g(k,x)-g(k,y)\\&-\Psi(k)(x-y)\big]\le 0\end{aligned}\tag{2}$$

for all $x,\ y\in\mathbb{R}^{n_x}$, where $\Phi(k)$ and $\Psi(k)$ are real matrices with appropriate dimensions.

The variable $\alpha(k)$ in (1), which accounts for the randomly occurring nonlinearity phenomena, is a Bernoulli distributed white sequences taking values on 0 or 1 with

$$\text{Prob}\{\alpha(k)=1\}=\bar{\alpha},\ \text{Prob}\{\alpha(k)=0\}=1-\bar{\alpha},\tag{3}$$

where $\bar{\alpha}\in[0,1]$ is a known constant.

In this paper, we consider an unreliable wireless network medium utilized for the signal transmission. In this case, the fading channels become a concern and the actually measured output $y(k)$ is described by

$$y(k)\ =\ \sum_{s=0}^{l_k}\beta_s(k)\tilde{y}(k-s)+D_3(k)\xi(k)\tag{4}$$

where $l_k=\min\{l,\ k\}$ with l being the given number of paths, $\xi(k)\in l_2[0,N]$ is an external disturbance, and $\beta_s(k)$ ($s=0,1,\cdots,l_k$) are the channel coefficients that are mutually independent random variables taking values on the interval $[0,1]$ with $\mathbb{E}\{\beta_s(k)\}=\bar{\beta}_s$ and $\text{Var}\{\beta_s(k)\}=\nu_s$.

For simplicity, we set $\{\tilde{y}(k)\}_{k\in[-l,-1]}=0$, $C(k)_{k\in[-l,-1]}=0$ and $[v^T(k)\ \ \xi^T(k)]_{k\in[-l,-1]}=0$.

Remark 1: The Rice fading model (4), which is capable of accounting for channel fading, time-delay and data dropout simultaneously, has been widely utilized in the area of signal processing and remote control. Also, it can be seen from (1) that both the parameter system matrices $A_i(k)$ ($i=1,2,\ldots,r$) and the nonlinear function $g(k,x(k))$ enter the system in probabilistic ways depicted by the random variable $w_i(k)$ and $\alpha(k)$, separately. As such, the system model described in (1)-(4) could better reflect the engineering practice in networked environments.

For the purpose of reducing data communication frequency, the event generator is constructed which uses the previously measurement output to determine whether the newly measurement output will be sent out to the filter or not. In this paper, such an event generator function $f(.,.)$ is defined as follows:

$$f(\sigma(k),\delta)=\sigma^T(k)\Omega\sigma(k)-\delta y^T(k)\Omega y(k)\tag{5}$$

where $\sigma(k):=y(k_i)-y(k)$ with $y(k_i)$ being the measurement at the latest event time k_i and $y(k)$ is the current measurement. Ω is a symmetric positive-definite weighting matrix and $\delta\in[0,1)$ is the threshold.

The execution (i.e. the transmission of the measurement output to the filter) is triggered as long as the condition

$$f(\sigma(k),\delta)>0\tag{6}$$

is satisfied. Therefore, the sequence of event-triggered instants $0\le k_0\le k_1\le\cdots\le k_i\le\cdots$ is determined iteratively by

$$k_{i+1}=\inf\{k\in\mathbb{N}|k>k_i,f(\sigma(k),\delta)>0\}.\tag{7}$$

Accordingly, any measurement data satisfying the event condition (6) will be transmitted to the filter.

Remark 2: Different from the traditional filtering problems, in this paper, the event trigger is adopted in order to reduce the data communication frequency and network bandwidth usages. With the event trigger applied here, unnecessarily frequent transmission could be avoided when the change rate of the measurement signals is relatively small. Obviously, the set of event instants is only a subset of the time sequences, i.e., $\{k_0,k_1,k_2,\ldots\}\in\{0,1,2,\ldots\}$. Note that, when $\delta=0$, all the measurement sequences would be transmitted, and the problem addressed reduces to the traditional filtering one.

For system (1), the following time-varying filter structure is proposed:

$$\begin{cases}\hat{x}(k+1)=A(k)\hat{x}(k)+\bar{\alpha}g(k,\hat{x}(k))-K(k)\Big(y(k_i)\\ \qquad\qquad -\sum_{s=0}^{l}\bar{\beta}_sC(k-s)\hat{x}(k-s)\Big)\\ \hat{z}(k)=L(k)\hat{x}(k)\end{cases}\tag{8}$$

where $\hat{x}(k)\in\mathbb{R}^{n_x}$ is the estimate of the state $x(k)$, $\hat{z}(k)\in\mathbb{R}^{n_z}$ represents the estimate of the output $z(k)$, and $K(k)$ is the filter gain matrix to be designed.

By letting $e(k)=x(k)-\hat{x}(k)$, $\eta(k)=\big[x^T(k)\quad e^T(k)\big]^T$, $\tilde{z}(k)=z(k)-\hat{z}(k)$, $\varpi(k)=\big[v^T(k)\quad\xi^T(k)\big]^T$, $\bar{g}(k)=\big[g^T(k,x(k))\quad g^T(k,x(k))-g^T(k,\hat{x}(k))\big]^T$, $\tilde{\alpha}(k)=\alpha(k)-\bar{\alpha}$ and $\tilde{\beta}_s(k)=\beta_s(k)-\bar{\beta}_s$, we have the following augmented system to be investigated:

$$\begin{cases}\eta(k+1)=\mathcal{Y}_l(k)+\Big(\sum_{i=1}^{r}w_i(k)\bar{A}_i(k)+\tilde{\beta}_0(k)\bar{C}_2(k)\Big)\\ \qquad\times\eta(k)+\tilde{\beta}_0(k)\bar{D}_2(k)\varpi(k)+\tilde{\alpha}(k)S_1\bar{g}(k)\\ \qquad+\sum_{s=1}^{l}\tilde{\beta}_s(k)\bar{C}_2(k-s)\eta(k-s)\\ \qquad+\sum_{s=1}^{l}\tilde{\beta}_s(k)\bar{D}_2(k-s)\varpi(k-s)\\ \tilde{z}(k)=\bar{L}(k)\eta(k)\end{cases}\tag{9}$$

where

$$\mathcal{Y}_l(k) = \bar{A}(k)\eta(k) + \bar{\alpha}\bar{g}(k) + \sum_{s=1}^{l}\bar{\beta}_s\bar{C}_1(k-s)\eta(k-s) + \bar{K}(k)\sigma(k) + \sum_{s=1}^{l}\bar{\beta}_s\bar{D}_2(k-s)\varpi(k-s) + \bar{D}_1(k)\varpi(k),$$

