Receding Horizon Filtering for a Class of Discrete Time-Varying Nonlinear Systems with Multiple Missing Measurements

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This paper is concerned with the receding horizon filtering problem for a class of discrete time-varying nonlinear systems with multiple missing measurements. The phenomenon of missing measurements occurs in a random way and the missing probability is governed by a set of stochastic variables obeying the given Bernoulli distribution. By exploiting the projection theory combined with stochastic analysis techniques, a Kalman-type receding horizon filter is put forward to facilitate the online applications. Furthermore, by utilizing the conditional expectation, a novel estimation scheme of state covariance matrices is proposed to guarantee the implementation of the filtering algorithm. Finally, a simulation example is provided to illustrate the effectiveness of the established filtering scheme.

Keywords: Receding horizon filtering; discrete time-varying systems; stochastic nonlinear; multiple missing measurements

1.   Introduction

In signal processing and control areas, the Kalman filter (KF) has been regarded as an efficiently recursive filter to estimate unknown system states by utilizing measurements. However, since undesirable signals with internal states may accumulate, the traditional KF algorithms might suffer from persistent modeling errors (Fitzgerald, 1971; Sangsuk & Bullock, 1990) or numerical errors (Grewal & Anderews, 1993). As such, it could take a long time to converge to a real state even though the temporary undesirable errors disappear. To improve its performance, an alternative filter, namely, the receding horizon filter, has been proposed in the past few years. The main idea of this kind of filter is to estimate the state vector in terms of finite measurement samples, and then discard the oldest measurement sample as a new sample becomes available. It is not difficult to see that the memory length of the filter is fixed. As discussed by Kwon et al. (1994), the receding horizon filter owns a series of good numerical characteristics such as the bounded input bounded output (BIBO) stability, the low round-off noise, the no limit cycles compared to the infinite impulse response(IIR), and so forth. Therefore, such

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a filtering approach has been widely applied in various engineering problems, for instance, signal processing, control system design, mobile target tracking, wireless positioning system, see Alessandri et al. (2008, 2003); Buxbaum et al. (1974); Friedlander et al. (1978); Jazwinski (1968); Kim (2010); Kwon et al. (1989, 1994, 2002); Park et al. (1997); Shmaliy et al. (2008); Shmaliy (2008) and the references therein.

As the ever-increasing popularity of communication networks, the limited bandwidth of the communication channel inevitably leads to some network induced phenomena which have gained much research effort in the past decade (Hounkpevi & Yaz, 2007; Kluge et al., 2010; Liang et al., 2014; Liu et al., 2014; Sahebsara et al., 2007b; Sinopoli et al., 2004; Wang et al., 2003). Recently, the so-called multiple missing measurement has been considered to account for the case that each sensor owns a different occurrence probability of missing measurements in an array of sensors, see (Bouibed et al., 2014; Dong et al., 2010; Hu et al., 2012; Shen et al., 2010; Shen, 2014; Wei et al., 2009) for more details. On the other hand, nonlinearities exist universally in practice that have long been one of the main research streams. In a networked environment, the nonlinearities might stem from the random fluctuation of the network load and the unreliability of the wireless links and therefore such nonlinearities themselves could experience random abrupt changes in their type or intensity, see e.g. (He et al., 2013; Hu et al., 2012, 2013; Jacobson, 1974; Wei & Yaz, 2001). As such, it seems more reasonable to design the receding horizon filter by taking both the multiple missing measurements and the stochastic nonlinearities into account.

To the best of the authors' knowledge, the receding horizon filtering problem for discrete time-varying systems with multiple missing measurements or stochastic nonlinearities has not been adequately investigated, despite its clear engineering insight. The main reason is that the introduced phenomena would pose significant challenges on the design of receding horizon filters. For instance, in this case, the mean and covariance of the initial state would be unknown in the sliding-window because *a priori* information is hardly available. Furthermore, traditional methods handling covariance matrices of filtering error, such as setting values (e.g. ∞I), least-squares techniques or maximum-likelihood methods, are no longer valid due primarily to the coupling between the filtering error and the system state. As such, the purpose of this paper is to shorten such a gap.

