Robust Reliable H_∞ Control for Uncertain Stochastic Spatial-Temporal Systems: The Output Feedback Case

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Abstract

This paper is concerned with the robust reliable H_∞ output-feedback control problem for a class of uncertain spatial-temporal systems. The system under consideration resides in a given discrete rectangular region and its states evolve not only over time but also over space. A set of sensors located at the specified points is used to measure the system outputs. Based on the available measurement outputs, the static output feedback control strategy is adopted where both the parameter uncertainties and the actuator failures are taken into account. By reorganizing the state variables, we first transform the closed-loop spatial-temporal system into an ordinary differential dynamic system. Then, by dint of the Lyapunov stability theory, a sufficient condition is given that ensures the globally asymptotical stability as well as the H_∞ performance requirement for all the possible uncertainties and actuator failures. According to the performance analysis, the desired robust reliable H_∞ controller is designed in terms of a matrix inequality which can be solved by available software. Finally, a numerical example is employed to demonstrate the effectiveness of the control scheme proposed.

Keywords

Actuator failures; output feedback control; reliable control; robust H_∞ control; spatial-temporal systems.

I. Introduction

The past few decades have witnessed constant research interests on spatial-temporal systems because of their enormous potential to represent the real physical systems in various areas such as the chemical engineering [23], molecular biology [8] and population dynamic systems [11]. So far, there has been a rich body of research results available in the literature with respect to various aspects of spatial-temporal systems. It is well known that, comparing with the systems dependent on time only, the spatial-temporal systems exhibit more complexities which bring significant difficulty in the analysis and synthesis of the system dynamics. A conventional approach is to transform the spatial-temporal systems into an equivalent ordinary differential system. For example, in [20], a linear parabolic stochastic spatial-temporal system has been converted into an equivalent infinite-dimensional ordinary differential system. Then, in terms of the separation of eigenvalues,

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the infinite-dimensional ordinary differential system has been further separated into a finite-dimensional slow mode and an infinite-dimensional fast mode. Finally, the desired stabilization controller has been derived via designing a finite-dimensional controller for the finite-dimensional slow mode system. Based on the similar idea, the robust H_∞ filter has been designed for linear stochastic partial differential systems in [6] and the robust H_∞ observer-based stabilization problem has been studied in [7] for the same systems. Moreover, in [34], the synchronization problem has been investigated for a class of coupled partial differential systems and the synchronization criteria have been established.

Note that the system parameters in all the literature mentioned above have been assumed to be known exactly. However, such an assumption is not always true in practical engineering and parameter uncertainties are usually inevitable due to the effects from the modeling error and external disturbance. Such uncertainties, if not taken into account appropriately, may deteriorate the prespecified control/estimation performance. Therefore, in the past decade, uncertain systems have received considerable attention and a number of papers have attempted to develop robust technologies to handle these uncertainties. For example, in [5, 18, 22, 26, 29, 32, 35, 37, 38], a variety of robust controllers have been designed for uncertain systems where the uncertainties are supposed be norm-bounded. In [13, 16, 19], the polytopic-type uncertainties have been considered and the controllers/filters have been designed when the uncertain parameters reside in a polytope. In [15], the robust technology has been developed to design the quadratic mini-max regulator for uncertain polynomial systems. It should be pointed out that, despite the numerous robust control/filtering technologies developed in the existing literature, the robust control problems for uncertain *spatial-temporal* systems have not been fully investigated. Therefore, the first motivation of this paper is to shorten such a gap by developing a robust H_∞ control scheme for the uncertain *spatial-temporal* systems.

On the other hand, in practical control systems, the actuator failures often occur due to the component aging and, in such a case, the performance of controlled systems can no longer be guaranteed by the control scheme designed under the failure-free conditions. Therefore, it is of great necessity to redesign a controller which can achieve the specified system performance in a reliable way. This is referred to as the reliable control problem that has received increasing research attention, see e.g. [10, 12, 17, 21, 25, 27, 30, 33, 36]. Among others, in [10, 30, 36], the reliable controller has been designed based on the state feedback control strategy. In many cases, however, the system states are not directly accessible and we can only obtain the system information from the measurable system output. In such a situation, in [12, 17], the output feedback reliable control schemes have been proposed. It is worth mentioning that, to date, almost all the results on the reliable control have been reported only for the systems dependent on time, and the corresponding results for the spatial-temporal systems have been very few, not to mention the case where the parameter uncertainties are also taken into account. Indeed, for spatial-temporal systems, due to the spatial-temporal nature, the design problem of reliable controllers would be more challenging and some fundamental difficulties can be identified. For example, does there exist a valid reliable controller for the spatial-temporal systems in the presence of actuator faults and how to design such a reliable controller based on the measurement outputs to achieve the prespecified system performance index? Therefore, the second motivation for our investigation is to offer satisfactory answers to these two questions.

In this paper, the robust reliable H_∞ output feedback control problem is addressed for a class of uncertain spatial-temporal systems. The system of interest is subject to parameter uncertainties and evolves over both time and space in a given discrete rectangular region. The measurements are received by a set of sensors located in the specified rectangular region. The static output feedback control strategy is adopted and the actuator failure is taken into account. By reorganizing the state variables, the spatial-temporal systems is first transformed into an ordinary differential dynamic system. Then, by using the Lyapunov stability theory, a sufficient condition is obtained under which the dynamics of the closed-loop system is globally asymptotically stable in probability and the controlled output satisfies the desired H_∞ performance index. Based on the established existence condition, the robust reliable H_∞ controller is designed that can effectively handle both the possible uncertainties and the actuator failures. Finally, a numerical simulation example is provided to show the effectiveness of the control scheme proposed.