$$S_1 = \begin{bmatrix} I & 0 \\ I & 0 \end{bmatrix}, \ \bar{C}_2(k-s) = \begin{bmatrix} 0 & 0 \\ K(k)C(k-s) & 0 \end{bmatrix},$$

$$\bar{D}_1(k) = \begin{bmatrix} D_1(k) & 0 \\ D_1(k) + \bar{\beta}_0 K(k)D_2(k) & K(k)D_3(k) \end{bmatrix},$$

$$\bar{K}(k) = \begin{bmatrix} 0 \\ K(k) \end{bmatrix}, \ \bar{D}_2(k-s) = \begin{bmatrix} 0 & 0 \\ K(k)D_2(k-s) & 0 \end{bmatrix},$$

$$\bar{A}(k) = \mathrm{diag}\{A(k), A(k) + \bar{\beta}_0 K(k)C(k)\},$$

$$\bar{C}_1(k-s) = \mathrm{diag}\{0, K(k)C(k-s)\},$$

$$\bar{A}_i(k) = \mathbf{1}_2 \otimes \begin{bmatrix} A_i(k) & 0 \end{bmatrix}, \ \bar{L}(k) = \begin{bmatrix} 0 & L(k) \end{bmatrix}.$$

Our objective of this paper is to design a time-varying filter of the form (8) such that, for the given positive scalar γ, the dynamic system (9) satisfies the following filtering performance requirement:

$$J := \mathbb{E}\left\{\sum_{k=0}^{N-1}\left(\|\tilde{z}(k)\|^2 - \gamma^2\|\varpi(k)\|_U^2\right)\right\} - \gamma^2\sum_{i=-l}^{0}\mathbb{E}\{\eta^T(i) \times V_i\eta(i)\} < 0 \quad (\forall\{\varpi(k)\}, \eta(i) \neq 0) \tag{10}$$

where U and V_i are some given positive definite weighted matrices. $\|\varpi(k)\|_U^2 = \varpi^T(k)U\varpi(k)$.

III. Main results

In this section, let us investigate both the event-based filtering performance analysis and filter design problems for system (9). Firstly, we propose the following event-based filtering performance analysis results for a class of nonlinear time-varying systems with multiplicative noises and fading channels.

Theorem 1: Consider the discrete time-varying nonlinear stochastic system described by (1)–(4). Let the disturbance attenuation level $\gamma > 0$, the positive definite weighted matrices $U > 0$, $V_i > 0$ $(i = -l, -l+1, \ldots, 0)$, the event weighted matrix $\Omega > 0$, the scalar $\delta \in [0,\ 1)$ and the filter gain matrix $\{K(k)\}_{k\in[0,\ N-1]}$ in (8) be given. For the augmented system (9), the performance criterion (10) is guaranteed for all nonzero $\varpi(k)$ if there exist families of positive scalars $\{\lambda(k)\}_{k\in[0,\ N-1]}$, positive definite matrices $\{P(k)\}_{k\in[0,\ N]} > 0$ and $\{Q(i,j)\}_{i\in[-l,\ N], j\in[1,\ l]} > 0$ satisfying

$$\Gamma(k) = \bar{\Gamma}(k) + \begin{bmatrix} \mathcal{T}_{11}(k) & * \\ \mathcal{T}_{21}(k) & \mathcal{T}_{22}(k) \end{bmatrix} < 0 \tag{11}$$

with the initial condition

$$\gamma^2 V_0 - P(0) > 0, \ \gamma^2 V_{-i} - \sum_{j=i}^{l} Q(-i,j) > 0 \quad (i = 1, 2, \ldots, l) \tag{12}$$

where

$$\mathcal{T}_{11}(k) = \begin{bmatrix} \Gamma_{11}(k) & * & * \\ \delta\bar{\beta}_0(\Lambda_\beta\bar{\mathcal{C}}_l(k))^T\Omega\bar{C}(k) & \Gamma_{22}(k) & * \\ \Gamma_{31}(k) & \Gamma_{32}(k) & \Gamma_{33}(k) \end{bmatrix},$$

$$\mathcal{T}_{21}(k) = \begin{bmatrix} \delta\bar{\beta}_0(\Lambda_\beta\bar{\mathcal{D}}_l(k))^T\Omega\bar{C}(k) & \Gamma_{42}(k) & \Gamma_{43}(k) \\ \lambda(k)\mathcal{U}_1(k) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{T}_{22}(k) = \mathrm{diag}\{\Gamma_{44}(k), -\lambda(k)I, -\Omega I\},$$

$$\mathcal{U}_1(k) = I \otimes (\Phi(k) + \Psi(k))/2,$$

$$\mathcal{U}_2(k) = I \otimes (\Phi^T(k)\Psi(k) + \Psi^T(k)\Phi(k))/2,$$

$$\bar{\Gamma}(k) = \begin{bmatrix} \bar{\Gamma}_{ij}(k) \end{bmatrix}_{\{i=1,2,\ldots,6; j=1,2,\ldots,6\}},$$

$$\bar{Q}(k,l) = \mathrm{diag}\{Q(k-1,1), Q(k-2,2), \cdots, Q(k-l,l)\},$$

$$\bar{\Gamma}_{11}(k) = \bar{A}^T(k)P(k+1)\bar{A}(k) - P(k) + \nu_0\bar{C}_2^T(k) \times P(k+1)\bar{C}_2(k) + \sum_{i=1}^{r}\bar{A}_i^T(k)P(k+1)\bar{A}_i(k), + \sum_{j=1}^{l}Q(k,j), \bar{P}(k+1) = I_l \otimes P(k+1),$$

$$\bar{\Gamma}_{21}(k) = (\Lambda_\beta\bar{\mathcal{C}}_{1l}(k))^T P(k+1)\bar{A}(k),$$

$$\bar{\Gamma}_{22}(k) = (\Lambda_\beta\bar{\mathcal{C}}_{1l}(k))^T P(k+1)\Lambda_\beta\bar{\mathcal{C}}_{1l}(k) - \bar{Q}(k,l) + (\bar{\Lambda}_\gamma\bar{\mathcal{C}}_{2l}(k))^T\bar{P}(k+1)\bar{\Lambda}_\gamma\bar{\mathcal{C}}_{2l}(k),$$

$$\bar{\Gamma}_{31}(k) = \bar{D}_1^T(k)P(k+1)\bar{A}(k) + \nu_0\bar{D}_2^T(k)P(k+1)\bar{C}_2(k),$$