In this paper, we aim to examine the impact of missing measurements and stochastic nonlinearities on the design of receding horizon filters. By employing the projection theory combined with the stochastic analysis techniques, a receding horizon Kalman-type filter is designed and a novel estimation method is provided to obtain the covariance matrix of the filtering error. Moreover, a simulation example is proposed to show the effectiveness of the established receding horizon filtering scheme. The main contributions of this paper are outlined as follows: *1) the system under consideration is comprehensive to cover several network-induced phenomena; 2) a kind of receding horizon filter is derived for the purpose of online applications; and 3) a novel estimation scheme for the state covariance matrix is provided by employing the augmented method in order to guarantee the implementation of the filtering algorithm.*

The rest of this paper is organized as follows. In Section 2, a class of discrete time-varying nonlinear systems with multiple missing measurements and stochastic nonlinearities are introduced. In Section 3, by using the projection theory, some recursive algorithms of receding horizon filtering are derived, and then the estimation of filtering error covariance matrix is carried out in terms of the conditional expectation. Furthermore, a simulation example is given in Section 4 to demonstrate

the effectiveness. Finally, we conclude the paper in Section 5.

Notation The notation used here is fairly standard except where otherwise stated. R^n and $R^{n \times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices. I denotes the identity matrix of compatible dimension. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). A^T and $r(A)$ represent the transpose and rank of the matrix A, respectively. Moreover, we may fix a probability space $(\Omega, \mathscr{F}, \mathrm{Prob})$, where Prob, the probability measure, has total mass 1. $E\{x\}$ stands for the expectation of the stochastic variable x with respect to the given probability measure Prob. $\mathrm{diag}\{\cdots\}$ stands for a block-diagonal matrix. The symbol $\otimes$ and $\circ$ denote the Kronecker product and the Hadamard product, respectively.

2.   Main strategies of distributed filtering

Consider the following discrete time-varying nonlinear system:

$$\begin{cases} x_{k+1} = A_k x_k + f(x_k, \vartheta_k) + D_k w_k \\ y_k = \Xi_k C_k x_k + E_k v_k \end{cases} \tag{1}$$

where $x_k \in R^{n_x}$ represents the state vector, $y_k \in R^{n_y}$ is the measurement output, and $w_k \in R^p$, $v_k \in R^q$ and $\vartheta_k \in R^s$ are mutually independent zero-mean white Gaussian sequences with covariances Q, R and I, respectively. A_k, C_k, D_k and E_k are known matrices with appropriate dimensions. $\Xi_k = \mathrm{diag}\{\alpha_{1,k}, \alpha_{2,k}, \cdots, \alpha_{n_y,k}\}$ with $\alpha_{i,k}$ $(i = 1, 2, \cdots, n_y)$ being n_y unrelated random variables, which are also unrelated with w_k, v_k, ϑ_k and x_0. Furthermore, the stochastic variable $\alpha_{i,k}$ is assumed to be a Bernoulli-distributed white sequence taking values on 0 or 1 with:

$$\mathrm{Prob}\{\alpha_{i,k} = 0\} = 1 - \bar{\alpha}_i, \quad \mathrm{Prob}\{\alpha_{i,k} = 1\} = \bar{\alpha}_i.$$

The function $f(x_k, \vartheta_k)$ with $f(0, \vartheta_k) = 0$ is a stochastic nonlinearity (Wei et al., 2009) having the following first moment for all x_k:

$$E\{f(x_k, \vartheta_k)|x_k\} = 0 \tag{2}$$

and the covariances given by

$$E\{f(x_k, \vartheta_k)f^T(x_i, \vartheta_i)|x_k\} = 0, \quad i \neq k, \tag{3}$$

and

$$E\{f(x_k, \vartheta_k)f^T(x_k, \vartheta_k)|x_k\} = \sum_{i=1}^m \Pi_i x_k^T \Gamma_i x_k, \tag{4}$$

where m is a known integer, Π_i and Γ_i are known matrices with appropriate dimensions.