Notation The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n\times m$ real matrices. $\|A\|$ refers to the norm of a matrix A defined by $\|A\| = \sqrt{\mathrm{trace}(A^TA)}$. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are real symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). M^T represents the transpose of the matrix M. I denotes the identity matrix of compatible dimension. $\mathrm{diag}\{\cdots\}$ stands for a block-diagonal matrix. $\mathbb{E}\{x\}$ stands for the expectation of the stochastic variable x. $\mathrm{P}\{\cdot\}$ means the occurrence probability of the event "$\cdot$". $\mathcal{L}_2([0,\infty);\mathbb{R}^n)$ is the space of square integrable vector-valued functions. "$\otimes$" and "$\circ$" represent the Kronecker and Hadamard products, respectively. In symmetric block matrices, "$*$" is used to denote a term induced by symmetry. Matrices, if they are not explicitly specified, are assumed to have compatible dimensions.

II. Problem Formulation

Given a rectangular region $[0,N_1]\times[0,N_2]$, consider the following class of uncertain spatial-temporal systems at location (k,l) ($k\in[0,N_1]$, $l\in[0,N_2]$)

$$\begin{cases} dx_{k,l}(t) = \Big\{(\kappa+\Delta\kappa)(x_{k+1,l}(t)+x_{k-1,l}(t)+x_{k,l+1}(t)+x_{k,l-1}(t)-4x_{k,l}(t)) \\ \qquad + (A+\Delta A)(x_{k+1,l}(t)+x_{k,l+1}(t)-2x_{k,l}(t)) + (B+\Delta B)x_{k,l}(t) \\ \qquad + Du^f_{k,l}(t) + Gv_{k,l}(t)\Big\}dt + Hx_{k,l}(t)dW_{k,l}(t), \\ y_{k_i,l_i}(t) = \bar{T}_{k_i,l_i}x_{k_i,l_i}(t), \quad i=1,2,\cdots,n_y \\ z_{k,l}(t) = Mx_{k,l}(t) + \tilde{N}u^f_{k,l}(t), \end{cases} \tag{1}$$

where $x_{k,l}(t)\in\mathbb{R}^n$ is the state vector, $y_{k_i,l_i}(t)\in\mathbb{R}^m$ is the measurement output received by the sensor at the location (k_i,l_i), $z_{k,l}(t)\in\mathbb{R}^{n_z}$ is the controlled output, $u^f_{k,l}(t)\in\mathbb{R}^{n_u}$ is the control input with actuator failure, $v_{k,l}(t)\in\mathbb{R}^{n_v}$ is the external disturbance input belonging to $\mathcal{L}_2([0,\infty];\mathbb{R}^n)$, and $W_{k,l}(t)\in\mathbb{R}$ is a zero-mean Brownian motion with unit covariance. The initial values of states at the inner points are given by $x_{k,l}(0) = x^0_{k,l}$. $\kappa\in\mathbb{R}^{n\times n}$, $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times n}$, $D\in\mathbb{R}^{n\times n_u}$, $G\in\mathbb{R}^{n\times n_v}$, $H\in\mathbb{R}^{n\times n}$, $\bar{T}_{k_i,l_i}\in\mathbb{R}^{m\times n}$, $M\in\mathbb{R}^{n_z\times n}$ and $\tilde{N}\in\mathbb{R}^{n_z\times n_u}$ are known real constant matrices, while $\Delta\kappa$, ΔA and ΔB are uncertain matrices representing parameter uncertainties which are assumed to be of the following form

$$\begin{bmatrix}\Delta\kappa & \Delta A & \Delta B\end{bmatrix} = \Gamma E(t)\begin{bmatrix}Q_1 & Q_2 & Q_3\end{bmatrix} \tag{2}$$

where Γ, Q_1, Q_2 and Q_3 are known real constant matrices with appropriate dimensions, and $E(t)$ is an unknown matrix satisfying $E(t)E(t)^T\leq I$.

Assumption 1: The values of states on the boundary satisfy the Dirichlet boundary condition, i.e., $x_{k,l}(t)=0$ for $k=0$, N_1 or $l=0$, N_2.

Assumption 2: The matrices $\bar{T}_{k_i,l_i}$ ($i=1,2,\cdots,n_y$) are of full row rank.

Remark 1: Recently, continuous-spatial temporal systems (or partial differential systems) have received much research attention, see. e.g. [6, 7]. Model (1) can be viewed as the discrete-space version of continuous-spatial temporal systems. Specifically, the first and second terms in model (1) are originally from the discretization of the second and first orders continuous partial derivatives, respectively.

The n_y sensors under consideration are fitted at the positions of interest in the rectangular region. The measurements are collected from all sensors and then sent to the controller at every position in the rectangular region. That is, each controller in the rectangular region can access all the measurements from sensors. Define $y(t) = [y^T_{k_1,l_1}(t), \dots, y^T_{k_{n_y},l_{n_y}}(t)]^T$. In this paper, the static output feedback control strategy is designed with possible actuator failures. Specifically, the controller is given by

$$u^f_{k,l}(t) = FK_{k,l}y(t) \tag{3}$$

where $F \in \mathbb{R}^{n_u \times n_u}$ is the actuator fault matrix and $K_{k,l} \in \mathbb{R}^{n_u \times mn_y}$ denotes the control gain to be determined. The actuator fault matrix F is defined as

$$F = \mathrm{diag}\{f_1, \dots, f_{n_u}\} \tag{4}$$

where f_i $(i = 1, \dots, n_u)$ describes the failures of actuators satisfying $\underline{f}_i \le f_i \le \overline{f}_i$.

Set

$$F_0 = \mathrm{diag}\left\{\frac{\underline{f}_1 + \overline{f}_1}{2}, \dots, \frac{\underline{f}_{n_u} + \overline{f}_{n_u}}{2}\right\},$$
$$\widetilde{F} = \mathrm{diag}\left\{\frac{\overline{f}_1 - \underline{f}_1}{2}, \dots, \frac{\overline{f}_{n_u} - \underline{f}_{n_u}}{2}\right\}.$$

The actuator fault matrix F can be written as $F = F_0 + \Delta F$ where $\Delta F \le \widetilde{F}$.