$$\bar{\Gamma}_{32}(k) = \bar{D}_1^T(k)P(k+1)\Lambda_\beta\bar{\mathcal{C}}_{1l}(k),$$

$$\bar{\Gamma}_{33}(k) = \bar{D}_1^T(k)P(k+1)\bar{D}_1(k) + \nu_0\bar{D}_2^T(k)P(k+1)\bar{D}_2(k),$$

$$\bar{\Gamma}_{41}(k) = (\Lambda_\beta\bar{\mathcal{D}}_{2l}(k))^T P(k+1)\bar{A}(k),$$

$$\bar{\Gamma}_{42}(k) = (\Lambda_\beta\bar{\mathcal{D}}_{2l}(k))^T P(k+1)\Lambda_\beta\bar{\mathcal{C}}_{1l}(k) + (\bar{\Lambda}_\gamma\bar{\mathcal{D}}_{2l}(k))^T\bar{P}(k+1)\bar{\Lambda}_\gamma\bar{\mathcal{C}}_{2l}(k),$$

$$\bar{\Gamma}_{43}(k) = (\Lambda_\beta\bar{\mathcal{D}}_{2l}(k))^T P(k+1)\bar{D}_1(k),$$

$$\bar{\Gamma}_{44}(k) = (\Lambda_\beta\bar{\mathcal{D}}_{2l}(k))^T P(k+1)\Lambda_\beta\bar{\mathcal{D}}_{2l}(k) + (\bar{\Lambda}_\gamma\bar{\mathcal{D}}_{2l}(k))^T \times \bar{P}(k+1)\bar{\Lambda}_\gamma\bar{\mathcal{D}}_{2l}(k),$$

$$\bar{\Gamma}_{51}(k) = \bar{\alpha}P(k+1)\bar{A}(k), \bar{\Gamma}_{52}(k) = \bar{\alpha}P(k+1)\Lambda_\beta\bar{\mathcal{C}}_{1l}(k),$$

$$\bar{\Gamma}_{53}(k) = \bar{\alpha}P(k+1)\bar{D}_1(k), \bar{\Gamma}_{54}(k) = \bar{\alpha}P(k+1)\Lambda_\beta\bar{\mathcal{D}}_{2l}(k),$$

$$\bar{\Gamma}_{55}(k) = \bar{\alpha}^2P(k+1) + \bar{\alpha}(1-\bar{\alpha})S_1^T P(k+1)S_1,$$

$$\bar{\Gamma}_{61}(k) = \bar{K}^T(k)P(k+1)\bar{A}(k),$$

$$\bar{\Gamma}_{62}(k) = \bar{K}^T(k)P(k+1)\Lambda_\beta\bar{\mathcal{C}}_{1l}(k),$$

$$\bar{\Gamma}_{63}(k) = \bar{K}^T(k)P(k+1)\bar{D}_1(k),$$

$$\bar{\Gamma}_{64}(k) = \bar{K}^T(k)P(k+1)\Lambda_\beta\bar{\mathcal{D}}_{2l}(k),$$

$$\bar{\Gamma}_{65}(k) = \bar{\alpha}\bar{K}^T(k)P(k+1),$$

$$\bar{\Gamma}_{66}(k) = \bar{K}^T(k)P(k+1)\bar{K}(k),$$

$$\Gamma_{11}(k) = -\lambda(k)\mathcal{U}_2(k) + \bar{L}^T(k)\bar{L}(k) + \delta(\bar{\beta}_0^2 + \nu_0)\bar{C}^T(k)\Omega\bar{C}(k),$$

$$\Gamma_{22}(k) = \delta(\Lambda_\beta\bar{\mathcal{C}}_l(k))^T\Omega\Lambda_\beta\bar{\mathcal{C}}_l(k) + \delta(\bar{\Lambda}_\gamma\bar{\mathcal{C}}_l(k))^T\Omega\bar{\Lambda}_\gamma\bar{\mathcal{C}}_l(k),$$

$$\Gamma_{31}(k) = \delta\big((\bar{\beta}_0^2 + \nu_0)\bar{D}(k) + \bar{\beta}_0\bar{D}_3(k)\big)^T\Omega\bar{C}(k),$$

$$\Gamma_{32}(k) = \delta(\bar{\beta}_0\bar{D}(k) + \bar{D}_3(k))^T\Omega\Lambda_\beta\bar{\mathcal{C}}_l(k),$$

$$\Gamma_{33}(k) = -\frac{\gamma^2}{l+1}U + \delta\big(\bar{\beta}_0\bar{D}(k) + \bar{D}_3(k)\big)^T\Omega\big(\bar{\beta}_0\bar{D}(k) + \bar{D}_3(k)\big) + \delta\nu_0\bar{D}^T(k)\Omega\bar{D}(k),$$
$$\Gamma_{42}(k) = \delta(\Lambda_\beta\bar{\mathcal{D}}_l(k))^T\Omega\Lambda_\beta\bar{\mathcal{C}}_l(k) + \delta(\bar{\Lambda}_\gamma\bar{\mathcal{D}}_l(k))^T\Omega\bar{\Lambda}_\gamma\bar{\mathcal{C}}_l(k),$$
$$\Gamma_{43}(k) = \delta(\Lambda_\beta\bar{\mathcal{D}}_l(k))^T\Omega(\bar{\beta}_0\bar{D}(k) + \bar{D}_3(k)),$$
$$\Gamma_{44}(k) = -\frac{\gamma^2}{l+1}I_l\otimes U + \delta(\Lambda_\beta\bar{\mathcal{D}}_l(k))^T\Omega\Lambda_\beta\bar{\mathcal{D}}_l(k) + \delta(\bar{\Lambda}_\gamma\bar{\mathcal{D}}_l(k))^T\Omega\bar{\Lambda}_\gamma\bar{\mathcal{D}}_l(k),$$
$$\bar{\mathcal{C}}_{1l}(k) = \mathrm{diag}\{\bar{C}_1(k-1), \bar{C}_1(k-2), \ldots, \bar{C}_1(k-l)\},$$
$$\bar{\mathcal{D}}_{2l}(k) = \mathrm{diag}\{\bar{D}_2(k-1), \bar{D}_2(k-2), \ldots, \bar{D}_2(k-l)\},$$
$$\bar{\mathcal{C}}_{2l}(k) = \mathrm{diag}\{\bar{C}_2(k-1), \bar{C}_2(k-2), \ldots, \bar{C}_2(k-l)\},$$
$$\bar{\mathcal{D}}_{l}(k) = \mathrm{diag}\{\bar{D}(k-1), \bar{D}(k-2), \ldots, \bar{D}(k-l)\},$$
$$\bar{\mathcal{C}}_{l}(k) = \mathrm{diag}\{\bar{C}(k-1), \bar{C}(k-2), \ldots, \bar{C}(k-l)\},$$
$$\bar{D}_3(k) = \begin{bmatrix}0 & D_3(k)\end{bmatrix}, \bar{C}(k-s) = \begin{bmatrix}C(k-s) & 0\end{bmatrix},$$
$$\bar{D}(k-s) = \begin{bmatrix}D_2(k-s) & 0\end{bmatrix}, \Lambda_\beta = \begin{bmatrix}\bar{\beta}_1 I & \bar{\beta}_2 I & \cdots & \bar{\beta}_l I\end{bmatrix},$$
$$\bar{\Lambda}_\gamma = \mathrm{diag}\{\sqrt{\nu_1}I, \sqrt{\nu_2}I, \ldots, \sqrt{\nu_l}I\}.$$