As discussed in Introduction, this paper focuses on the receding horizon filtering problem, where the length of sliding-window is $N + 1$. Generally speaking, the receding horizon filter employs only the finite measurements on the interval $[k - N, k]$ and discards the oldest measurements when a new measurement is available. Denoted the information set as $\mathcal{I}_k^N = \{y_{k_N}, y_{k_N+1}, \cdots, y_k\}$ with $k_N = k - N$, the

filter is described by

$$\hat{x}_{(k|k;N)} = \sum_{i=k_N}^{k} H_{[k,i;N]}y_i, \quad y_i \in \mathcal{I}_k^N \tag{5}$$

where $\hat{x}_{(k|k;N)}$ denotes the state estimate of system (1) at time k and $H_{[k,i;N]}$ is the filter parameter to be designed.

Our aim in this paper is to design a filter with the form (5) for system (1) with stochastic nonlinearities and missing measurements. In other words, we are going to determine the filter parameters $H_{[k,i;N]}$ ($k_N \leq i \leq k$) such that the following orthogonality is satisfied:

$$\mathrm{E}\Big\{ (x_k - \hat{x}_{(k|k;N)})y_i^T \Big\} = 0, \quad k_N \leq i \leq k. \tag{6}$$

3. Main results

In this section, we will provide a recursive algorithm of the receding horizon filter and then derive an estimation of state covariance matrix for the discrete time-varying nonlinear systems (1) to apply the established recursive algorithm.

3.1 *The design of receding horizon filter*

Before proceeding further, we introduce the following lemma which provides a fundamental iteration algorithm to design the filter parameter $H_{[k,i;N]}$ by handling a set of Riccati-type difference equations. Note that this lemma can be showed along the similar lines of the proofs of Theorem 2 in Kwon et al. (1989) and Theorem 2.1 in Kwon et al. (1994), and therefore the proof is omitted to the space saving.

Lemma 1. *Let the state covariance matrix* $\mathrm{E}\{x_{k_N-1}x_{k_N-1}^T | \mathcal{I}_k^N\} := \mathcal{P}_{[k_N-1,k_N-1;N]}$ *be given. For the discrete time-varying nonlinear system (1) with multiple missing measurements, the filter parameters are uniquely determined by the following iteration algorithms*
1) $H_{[k,i;N]} = H_{[k,i;n]}|_{n=N}$ *for the fixed k and N, where $H_{[k,i;n]}$ satisfies the recursive matrix difference equations*

$$\begin{cases} H_{[k,i;i-k_N]} = \mathcal{R}_{[k,i-k_N;N]}C_i^T \Xi \mathcal{S}_{[i;N]}^{-1}, & \text{(7a)} \\[2mm] H_{[k,i;n+1]} = \Big(I - \mathcal{R}_{[k,n+1;N]}C_{k_N+n+1}^T \Xi \mathcal{S}_{[k_N+n+1;N]}^{-1} \Xi C_{k_N+n+1} \Big) A_{k_N+n}H_{[k,i;n]}, \\[2mm] \qquad\qquad\qquad\qquad k_N \leq i \leq k, \quad i - k_N \leq n \leq N-1; & \text{(7b)} \end{cases}$$

or 2) $H_{[k,i;N]}$ *calculated directly by*

$$\begin{cases} H_{[k,i;N]} = \Phi_{[N,i-k_N]}^k \mathcal{R}_{[k,i-k_N;N]}C_i^T \Xi \mathcal{S}_{[i;N]}^{-1}, & k_N \leq i \leq k, \quad \text{(8a)} \\[2mm] \Phi_{[N,m]}^k = \Phi_{[N,m+1]}^k \Big(I - \mathcal{R}_{[k,m+1;N]}C_{k_N+m+1}^T \Xi \mathcal{S}_{[k_N+m+1;N]}^{-1} \Xi C_{k_N+m+1} \Big) A_{k_N+m}, \\[2mm] \qquad\qquad\qquad\qquad 0 \leq m \leq N-1, & \text{(8b)} \\[2mm] \Phi_{[N,N]}^k = I, & \text{(8c)} \end{cases}$$