Remark 2: Different from traditional Itô stochastic systems, in the stochastic spatial-temporal systems under consideration, the evolution of the system states is dependent on not only time but also the space. In this paper, we make the first attempt to develop a robust reliable H_∞ output-feedback control approach for the uncertain spatial-temporal systems to cope with the difficulties resulting from such dual natures of time and space.

In what follows, we transform the spatial-temporal system (1) into an ordinary differential dynamic system. Collect all the state variable $x_{k,l}(t)$ $(k = 0, 1, \cdots, N_1,\ l = 0, 1, \cdots, N_2)$ and reorganize them into a new vector as follows:

$$x(t) = \left[x^T_{0,0}(t) \dots x^T_{k,0}(t) \dots x^T_{N_1,0}(t) \dots x^T_{k,l}(t) \dots x^T_{0,N_2}(t) \dots x^T_{k,N_2}(t) \dots x^T_{N_1,N_2}(t)\right]^T. \tag{5}$$

For the purpose of notation simplicity, we denote $x_{k,l}(t)$ in the above vector by $x_j(t)$ where $j = l(N_1+1)+k+1$. Obviously, j can take the values from 1 to $N = (N_1+1)(N_2+1)$ and the set of boundary points can be denoted by $\mathfrak{J} := \{j = l(N_1+1)+k+1 | k = 0 \text{ or } k = N_1 \text{ or } l = 0 \text{ or } l = N_2\}$. Similarly, we can define $u^f(t)$, $v(t)$, $W(t)$, $z(t)$, $u^f_j(t)$, $v_j(t)$, $W_j(t)$ and $z_j(t)$.

By using the above notations, the spatial-temporal system (1) can be represented by a stochastic differential equation as follows:

$$\begin{cases} dx_j(t) =[(\kappa+\Delta\kappa)T_{1j}x(t)+(A+\Delta A)T_{2j}x(t)+(B+\Delta B)x_j(t)+Du_j^f(t)+Gv_j(t)]dt \\ \qquad\qquad +Hx_j(t)dW_j(t), \\ y(t) =\bar{C}x(t), \\ z_j(t) =Mx_j(t)+\tilde{N}u_j^f(t), \end{cases} \tag{6}$$

where

$$T_{1j}=\begin{cases} [\underbrace{\overbrace{0\dots0}^{j-N_1-2}\quad I_n\quad \overbrace{0\dots0}^{N_1-1}\quad I_n\quad -4I_n\quad I_n\quad \overbrace{0\dots0}^{N_1-1}\quad I_n\quad \overbrace{0\dots0}^{N-j-N_1-1}}_{N}], & j\notin\mathfrak{J}, \\ 0, \quad j\in\mathfrak{J}, \end{cases}$$

$$T_{2j}=\begin{cases} [\underbrace{\overbrace{0\dots0}^{j-1}\quad -2I_n\quad I_n\quad \overbrace{0\dots0}^{N_1-1}\quad I_n\quad \overbrace{0\dots0}^{N-j-N_1-1}}_{N}], & j\notin\mathfrak{J}, \\ 0, \quad j\in\mathfrak{J}, \end{cases}$$

$$\bar{C}=[C_1^T\quad C_2^T\quad \dots\quad C_{n_y}^T]^T$$

with

$$C_i=[\underbrace{\overbrace{0\dots0}^{j_i-1}\quad \bar{T}_{k_i,l_i}\quad 0\dots0}_{N}],\quad j_i=l_i(N_1+1)+k_i+1.$$

The controller can be described as

$$u_j^f(t)=FK_jy(t)$$

where K_j is a matrix obtained from all $K_{k,l}$ $(k=0,1,\cdots,N_1,\ l=0,1,\cdots,N_2)$ by using the constructing method similar to that of $x_j(t)$.

By setting

$$\begin{aligned} &T_1=\begin{bmatrix}T_{11}^T & T_{12}^T & \dots & T_{1N}^T\end{bmatrix}^T,\quad T_2=\begin{bmatrix}T_{21}^T & T_{22}^T & \dots & T_{2N}^T\end{bmatrix}^T, \\ &\bar{K}=\begin{bmatrix}K_1^T & K_2^T & \dots & K_N^T\end{bmatrix}^T,\quad \bar{B}=I_N\otimes D,\quad \bar{G}=I_N\otimes G, \\ &\bar{A}=(I_N\otimes\kappa)T_1+(I_N\otimes A)T_2+I_N\otimes B,\quad \bar{H}=I_N\otimes H, \\ &\Delta\bar{A}=(I_N\otimes\Gamma)(I_N\otimes E(t))[(I_N\otimes Q_1)T_1+(I_N\otimes Q_2)T_2+I_N\otimes Q_3], \\ &\breve{F}=I_N\otimes F,\quad \bar{W}(t)=\begin{bmatrix}W_1(t)J^T & W_2(t)J^T & \dots & W_N(t)J^T\end{bmatrix}^T, \\ &J=\begin{bmatrix}1 & 1 & \dots & 1\end{bmatrix}^T,\quad \bar{M}=I_N\otimes M,\quad \bar{N}=I_N\otimes\tilde{N}, \\ &\bar{F}_0=I_N\otimes F_0,\quad \hat{F}=I_N\otimes\tilde{F},\quad \bar{\Gamma}=I_N\otimes\Gamma,\quad \bar{E}(t)=I_N\otimes E(t), \\ &\bar{Q}=(I_N\otimes Q_1)T_1+(I_N\otimes Q_2)T_2+I_N\otimes Q_3, \end{aligned}$$

we arrive at the following augmented system

$$\begin{cases} dx(t) =[(\breve{A}+\Delta\bar{A})x(t)+\bar{G}v(t)]dt+\bar{H}x(t)\circ d\bar{W}(t), \\ z(t) =\breve{M}x(t) \end{cases} \tag{7}$$

where

$$\breve{A} = \bar{A} + \bar{B}\breve{F}\bar{K}\bar{C}, \quad \breve{M} = \bar{M} + \bar{N}\breve{F}\bar{K}\bar{C}. \tag{8}$$

Note that the augmented system (7) is inherently a stochastic system due to the existence of stochastic process $\bar{W}(t)$. Therefore, we need to introduce the following stochastic stability concept.