Proof: Consider the following Lyapunov functional candidate for system (9):

$$V(k) = V_1(k) + V_2(k) = \eta^T(k)P(k)\eta(k) + \sum_{j=1}^{l}\sum_{i=k-j}^{k-1}\eta^T(i)Q(i,j)\eta(i) \quad (13)$$

where $P(k) > 0$ and $Q(i,j) > 0$ are symmetric positive definite matrices with appropriate dimensions. Calculate the difference of $V(k)$ along the solution of system (9) and take the mathematical expectation. Then, we have

$$\begin{aligned}&\mathbb{E}\{\Delta V_1(k)\} = \mathbb{E}\{V_1(k+1) - V_1(k)\}\\ =&\mathbb{E}\bigg\{\bigg(\mathcal{Y}_l^T(k)P(k+1)\mathcal{Y}_l(k) + \bar{\alpha}(1-\bar{\alpha})\bar{g}^T(k)S_1^TP(k+1)\\ &\times S_1\bar{g}(k) + \eta^T(k)\bigg(\sum_{i=1}^{r}\bar{A}_i^T(k)P(k+1)\bar{A}_i(k)\bigg)\eta(k)\\ &+\sum_{s=0}^{l}\nu_s\Big(\bar{C}_2(k-s)\eta(k-s) + \bar{D}_2(k-s)\varpi(k-s)\Big)^T\\ &\times P(k+1)\Big(\bar{C}_2(k-s)\eta(k-s) + \bar{D}_2(k-s)\varpi(k-s)\Big)\\ &-\eta^T(k)P(k)\eta(k)\bigg\}\end{aligned} \quad (14)$$

Similarly, by noting the equation (13), one has

$$\begin{aligned}&\mathbb{E}\{\Delta V_2(k)\}\\ =&\mathbb{E}\bigg\{\sum_{j=1}^{l}\eta^T(k)Q(k,j)\eta(k) - \eta_l^T(k)\bar{Q}(k,l)\eta_l(k)\bigg\}\end{aligned} \quad (15)$$

where $\eta_l(k) = \begin{bmatrix}\eta^T(k-1) & \eta^T(k-2) & \cdots & \eta^T(k-l)\end{bmatrix}^T$. Therefore, by denoting

$$\varpi_l(k) = \begin{bmatrix}\varpi^T(k-1) & \cdots & \varpi^T(k-l)\end{bmatrix}^T,$$
$$\tilde{\eta}(k) = \begin{bmatrix}\eta^T(k) & \eta_l^T(k) & \varpi^T(k) & \varpi_l^T(k) & \bar{g}^T(k) & \sigma^T(k)\end{bmatrix}^T$$

and combining (13)–(15), one immediately obtains

$$\mathbb{E}\{\Delta V(k)\} = \mathbb{E}\left\{\tilde{\eta}^T(k)\bar{\Gamma}(k)\tilde{\eta}(k)\right\}. \quad (16)$$

Moreover, it follows from the constraint (2) that

$$\big[\bar{g}(k) - (I\otimes\Phi(k))\eta(k)\big]^T\big[\bar{g}(k) - (I\otimes\Psi(k))\eta(k)\big] \le 0. \quad (17)$$

Then, substituting (17) into (16) results in

$$\begin{aligned}\mathbb{E}\{\Delta V(k)\} \le \mathbb{E}\Big\{&\tilde{\eta}^T(k)\bar{\Gamma}(k)\tilde{\eta}(k) - \lambda(k)\big[\bar{g}(k) - (I\otimes\Phi(k))\\ &\times\eta(k)\big]^T\big[\bar{g}(k) - (I\otimes\Psi(k))\eta(k)\big]\Big\}.\end{aligned} \quad (18)$$

Considering the event condition (6), we have

$$\begin{aligned}&\mathbb{E}\{\Delta V(k)\}\\ \le\ &\mathbb{E}\Big\{\tilde{\eta}^T(k)\bar{\Gamma}(k)\tilde{\eta}(k) - \lambda(k)\big[\bar{g}(k) - (I\otimes\Phi(k))\eta(k)\big]^T\\ &\times\big[\bar{g}(k) - (I\otimes\Psi(k))\eta(k)\big] - \sigma^T(k)\Omega\sigma(k)\\ &+\delta y^T(k)\Omega y(k)\Big\}.\end{aligned} \quad (19)$$

Due to $\{\varpi(k)\}_{k\in[-l,\ -1]} = 0$, adding the zero term

$$\begin{aligned}&\tilde{z}^T(k)\tilde{z}(k) - \gamma^2\varpi^T(k)U\varpi(k) - (\tilde{z}^T(k)\tilde{z}(k)\\ &-\gamma^2\varpi^T(k)U\varpi(k))\end{aligned} \quad (20)$$

to (19) results in

$$\begin{aligned}&\mathbb{E}\{\Delta V(k)\}\\ \le&\mathbb{E}\Big\{\tilde{\eta}^T(k)\Gamma(k)\tilde{\eta}(k)\Big\} + \mathbb{E}\bigg\{\frac{\gamma^2}{l+1}\sum_{s=0}^{l}\|\varpi(k-s)\|_U^2\\ &-\gamma^2\|\varpi(k)\|_U^2\bigg\} - \mathbb{E}\bigg\{\|\tilde{z}(k)\|^2 - \gamma^2\|\varpi(k)\|_U^2\bigg\}.\end{aligned} \quad (21)$$