where

$$\mathcal{R}_{[k,s+1;N]} = \bar{\mathcal{R}}_{[k,s;N]} - \bar{\mathcal{R}}_{[k,s;N]}C_{k_N+s+1}^T \Xi \tilde{\mathcal{S}}_{[k_N+s+1;N]}^{-1} \Xi C_{k_N+s+1}\bar{\mathcal{R}}_{[k,s;N]},$$

$$\bar{\mathcal{R}}_{[k,s;N]} = A_{k_N+s}\mathcal{R}_{[k,s;N]}A_{k_N+s}^T + \sum_{l=1}^{m}\Pi_l tr(\mathcal{P}_{[k_N+s,k_N+s;N]}\Gamma_l) + D_{k_N+s}QD_{k_N+s}^T,$$

$$\tilde{\mathcal{S}}_{[k_N+n+1;N]} = \mathcal{S}_{[k_N+n+1;N]} + \Xi C_{k_N+n+1}\bar{\mathcal{R}}_{[k,n;N]}C_{k_N+n+1}^T\Xi, \quad s \geq -1,$$

$$\mathcal{P}_{[r+1,r+1;N]} = A_r\mathcal{P}_{[r,r;N]}A_r^T + \sum_{l=1}^{m}\Pi_l tr(\mathcal{P}_{[r,r;N]}\Gamma_l) + D_r QD_r^T, \quad r \geq k_N - 1,$$

$$\mathcal{R}_{[k,-1;N]} = \mathcal{P}_{[k_N-1,k_N-1;N]}, \quad \mathcal{S}_{[r;N]} = \tilde{\Xi} \circ (C_r\mathcal{P}_{[r,r;N]}C_r^T) + E_r RE_r^T,$$

$$\Xi = diag\{\bar{\alpha}_1, \bar{\alpha}_2, \cdots, \bar{\alpha}_{n_y}\}, \quad \tilde{\Xi} = diag\{\bar{\alpha}_1(1-\bar{\alpha}_1), \ \bar{\alpha}_2(1-\bar{\alpha}_2), \cdots, \bar{\alpha}_{n_y}(1-\bar{\alpha}_{n_y})\}.$$

In the above lemma, two algorithms of the receding horizon filter have been provided. Now, it is interesting to investigate how to modify the filter parameters by using the obtained parameters when the length of sliding-windows is increased, and how to design a Kalman-type receding horizon filter for the easy implementation. As such, in terms of the information sets $\mathcal{I}_{k+1}^{N+1}$ and $\mathcal{I}_k^N$, the relationship on the designed filter parameters is revealed in the following theorem.

Theorem 1. *Let $\mathcal{P}_{[k_N-1,k_N-1;N]}$ and $\mathcal{P}_{[k_N-1,k_N-1;N+1]}$ be given. For the discrete time-varying nonlinear systems (1) with multiple missing measurements, there exists the following Kalman-type receding horizon filter*

$$\hat{x}_{(k+1|k+1;N+1)} = A_k\hat{x}_{(k|k;N)} + K_{k+1}(y_{k+1} - \Xi C_{k+1}A_k\hat{x}_{(k|k;N)}) \tag{9}$$

with the filter gain

$$K_{k+1} = (\bar{\mathcal{R}}_{[k,N;N]} + \Delta_{k+1}^{\mathcal{H}})C_{k+1}^T \Xi \Upsilon_{k+1}^{-1} \tag{10}$$

and the filtering error covariance

$$\mathcal{R}_{[k+1,N+1;N+1]} = \frac{1}{2}\left\{(I - K_{k+1}\Xi C_{k+1})\Delta_{\mathcal{R}}^{\mathcal{H}} + (\Delta_{\mathcal{R}}^{\mathcal{H}})^T(I - K_{k+1}\Xi C_{k+1})^T\right\}$$