Definition 1: [28] The zero solution of the system (7) with $v(t) = 0$ is said to be globally asymptotically stable in probability if, (i) for any $\varepsilon > 0$, $\lim_{x_0 \to 0} \mathrm{P}\{\sup_{t \geq 0} \|x(t)\| > \varepsilon\} = 0$ and (ii) for any initial condition $x(0)$, $\mathrm{P}\{\lim_{t \to 0} \|x(t)\| = 0\} = 1$.

We are now ready to state the robust reliable H_∞ output feedback control problem as follows. For the spatial-temporal system (1), we are interested in finding the control gain $\bar{K}$ such that, for all the possible uncertainties and actuator failures, the following two requirements are simultaneously satisfied.

1) The zero solution of the system (7) with $v(t) = 0$ is globally asymptotically stable in probability;

2) Under the zero initial condition, the controlled output $z(t)$ satisfies

$$\mathbb{E}\int_0^\infty \|z(t)\|^2 dt < \gamma^2 \mathbb{E}\int_0^\infty \|v(t)\|^2 dt \tag{9}$$

for all nonzero $v(t)$, where $\gamma > 0$ is a given disturbance attenuation level.

III. Main Results

In this section, the H_∞ performance analysis is first conducted and a sufficient condition is given which guarantees the H_∞ performance as well as the stability of the augmented system (7). Then, according to the analysis results derived, the desired robust reliable H_∞ output feedback controller is designed.

The following lemmas are needed in deriving our main results.

Lemma 1: [4] Given constant matrices S_1, S_2 and S_3, where $S_1 = S_1^T$ and $S_2 = S_2^T > 0$, then $S_1 + S_3^T S_2^{-1} S_3 < 0$ if and only if

$$\begin{bmatrix} S_1 & S_3^T \\ S_3 & -S_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -S_2 & S_3 \\ S_3^T & S_1 \end{bmatrix} < 0. \tag{10}$$

Lemma 2: [4] Let $J = J^T$, M and N be real matrices of appropriate dimensions with F satisfying $FF^T \leq I$. Then $J + MFN + N^T F^T M^T < 0$ if and only if there exists a positive scalar σ such that $J + \sigma M M^T + \sigma^{-1} N^T N < 0$ or, equivalently,

$$\begin{bmatrix} J & \sigma M & N^T \\ \sigma M^T & -\sigma I & 0 \\ N & 0 & -\sigma I \end{bmatrix} < 0. \tag{11}$$

In the following theorem, a sufficient condition is derived under which the requirements 1) and 2) given in Section II are simultaneously met.

Theorem 1: Let the controller parameter $\bar{K}$ and the disturbance attenuation level $\gamma > 0$ be given. Then, the zero solution of the system (7) with $v(t) = 0$ is globally asymptotically stable in probability and the controlled output $z(t)$ satisfies the H_∞ performance constraint (9) for all nonzero exogenous disturbances under the zero initial condition if there exists a positive definite matrix $P > 0$ such that the following matrix inequality holds:

$$\Xi := \begin{bmatrix} \Xi_{11} & P\bar{G} \\ \bar{G}^T P & -\gamma^2 I \end{bmatrix} < 0 \tag{12}$$

where $\Xi_{11} = P(\breve{A} + \Delta\bar{A}) + (\breve{A} + \Delta\bar{A})^T P + \breve{M}^T \breve{M} + \bar{H}^T P \bar{H}$.

Proof: Let the Lyapunov function be $V(x(t)) = x^T(t)Px(t)$ and its infinitesimal operator $\mathbb{L}V(x(t))$ be given by

$$\mathbb{L}V(x(t)) := \frac{\partial V^T(x(t))}{\partial x}[(\breve{A} + \Delta\bar{A})x(t)] + \frac{1}{2}[\bar{H}x(t)]^T \frac{\partial^2 V(x(t))}{\partial x^2}[\bar{H}x(t)].$$

Noting that

$$\frac{\partial V(x(t))}{\partial x} = 2Px(t), \quad \frac{\partial^2 V(x(t))}{\partial x^2} = 2P,$$

we have

$$\mathbb{L}V(x(t)) = x^T(t)[P(\breve{A} + \Delta\bar{A}) + (\breve{A} + \Delta\bar{A})^T P + \bar{H}^T P \bar{H}]x(t).$$

On the other hand, it is easily known from inequality (12) that

$$P(\breve{A} + \Delta\bar{A}) + (\breve{A} + \Delta\bar{A})^T P + \bar{H}^T P \bar{H} < 0,$$

which implies

$$\mathbb{L}V(x(t)) < 0.$$

From the stochastic Lyapunov stability theory [14], it can be shown that the system (7) with $v(t) = 0$ is globally asymptotically stable in probability.

Next, we shall show that, under zero initial conditions, the controlled output $z(t)$ satisfies the H_∞ performance constraint (9) for all nonzero $v(t)$. By employing the Itô formula, it can be obtained that

$$\begin{aligned}
&\mathbb{E}\int_0^{t_f} \|z(t)\|^2 dt - \gamma^2 \mathbb{E}\int_0^{t_f} \|v(t)\|^2 dt \\
=&\mathbb{E}\int_0^{t_f} (z^T(t)z(t) - \gamma^2 v^T(t)v(t))dt \\
=&\mathbb{E}\int_0^{t_f} [(x^T(t)\breve{M}^T\breve{M}x(t) - \gamma^2 v^T(t)v(t))dt + d(x^T(t)Px(t))] - \mathbb{E}(x^T(t_f)Px(t_f)) \\
\le&\mathbb{E}\int_0^{t_f} x^T(t)(P(\breve{A} + \Delta\bar{A}) + (\breve{A} + \Delta\bar{A})^T P + \breve{M}^T\breve{M} + \bar{H}^T P\bar{H})x(t) + v^T(t)\bar{G}^T Px(t) \\
&+ x^T(t)P\bar{G}v(t) - \gamma^2 v^T(t)v(t)]dt \\
=&\mathbb{E}\int_0^{t_f} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}^T \Xi \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} dt.
\end{aligned}$$