Summing up (21) on both sides from 0 to $N-1$ with respect to k, we obtain

$$\begin{aligned}&\sum_{k=0}^{N-1}\mathbb{E}\{\Delta V(k)\}\\ \le&\mathbb{E}\Big\{\sum_{k=0}^{N-1}\tilde{\eta}^T(k)\Gamma(k)\tilde{\eta}(k)\Big\} + \mathbb{E}\bigg\{\frac{\gamma^2}{l+1}\sum_{s=0}^{l}\sum_{k=0}^{N-1}(\|\varpi(k-s)\|_U^2\\ &-\|\varpi(k)\|_U^2)\bigg\} - \mathbb{E}\bigg\{\sum_{k=0}^{N-1}(\|\tilde{z}(k)\|^2 - \gamma^2\|\varpi(k)\|_U^2)\bigg\}\end{aligned} \quad (22)$$

It can be obtained from (11) and (12) that

$$\begin{aligned}&\mathbb{E}\left\{\sum_{k=0}^{N-1}\Big(\gamma^2\|\varpi(k)\|_U^2 - \|\tilde{z}(k)\|^2\Big) + \gamma^2\sum_{i=-l}^{0}\eta^T(i)V_i\eta(i)\right\}\\ &> \mathbb{E}\{V(N)\} + \mathbb{E}\left\{\gamma^2\sum_{k=-l}^{0}\eta^T(i)V_i\eta(i) - V(0)\right\} \ge 0\end{aligned} \quad (23)$$

which is equivalent to (10), and the proof is now complete. ■

Based on the analysis results, we are now ready to solve the filter design problem for system (9) in the following theorem.

For convenience of later analysis, we denote

$$
\begin{aligned}
\hat{\Gamma}_{11}(k) &= \begin{bmatrix} \mathscr{T}_{11}(k) & * & * \\ \mathscr{T}_{21}(k) & -\bar{Q}(k,l)+\Gamma_{22}(k) & * \\ \Gamma_{31}(k) & \Gamma_{32}(k) & \Gamma_{33}(k) \end{bmatrix}, \\
\mathscr{T}_{11}(k) &= -P(k) + \sum_{j=1}^{l} Q(k,j) + \Gamma_{11}(k), \\
\mathscr{T}_{21}(k) &= \delta\bar{\beta}_0(\Lambda_\beta \bar{\mathcal{C}}_l(k))^T \Omega \bar{C}(k), \\
\hat{\Gamma}_{21}(k) &= \begin{bmatrix} \delta\bar{\beta}_0(\Lambda_\beta \bar{\mathcal{D}}_l(k))^T \Omega \bar{C}(k) & \Gamma_{42}(k) & \Gamma_{43}(k) \\ \lambda(k)\mathcal{U}_1(k) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\
\hat{\Gamma}_{22}(k) &= \mathrm{diag}\{\Gamma_{44}(k), -\lambda(k)I, -\Omega I\}, \\
\hat{\Gamma}_{32}(k) &= \begin{bmatrix} \Lambda_\beta \hat{K}(k)\bar{\mathcal{D}}_l(k) & \bar{\alpha} I & H_0K(k) \end{bmatrix}, \\
\hat{\Gamma}_{31}(k) &= \begin{bmatrix} \mathscr{T}_{311}(k) & \hat{D}_0(k) + H_0K(k)\hat{D}_3(k) \end{bmatrix}, \\
\mathscr{T}_{311}(k) &= \begin{bmatrix} \hat{A}_0(k) + \bar{\beta}_0 H_0 K(k)\hat{C}_0(k) & \Lambda_\beta \hat{K}(k)\hat{\mathcal{C}}_{0l}(k) \end{bmatrix}, \\
\hat{\Gamma}_{41}(k) &= \begin{bmatrix} \mathscr{C}(k) & 0 & \mathscr{D}(k) \\ 0 & \Lambda_\beta \hat{K}(k)\hat{\mathcal{C}}_{0l}(k) & 0 \\ 0 & \bar{\Lambda}_\gamma \hat{K}(k)\bar{\mathcal{C}}_l(k) & 0 \end{bmatrix}, \\
\mathscr{C}(k) &= \sqrt{\nu_0}H_0K(k)\bar{C}(k), \quad \mathscr{D}(k) = \sqrt{\nu_0}H_0K(k)\bar{D}(k), \\
\hat{\Gamma}_{44}(k) &= \mathrm{diag}\{-R(k+1), -R(k+1), -\bar{R}(k+1)\}, \\
\hat{\Gamma}_{51}(k) &= \begin{bmatrix} \hat{\Gamma}_{511}(k) & \hat{\Gamma}_{512}(k) & 0 \end{bmatrix}, \ \hat{K}(k) = I_l \otimes H_0K(k), \\
\hat{\Gamma}_{511}(k) &= \begin{bmatrix} 0 & 0 & \bar{A}_r^T(k) \end{bmatrix}^T, \ \bar{R}(k+1) = I_l \otimes R(k+1), \\
\hat{\Gamma}_{512}(k) &= \begin{bmatrix} \left(\bar{\Lambda}_\gamma \hat{K}(k)\bar{\mathcal{C}}_l(k)\right)^T & 0 & 0 \end{bmatrix}^T, \\
\hat{\Gamma}_{52}(k) &= \mathrm{diag}\{\bar{\Lambda}_\gamma \hat{K}(k)\bar{\mathcal{D}}_l(k), \sqrt{\bar{\alpha}(1-\bar{\alpha})}S_1, 0\}, \\
\hat{\Gamma}_{55}(k) &= \mathrm{diag}\{-\bar{R}(k+1), -R(k+1), -\hat{R}(k+1)\}, \\
\bar{A}_r(k) &= \begin{bmatrix} \bar{A}_1^T(k) & \bar{A}_2^T(k) & \cdots & \bar{A}_r^T(k) \end{bmatrix}^T, \\
\hat{A}_0(k) &= I_2 \otimes A(k), \ \hat{R}(k+1) = I_r \otimes R(k+1), \\
\hat{C}_0(k) &= \begin{bmatrix} 0 & C(k) \end{bmatrix}, \ \hat{D}_0(k) = \mathbf{1}_2 \otimes \begin{bmatrix} D_1(k) & 0 \end{bmatrix}, \\
\hat{D}_3(k) &= \begin{bmatrix} \bar{\beta}_0 D_2(k) & D_3(k) \end{bmatrix}, \ H_0 = \begin{bmatrix} 0 & I \end{bmatrix}^T, \\
\hat{\mathcal{C}}_{0l}(k) &= \mathrm{diag}\left\{\hat{C}_0(k-1), \hat{C}_0(k-2), \ldots, \hat{C}_0(k-l)\right\}.
\end{aligned} \tag{24}
$$

Theorem 2: Consider the discrete time-varying nonlinear stochastic system (1) with the time-varying filter (8). For the given disturbance attenuation level $\gamma > 0$, the positive definite weighted matrices $U > 0$, $V_i > 0$ $(i = -l, -l+1, \ldots, 0)$, the event weighted matrix $\Omega > 0$ and the scalar $\delta \in [0,\ 1)$, the filtering error $\tilde{z}(k)$ satisfies the performance criterion (10) if there exist families of positive scalars $\{\lambda(k)\}_{k\in[0,N-1]}$, positive definite matrices $\{P(k)\}_{k\in[0,N]} > 0$, $\{Q(i,j)\}_{i\in[-l,N],j\in[1,l]} > 0$, $\{R(k)\}_{k\in[0,N]} > 0$ and real-valued matrices $K(k)_{k\in[0,N-1]}$ satisfying

$$
\hat{\Gamma}(k) = \begin{bmatrix} \hat{\Gamma}_{11}(k) & * & * & * & * \\ \hat{\Gamma}_{21}(k) & \hat{\Gamma}_{22}(k) & * & * & * \\ \hat{\Gamma}_{31}(k) & \hat{\Gamma}_{32}(k) & -R(k+1) & * & * \\ \hat{\Gamma}_{41}(k) & 0 & 0 & \hat{\Gamma}_{44}(k) & * \\ \hat{\Gamma}_{51}(k) & \hat{\Gamma}_{52}(k) & 0 & 0 & \hat{\Gamma}_{55}(k) \end{bmatrix} < 0 \tag{25}
$$

and the initial condition

$$
\gamma^2 V_0 - P(0) > 0, \ \gamma^2 V_{-i} - \sum_{j=i}^{l} Q(-i,j) > 0 \quad (i = 1, 2, \ldots, l) \tag{26}
$$

with the parameters updated by $P(k+1) = R^{-1}(k+1)$.

Proof: In order to avoid partitioning the positive define matrices $\{P(k)\}_{k\in[0,N]}$, $\{Q(i,j)\}_{i\in[-l,N],j\in[1,l]}$ and $\{R(k)\}_{k\in[0,N]}$, we rewrite the parameters in Theorem 1 in the following form:

$$
\begin{aligned}
&\bar{A}(k) = \hat{A}_0(k) + \bar{\beta}_0 H_0 K(k)\hat{C}_0(k), \ \bar{\mathcal{C}}_{1l}(k) = \hat{K}(k)\hat{\mathcal{C}}_{0l}(k), \\
&\bar{C}_1(k-s) = H_0K(k)\hat{C}_0(k-s), \ \bar{\mathcal{C}}_{2l}(k) = \hat{K}(k)\bar{\mathcal{C}}_l(k), \\
&\bar{C}_2(k-s) = H_0K(k)\bar{C}(k-s), \ \bar{\mathcal{D}}_{2l}(k) = \hat{K}(k)\bar{\mathcal{D}}_l(k), \\
&\bar{D}_1(k) = \hat{D}_0(k) + H_0K(k)\hat{D}_3(k), \ \bar{K}(k) = H_0K(k), \\
&\bar{D}_2(k) = H_0K(k)\bar{D}(k-s).
\end{aligned} \tag{27}
$$

Noticing (27) and using the Schur Complement Lemma [2], (25) can be obtained by (11) after some straightforward algebraic manipulations. The proof of this theorem is now complete. ∎

Remark 3: Theorem 1 presents sufficient conditions for the existence of admissible filters. It is worth noting that the technique used for deriving these conditions is quite different from the previous results in the filtering area, e.g. [10], [23], [24]. In this paper, to reduce the design conservatism, the positive definite matrices $\{P(k)\}_{k\in[0,N]}$, $\{Q(i,j)\}_{i\in[-l,N],j\in[1,l]}$ and $\{R(k)\}_{k\in[0,N]}$ remain in its original form. Therefore, the difficulty of dilating positive definite matrices does not occur in our result. Besides, it can be observed from Theorem 2 that the main results established contain all the information of the addressed general systems including the time-varying systems parameters, multiplicative noise, the threshold of event trigger, the occurrence probabilities of the random nonlinearity as well as the statistics characteristics of the channel coefficients. In the next section, a simulation example is provided to show the effectiveness of the proposed finite-horizon filtering technique.

For implementation purpose and based on Theorem 2, we can summarize the Finite-Horizon Filter Design (FHFD) algorithm at the top of the next page.

IV. An Illustrative Example

In this section, we aim to demonstrate the effectiveness and applicability of the proposed method. The system model is concerned with one of the test runs of an aircraft which is powered by energy from two F-404 engines. Both engines are mounted close together in the aft fuselage. We are interested in tracking such an aircraft through wireless communications subject to fading channels and multiplicative noises. In this simulation, the nominal system matrix A and the measurement output matrix C are taken from the linearized model of an F-404 aircraft engine system in [8]:

$$
\begin{aligned}
A(k) &= \begin{bmatrix} -1.4600 & 0 & 2.4280 \\ 0.1643 & -0.4000 & -0.3788 \\ 0.3107 & 0 & -2.2300 \end{bmatrix}, \\
C(k) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.
\end{aligned}
$$

The Finite-Horizon Filter Design (FHFD) Algorithm:

Step 1.	Given the disturbance attenuation level γ, the positive definite weighted matrices $U > 0$, $V_i > 0$ $(i = -l, -l+1, \ldots, 0)$, the event weighted matrix $\Omega > 0$ and the saclar $\delta \in [0, 1)$.
Step 2.	Set $k = 0$. Solve the matrix inequalities (25) and the recursive matrix inequalities (12) to obtain the values of matrices $P(0)$, $\sum_{j=i}^{l} Q(-i, j)$ $(i = 1, 2, \ldots, l)$, $R(1)$ and the filter gain matrix $K(0)$.
Step 3.	Set $k = k + 1$, update the matrices $P(k + 1) = R^{-1}(k + 1)$ and then obtain the filter gain matrix $K(k)$ by solving the recursive matrix inequalities (25).
Step 4.	If $k < N$, then go to Step 3, else go to Step 5.
Step 5.	Stop.