By considering condition (12), we have $\mathbb{E}\int_0^{t_f} \|z(t)\|^2 dt < \gamma^2 \mathbb{E}\int_0^{t_f} \|v(t)\|^2 dt$, from which the H_∞ performance index can be ensured by letting $t_f \to \infty$. The proof of Theorem 1 is accomplished. ∎

Having conducted the performance analysis in Theorem 1, we are now in a position to deal with the problem of designing the robust reliable H_∞ controller for the spatial-temporal system (1). The following theorem shows that the addressed controller design problem is solvable if a matrix inequality is feasible.

Theorem 2: Let the disturbance attenuation level $\gamma > 0$ be given. The robust reliable H_∞ control problem is solvable for the system (1) if there exist a positive definite matrix $\hat{P}$, real matrices Y and Z, positive scalars ε_1 and ε_2 such that

$$\breve{\Psi} = \begin{bmatrix} \hat{\Psi}_0 & \hat{\Pi}_{12} \\ * & \hat{\Pi}_{22} \end{bmatrix} < 0, \tag{13}$$

$$Z\bar{C} = \bar{C}\hat{P} \tag{14}$$

where

$$\hat{\Psi}_0 = \begin{bmatrix} \hat{\Psi}_0^{11} & \bar{G} & \hat{P}\bar{M}^T + \bar{C}^T Y^T \bar{F_0}^T \bar{N}^T & \hat{P}\bar{H}^T & \varepsilon_1 \bar{\Gamma} & \hat{P}\bar{Q}^T \\ * & -\gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -\hat{P} & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & -\varepsilon_1 I \end{bmatrix},$$

$$\begin{aligned} \hat{\Psi}_0^{11} =& \bar{A}\hat{P} + \hat{P}\bar{A}^T + \bar{B}\bar{F}_0 Y\bar{C} + \bar{C}^T Y^T \bar{F_0}^T \bar{B}^T, \quad \hat{\Pi}_{22} = \mathrm{diag}\{-\varepsilon_2 I, -\varepsilon_2 I\}, \\ \hat{\Pi}_{12} =& \begin{bmatrix} \varepsilon_2 \hat{F}^T \bar{B}^T & 0 & \varepsilon_2 \hat{F}^T \bar{N}^T & 0 & 0 & 0 \\ Y\bar{C} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T. \end{aligned} \tag{15}$$

Moreover, if inequality (13) with (14) is feasible, the desired parameter of the desired controller $\bar{K}$ is given as follows:

$$\bar{K} = YZ^{-1}. \tag{16}$$

Proof: Pre and post-multiplying (12) in Theorem 1 by $\mathrm{diag}\{\hat{P}, I\}$ where $\hat{P} = P^{-1}$, we can obtain that

$$\begin{bmatrix} (\breve{A} + \Delta\bar{A})\hat{P} + \hat{P}(\breve{A} + \Delta\bar{A})^T + \hat{P}\breve{M}^T\breve{M}\hat{P} + \hat{P}\bar{H}^T P\bar{H}\hat{P} & \bar{G} \\ * & -\gamma^2 I \end{bmatrix} < 0. \tag{17}$$

By using Lemma 1, it can be seen that (17) is equivalent to

$$\Pi := \begin{bmatrix} (\breve{A} + \Delta\bar{A})\hat{P} + \hat{P}(\breve{A} + \Delta\bar{A})^T & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} < 0 \tag{18}$$

where

$$\Pi_{22} = \begin{bmatrix} -\gamma^2 I & 0 & 0 \\ * & -I & 0 \\ * & * & -\hat{P} \end{bmatrix}, \quad \Pi_{12} = \begin{bmatrix} \bar{G} & \hat{P}\breve{M}^T & \hat{P}\bar{H}^T \end{bmatrix}.$$

Next, we rewrite Π as follows:

$$\begin{aligned} \Pi =& \begin{bmatrix} \breve{A}\hat{P} + \hat{P}\breve{A}^T & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} + \begin{bmatrix} \Delta\bar{A}\hat{P} + \hat{P}\Delta\bar{A}^T & 0 \\ * & 0 \end{bmatrix} \\ =& \begin{bmatrix} \breve{A}\hat{P} + \hat{P}\breve{A}^T & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} + \begin{bmatrix} \bar{\Gamma} \\ 0 \end{bmatrix} \bar{E}(t) \begin{bmatrix} \bar{Q}\hat{P} & 0 \end{bmatrix} + \begin{bmatrix} \hat{P}\bar{Q}^T \\ 0 \end{bmatrix} \bar{E}^T(t) \begin{bmatrix} \bar{\Gamma}^T & 0 \end{bmatrix}. \end{aligned} \tag{19}$$

From Lemma 2, it can be easily seen that $\Pi < 0$ if and only if there exists a positive scalar ε_1 such that

$$\Psi := \begin{bmatrix} \breve{A}\hat{P} + \hat{P}\breve{A}^T & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} + \varepsilon_1 \begin{bmatrix} \bar{\Gamma} \\ 0 \end{bmatrix} \begin{bmatrix} \bar{\Gamma}^T & 0 \end{bmatrix} + {\varepsilon_1}^{-1} \begin{bmatrix} \hat{P}\bar{Q}^T \\ 0 \end{bmatrix} \begin{bmatrix} \bar{Q}\hat{P} & 0 \end{bmatrix} < 0 \tag{20}$$

or, equivalently,

$$\hat{\Psi} := \begin{bmatrix} \breve{A}\hat{P} + \hat{P}\breve{A}^T & \Pi_{12} & \varepsilon_1 \bar{\Gamma} & \hat{P}\bar{Q}^T \\ * & \Pi_{22} & 0 & 0 \\ * & * & -\varepsilon_1 I & 0 \\ * & * & * & -\varepsilon_1 I \end{bmatrix} < 0. \tag{21}$$