Setting the sampling time $T = 0.5s$, we obtain the following discretized nominal system matrices

$$
\begin{aligned}
A(k) &= \begin{bmatrix} 0.5227 & 0 & 0.5009 \\ 0.0458 & 0.8187 & -0.0783 \\ 0.0641 & 0 & 0.3638 \end{bmatrix}, \\
C(k) &= \begin{bmatrix} 0.6487 & 0 & 0 \\ 0 & 0.6487 & 0 \end{bmatrix}.
\end{aligned}
$$

As discussed in [27], virtually all aircraft engine systems are in some way disturbed by uncontrolled external forces. The disturbances may assume a myriad of forms such as wind gusts, gravity gradients, structural vibrations, or sensor and actuator noise, and may enter the systems in many different ways. These perturbations generally degrade the performance of the system and, in some cases, may even jeopardize the outcome of the engineering task. For example, the random vibration of an aircraft engine system would have a major impact on the accurate fatigue analysis as well as the design of engine control systems [14]. As in [11], we suppose that the motion of the F-404 aircraft engine can be determined by the system of stochastic differential equations derived from the basic aerodynamics, and the stochastic part of the motion is due to the changing wind.

In the F-404 aircraft engine model, $x_1(k)$ and $x_2(k)$ represent the horizontal position and $x_3(k)$ is the altitude of the aircraft. Our purpose is to design a time-varying filter in the form of (8) in a network environment. The movement of the aircraft is affected by the wind that acts as stochastic disturbances. In fact, when modeling the aircraft engine system, there exist modeling errors (state-multiplicative noises) and linearization errors (nonlinear disturbances). Moreover, in the scenario of tracking the aircraft through wireless communications, both fading channels and multiplicative noises are often unavoidable. To this end, the corresponding parameters are given as follows:

$$
\begin{aligned}
A_1(k) &= \begin{bmatrix} 0.05 & -0.1 & 0 \\ 0 & 0.02\sin(k) & 0.1 \\ 0.01 & 0 & 0.2 \end{bmatrix}, \\
A_2(k) &= \begin{bmatrix} 0.05\sin(k) & 0 & 0 \\ 0 & 0.02 & 0 \\ 0.1 & 0.01 & 0.02 \end{bmatrix}, \\
D_1(k) &= \begin{bmatrix} 0.2 & -0.05 & 0.01 \end{bmatrix}^T, \; L(k) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}, \\
D_2(k) &= \begin{bmatrix} 0.3 & -0.05 \end{bmatrix}^T, \; D_3(k) = \begin{bmatrix} 0 & 0.1 \end{bmatrix}^T.
\end{aligned}
$$

To track the state of the F-404 aircraft engine system, the RONs should be taken into account due to the unpredictable changes of the environmental circumstances. In practice, the probability $\alpha(k)$ can be determined beforehand thorough statistical tests. In this illustrative example, the probability of randomly occurring nonlinearities is taken as $\bar{\alpha} = 0.7$ and the nonlinear vector-valued function $g(k, x(k))$ is chosen as

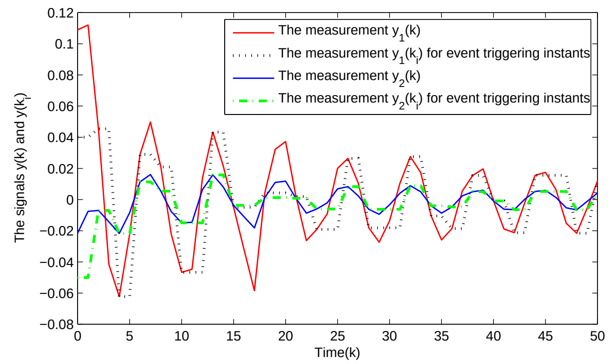

Fig. 1. The measurement $y(k)$ and the measurement $y(k_i)$ for event-triggered instants when $\delta = 0.6$

$$
g(k, x(k)) = \begin{bmatrix} -0.5x_1(k) + 0.4x_2(k) + 0.1x_3(k) \\ 0.1x_1(k) + \frac{\sin x_1(k)}{\sqrt{x_1^2(k)+x_2^2(k)+10}} \\ 0.5x_2(k) \end{bmatrix}
$$

where $x_i(k)$ $(i = 1, 2, 3)$ denotes the i-th element of the system state $x(k)$. It is easy to see that the constraint (2) is met with

$$
\Phi(k) = \begin{bmatrix} -0.2 & 0.4 & 0.1 \\ 0.05 & 0 & 0 \\ 0 & 0.2 & 0 \end{bmatrix}, \quad \Psi(k) = \begin{bmatrix} -0.8 & 0.4 & 0.1 \\ 0.15 & 0 & 0 \\ 0 & 0.8 & 0 \end{bmatrix}.
$$

The order of the fading model is $l = 1$ and the probability density functions of channel coefficients are as follows

$$
\begin{cases} \varrho(\beta_0(k)) = 0.0005(e^{9.89\beta_0(k)} - 1), & 0 \le \beta_0(k) \le 1, \\ \varrho(\beta_1(k)) = 8.5017e^{-8.5\beta_1(k)}, & 0 \le \beta_1(k) \le 1. \end{cases}
$$

The mathematical expectation $\bar{\beta}_s$ and variance ν_s $(s = 0, 1)$ can be obtained as 0.8991, 0.1174, 0.0133 and 0.01364, respectively.

The H_∞ performance level γ, the positive definite weighted matrices U, V_i $(i = -1, 0)$ are chosen as $\gamma = 1$, $U = I$, $V_{-1} = V_0 = 5I$, respectively. Choose event weighted matrix $\Omega = I$ and the threshold $\delta = 0.6$. As long as it goes beyond the established threshold, updates are triggered such that the value $\|\sigma(k)\|$ is reset to zero again. By applying Algorithm FHFD, the desired filter parameters are obtained and listed in Table I.