Noting (8), we can rewrite (21) as

$$\hat{\Psi} = \begin{bmatrix} \hat{\Psi}_{11} & \bar{G} & \hat{P}\bar{M}^T + \hat{P}(\bar{N}\breve{F}\bar{K}\bar{C})^T & \hat{P}\bar{H}^T & \varepsilon_1\bar{\Gamma} & \hat{P}\bar{Q}^T \\ * & -\gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -\hat{P} & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & -\varepsilon_1 I \end{bmatrix} < 0,$$
$$\hat{\Psi}_{11} = \bar{A}\hat{P} + \hat{P}\bar{A}^T + (\bar{B}\breve{F}\bar{K}\bar{C})\hat{P} + \hat{P}(\bar{B}\breve{F}\bar{K}\bar{C})^T. \tag{22}$$

Now, rewrite F as the following form

$$F = F_0 + \tilde{F}\chi$$

with $\chi = \tilde{F}^{-1}\Delta F$ satisfying $\chi\chi^T \leq I$. Letting $\bar{\chi} = I_N \otimes \chi$, we can obtain that

$$\breve{F} = \bar{F}_0 + \hat{F}\bar{\chi}. \tag{23}$$

By considering the fact that equations (14) and (16) mean $Y\bar{C} = \bar{K}\bar{C}\hat{P}$, it follows from (23) that

$$\begin{aligned} \bar{A}\hat{P} + \hat{P}\bar{A}^T + \bar{B}\breve{F}Y\bar{C} + \bar{C}^TY^T\breve{F}^T\bar{B}^T =& \bar{A}\hat{P} + \hat{P}\bar{A}^T + \bar{B}\bar{F}_0Y\bar{C} + \bar{C}^TY^T\bar{F}_0{}^T\bar{B}^T + \bar{B}\hat{F}\bar{\chi}Y\bar{C} + \bar{C}^TY^T\bar{\chi}^T\hat{F}^T\bar{B}^T, \\ \hat{P}\bar{M}^T + \bar{C}^TY^T\breve{F}^T\bar{N}^T =& \hat{P}\bar{M}^T + \bar{C}^TY^T\bar{F}_0{}^T\bar{N}^T + \bar{C}^TY^T\bar{\chi}^T\hat{F}^T\bar{N}^T. \end{aligned}$$

Then, the matrix $\hat{\Psi}$ in (22) can be rewritten as

$$\hat{\Psi} = \hat{\Psi}_0 + \Theta\bar{\chi}\tilde{Y} + \tilde{Y}^T\bar{\chi}^T\Theta^T \tag{24}$$

where

$$\begin{aligned} \Theta &= \begin{bmatrix} \hat{F}^T\bar{B}^T & 0 & \hat{F}^T\bar{N}^T & 0 & 0 & 0 \end{bmatrix}^T, \\ \tilde{Y} &= \begin{bmatrix} Y\bar{C} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

By using Lemma 2 again, it can be obtained that $\hat{\Psi} < 0$ if and only if there exists a positive scalar ε_2 such that

$$\hat{\Psi}_0 + \varepsilon_2\Theta\Theta^T + \varepsilon_2{}^{-1}\tilde{Y}^T\tilde{Y} < 0$$

which is equivalent to $\breve{\Psi} < 0$. The rest of proof follows directly from Theorem 1 and hence the proof of Theorem 2 is complete. ■

To this end, we have derived a sufficient condition under which 1) the system (7) with $v(t) = 0$ is globally asymptotically stable in probability, and 2) the H_∞ performance constraint is satisfied in the presence of all possible uncertainties and actuator failure. Based on the condition obtained, the desired robust reliable H_∞ output feedback controller has been designed. In the next section, a simulation example is provided to illustrate the effectiveness of the proposed control scheme.

IV. Illustrative Examples

In this section, a numerical simulation example is presented to demonstrate the effectiveness of the methods proposed in this paper.

Consider a second-order uncertain spatial-temporal system described by (1) with the following parameters

$$
\begin{aligned}
&\kappa = \begin{bmatrix} 6 & -10 \\ 5 & 8 \end{bmatrix}, \quad A = \begin{bmatrix} -30 & -37 \\ 1 & 5 \end{bmatrix}, \\
&B = \begin{bmatrix} -40 & -10 \\ -15 & -20 \end{bmatrix}, \quad H = \begin{bmatrix} 0.8 & 0 \\ 0.5 & 0.9 \end{bmatrix}, \\
&D = \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \quad G = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \\
&Q_1 = \begin{bmatrix} 0.3 & 0.3 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 0.2 & 0.2 \end{bmatrix}, \\
&Q_3 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \quad M = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad \tilde{N} = 3.
\end{aligned}
$$

The rectangular region is given by $[0,3] \times [0,3]$. The initial conditions at the inner points are chosen as $x_{k,l}^0 = \begin{bmatrix} 100 \times e^{-10\times|0.06-0.03k|} \times e^{-30\times|0.06-0.03l|} & 50 \times e^{-10\times|0.06-0.03k|} \times e^{-30\times|0.06-0.03l|} \end{bmatrix}^T$ where $k = 1,2$ and $l = 1,2$. The boundary condition is assumed to be $x_{k,l}(t) = 0$ for $k = 0,\ 3$ or $l = 0,\ 3$. We consider $n_y = 4$ sensors which are located at points $(1,1)$, $(1,2)$, $(2,1)$ and $(2,2)$. The parameters of the sensors are given by

$$
\begin{aligned}
&\bar{T}_{k_1,l_1} = \bar{T}_{1,1} = \begin{bmatrix} 0.9 & 0.8 \end{bmatrix}, \quad \bar{T}_{k_2,l_2} = \bar{T}_{2,1} = \begin{bmatrix} 0.7 & 1 \end{bmatrix}, \\
&\bar{T}_{k_3,l_3} = \bar{T}_{1,2} = \begin{bmatrix} 0.6 & 0.9 \end{bmatrix}, \quad \bar{T}_{k_4,l_4} = \bar{T}_{2,2} = \begin{bmatrix} 1.2 & 0.9 \end{bmatrix}.
\end{aligned}
$$

The failures of actuators f_1 satisfy $\underline{f}_1 \leq f_1 \leq \overline{f}_1$ with $\underline{f}_1 = 0.1$ and $\overline{f}_1 = 0.9$. Therefore, it is easy to obtain that $F_0 = 0.5$ and $\tilde{F} = 0.4$. Moreover, the disturbance attenuation level is selected as $\gamma = 0.2$.