TABLE I
THE FILTER PARAMETERS $K(k)$

k	0	1	2	$\cdots$	50
$K(k)$	$\begin{bmatrix} 0.3376 & 0.4775 \\ 0.4476 & 0.4285 \\ 0.4575 & 0.4726 \end{bmatrix}$	$\begin{bmatrix} 0.3302 & 0.4091 \\ 0.4149 & 0.2967 \\ 0.4657 & 0.4363 \end{bmatrix}$	$\begin{bmatrix} 0.1377 & 0.0046 \\ 0.2056 & -0.0236 \\ 0.3276 & 0.0040 \end{bmatrix}$	$\cdots$	$\begin{bmatrix} 0.1270 & -0.1241 \\ 0.3614 & -0.2708 \\ 0.1956 & -0.1346 \end{bmatrix}$

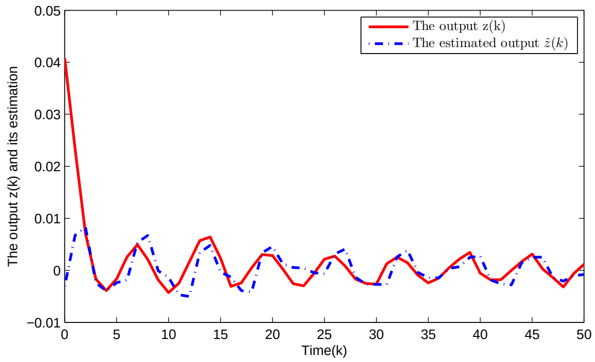

Fig. 2. The output $z(k)$ and its estimation when $\delta = 0.6$

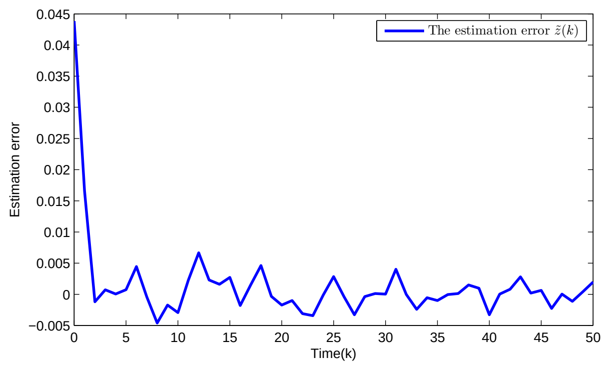

Fig. 3. The estimation error $\tilde{z}(k)$ when $\delta = 0.6$

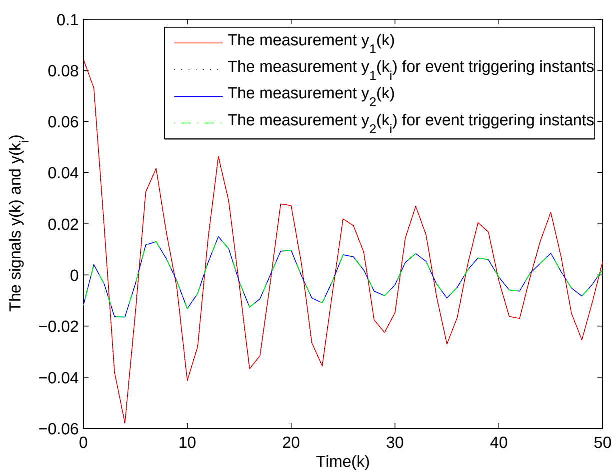

Fig. 4. The measurement $y(k)$ and the measurement $y(k_i)$ for event-triggered instants when $\delta = 0$

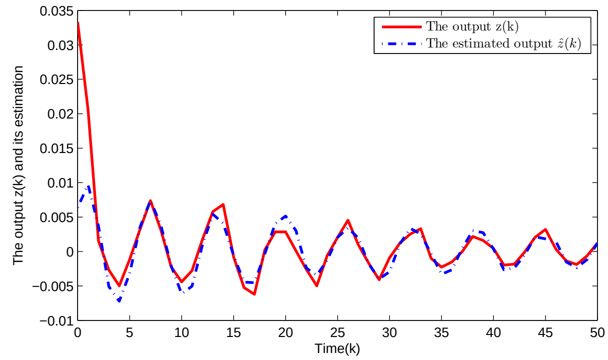

Fig. 5. The output $z(k)$ and its estimation when $\delta = 0$

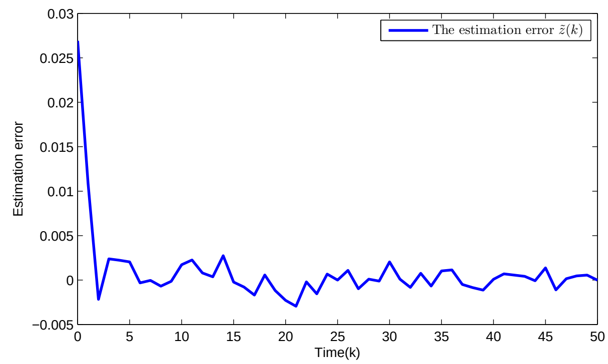

Fig. 6. The estimation error $\tilde{z}(k)$ when $\delta = 0$

In the simulation, the initial value of the state is $x(0) = \begin{bmatrix} -0.55 & -0.16 & 0 \end{bmatrix}^T$ and the exogenous disturbance inputs are selected as

$$\xi(k) = 0.5e^{-2k}\sin(4k), \quad v(k) = \frac{4}{k+20}\sin(k). \tag{28}$$

Fig. 1 plots the measurement $y(k)$ and the measurement $y(k_i)$ for event-triggered instants, and the outputs $z(k)$ and the filtering errors $\tilde{z}(k)$ are depicted in Fig. 2 and Fig. 3, respectively.

For $\delta = 0$, that is, no event triggering happens, Fig. 4 plots the measurement $y(k)$ and the measurement $y(k_i)$ for event-triggered instants. The corresponding outputs $z(k)$ and the filtering errors $\tilde{z}(k)$ are depicted in Fig. 5 and Fig. 6, respectively. It can be seen from the simulation results that the larger δ the worse the filtering performance, which is in agreement with the fact that event triggering is based on the relative error with respect to the output signal. Clearly, the bandwidth utilization cannot be reduced too much in order to guarantee certain filtering performance. All the simulation results confirm that the approach addressed in this paper provides a satisfactory filtering performance.

V. Conclusion

In this paper, we have dealt with the event-based filtering problem for time-varying systems with fading channels, randomly occurring nonlinearities and multiplicative noise. An event indicator variable has been constructed and the corresponding event-triggered scheme has been proposed to determine whether the measurement output is transmitted to the filter or not. The event-triggered scheme has been based on the relative error with respect to the measurement signal, and the fading channels have been described by modified stochastic Rice fading models. Some uncorrelated random variables have been introduced, respectively, to govern the phenomena of state-multiplicative noises, randomly occurring nonlinearities and fading measurements. By employing the stochastic analysis techniques, some sufficient conditions have been provided to ensure that the dynamic system under consideration satisfies the filtering performance constraint. Furthermore, the explicit expression of the desired filter gains have been derived in terms of solving recursive matrix inequalities. Finally, an illustrative example has highlighted the effectiveness of the event-based filtering technology presented in this paper.

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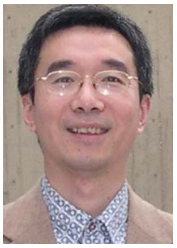

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