With the above parameters, we solve (13) and (14) by using LMI toolbox in Matlab and obtain the following parameters

$$Z=\begin{bmatrix}17.9674 & 0.7383 & 0.8162 & 0.2194\\ 0.7587 & 12.2720 & 0.3375 & 1.1050\\ 0.6586 & 0.2650 & 11.9191 & 1.0282\\ 0.3404 & 1.6687 & 1.9773 & 19.4544\end{bmatrix},$$

$$Y=\begin{bmatrix}0 & 0 & 0 & 0\\ -0.8483 & -0.1008 & -0.1136 & -0.0132\\ -0.0563 & -0.8169 & -0.0153 & -0.0781\\ 0 & 0 & 0 & 0\\ -0.8483 & -0.1008 & -0.1136 & -0.0132\\ -4.7222 & -0.1848 & -0.2081 & 0.0766\\ -0.3650 & -4.0532 & 0.0168 & -0.1131\\ 0.0561 & 0.1638 & -0.0377 & -0.0413\\ -0.0565 & -0.0133 & -0.9230 & -0.0779\\ -0.3608 & 0.0162 & -4.5330 & -0.1107\\ -0.1701 & -0.4387 & -0.5048 & -3.6865\\ 0.0356 & 0.0132 & 0.0127 & 0.2603\\ 0 & 0 & 0 & 0\\ 0.0561 & -0.0328 & 0.1731 & -0.0415\\ 0.0356 & 0.0132 & 0.0127 & 0.2603\\ 0 & 0 & 0 & 0\end{bmatrix}.$$

From (16), the desired controller parameters can then be obtained as follows:

$$\bar{K}=YZ^{-1}=\begin{bmatrix}0 & 0 & 0 & 0\\ -0.0468 & -0.0053 & -0.0063 & 0.0005\\ -0.0003 & -0.0665 & 0.0007 & -0.0003\\ 0 & 0 & 0 & 0\\ -0.0468 & -0.0053 & -0.0063 & 0.0005\\ -0.2629 & -0.0002 & -0.0006 & 0.0069\\ -0.0069 & -0.3318 & 0.0092 & 0.0126\\ 0.0027 & 0.0136 & -0.0033 & -0.0028\\ -0.0003 & 0.0006 & -0.0774 & 0.0001\\ -0.0067 & 0.0081 & -0.3824 & 0.0141\\ -0.0051 & -0.0096 & -0.0105 & -0.1883\\ 0.0018 & -0.0008 & -0.0013 & 0.0135\\ 0 & 0 & 0 & 0\\ 0.0027 & -0.0028 & 0.0149 & -0.0028\\ 0.0018 & -0.0008 & -0.0013 & 0.0135\\ 0 & 0 & 0 & 0\end{bmatrix}.$$

In the simulation, the external disturbance is taken as $v_{k,l}(t)=\sin(0.2t)\times e^{-0.2t-0.1\times 0.03k}$. The parameter uncertainty is set as $E(t)=0.5\sin(t)$. The actuator fault matrix is chosen as $F=0.75$. The simulation results are shown in Figs. 1-6. Figs. 1 and 2 depict the state trajectories at all inner points without the control inputs. Figs. 3 and 4 show state trajectories at all inner points with the designed control inputs. Figs. 5 and 6 present state plans when time instant $t = 0$ and $t = 30$, respectively. It can be observed from Figs. 5 and 6 that all the state trajectories of controlled system converge to an equilibrium point. The simulation results have confirmed that the designed controller performs very well.

V. Conclusions

In this paper, we have studied the robust reliable H_∞ output-feedback control problem for a class of stochastic spatial-temporal systems in a given discrete rectangular region. Both actuator failures and parameter uncertainties have been taken into consideration. Sensors have been fitted at the positions of interest in the rectangular region and each sensor can receive the measurements from the node in which the sensor has been located. Based on these measurements, the static output-feedback reliable control scheme has been employed. With the help of the Lyapunov stability theory, a sufficient condition has been obtained under which the closed-loop spatial-temporal system is globally asymptotically stable in probability and the H_∞ performance requirements are met. Then, if the existence condition is satisfied, we have designed the desired robust reliable H_∞ controller by solving a matrix inequality. Finally, we have discussed the effectiveness of the proposed control scheme by a numerical simulation example. Our future research topic would be to study more phenomena of incomplete information (see, e.g., polynomial stochastic systems [1–3] and missing measurements in [9] with probability-dependent descriptions [24, 31, 33]) in the spatial-temporal systems and give the corresponding the control schemes.

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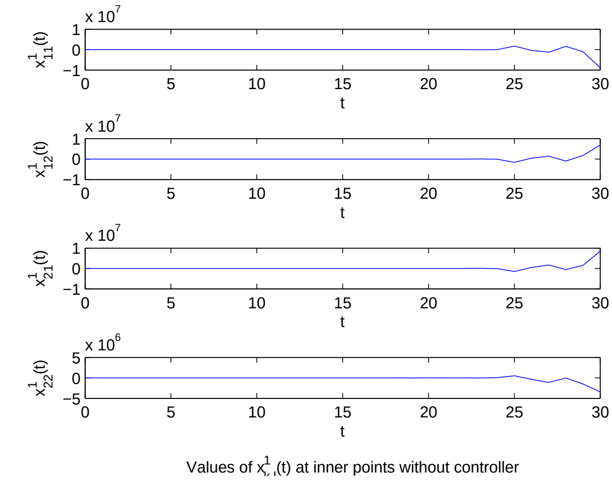

Fig. 1. Values of $x^1_{k,l}(t)$ at inner points without controller.

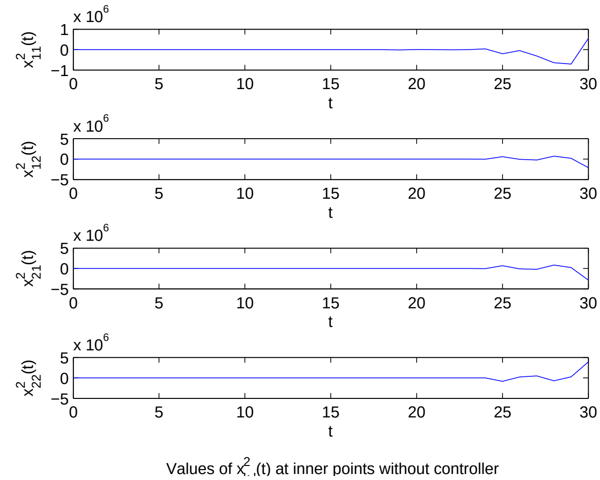

Fig. 2. Values of $x^2_{k,l}(t)$ at inner points without controller.

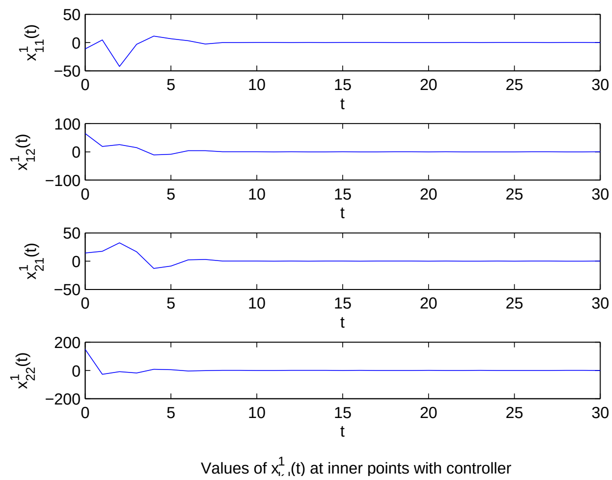

Fig. 3. Values of $x^1_{k,l}(t)$ at inner points with controller.

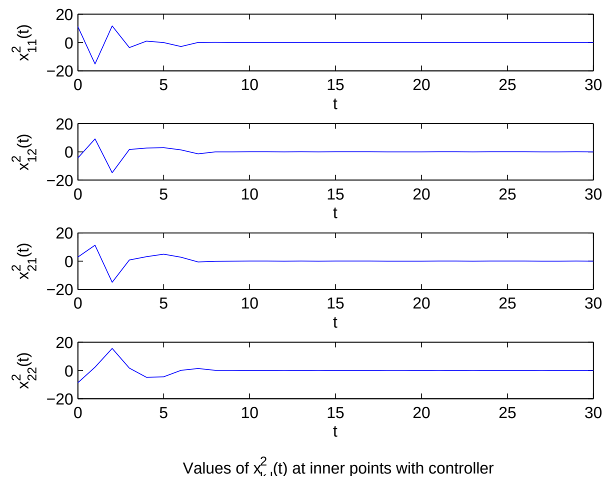

Fig. 4. Values of $x^2_{k,l}(t)$ at inner points with controller.

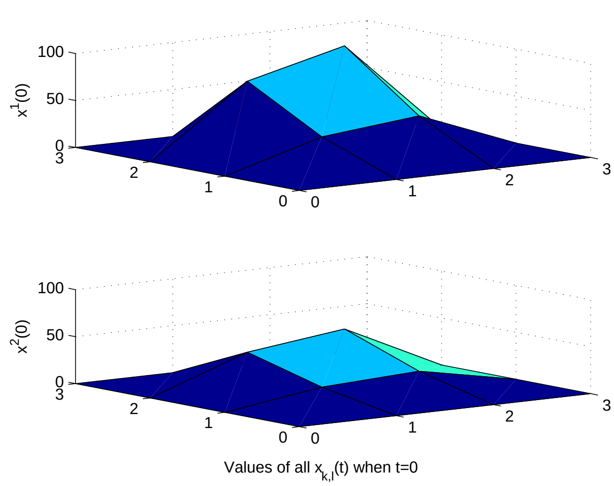

Fig. 5. Values of all $x_{k,l}(t)$ when $t = 0$.

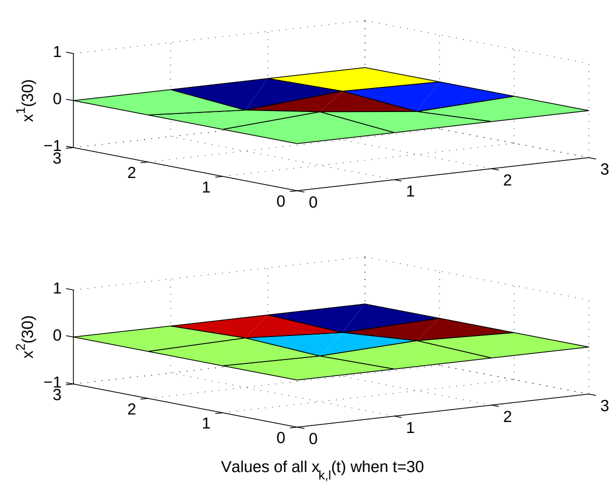

Fig. 6. Values of all $x_{k,l}(t)$ when $t = 30$.