where

$$\mathcal{A}_{(k:i)} := A_k A_{k-1}\cdots A_i \ (\forall k > i), \quad \mathcal{A}_{(i-1:i)} := I,$$

$$\Delta_{[s,k+1]}^{\mathcal{P}} = \mathcal{P}_{[s,k+1;N+1]} - \mathcal{P}_{[s,k+1;N]} = (\mathcal{P}_{[s,s;N+1]} - \mathcal{P}_{[s,s;N]})\mathcal{A}_{(k:s)}^T,$$

$$\Delta_{k+1}^{\mathcal{H}} = \Delta_{[k+1,k+1]}^{\mathcal{P}} - A_k\sum_{s=k_N}^{k} H_{[k,s;N]}\Xi C_s\Delta_{[s,k+1]}^{\mathcal{P}},$$

$$\Delta_{\mathcal{R}}^{\mathcal{H}} = A_k\mathcal{R}_{[k,N;N]}A_k^T + \sum_{l=1}^{m}\Pi_l tr(\mathcal{P}_{[k,k;N]}\Gamma_l) + D_k QD_k^T + \Delta_{k+1}^{\mathcal{H}}$$

$$\Upsilon_{k+1} = \Xi C_{k+1}(\bar{\mathcal{R}}_{[k,N;N]} + \Delta_{k+1}^{\mathcal{H}})C_{k+1}^T\Xi + \mathcal{S}_{[k+1;N+1]}.$$

Furthermore, the relationship between $H_{[k+1,i;N+1]}$ and $H_{[k,i;N]}$ can be described by

$$\begin{cases} H_{[k+1,k+1;N+1]} = K_{k+1}, \\ H_{[k+1,i;N+1]} = (A_k - K_{k+1}\Xi C_{k+1}A_k)H_{[k,i;N]}, \quad k_N \le i \le k. \end{cases} \tag{11}$$

Proof. First, (9) can be rewritten as

$$\begin{aligned} \hat{x}_{(k+1|k+1;N+1)} &= (A_k - K_{k+1}\Xi C_{k+1}A_k)\hat{x}_{(k|k;N)} + K_{k+1}y_{k+1} \\ &= \sum_{i=k_N}^{k} (A_k - K_{k+1}\Xi C_{k+1}A_k)H_{[k,i;N]}y_i + K_{k+1}y_{k+1}. \end{aligned} \tag{12}$$

Then, for $k_N \le i \le k$, it is not difficult to see that

$$\begin{aligned} &\mathrm{E}\Big\{ (x_{k+1} - \hat{x}_{(k+1|k+1;N+1)})y_i^T \big| \mathcal{I}_{k+1}^{N+1} \Big\} \\ &= \mathrm{E}\Big\{ (x_{k+1} - A_k\hat{x}_{(k|k;N)})y_i^T \big| \mathcal{I}_{k+1}^{N+1} \Big\} \\ &\quad - K_{k+1}\mathrm{E}\Big\{ (y_{k+1} - \Xi C_{k+1}A_k\hat{x}_{(k|k;N)})y_i^T \big| \mathcal{I}_{k+1}^{N+1} \Big\} \\ &= \mathrm{E}\Big\{ A_k(x_k - \hat{x}_{(k|k;N)})y_i^T \big| \mathcal{I}_{k+1}^{N+1} \Big\} + \mathrm{E}\Big\{ (f(x_k,\vartheta_k) + D_k w_k)y_i^T \big| \mathcal{I}_{k+1}^{N+1} \Big\} \\ &\quad - K_{k+1}\mathrm{E}\Big\{ \Xi C_{k+1}A_k(x_k - \hat{x}_{(k|k;N)})y_i^T \big| \mathcal{I}_{k+1}^{N+1} \Big\} \\ &\quad - K_{k+1}\mathrm{E}\Big\{ \Big((\Xi_{k+1} - \Xi)C_{k+1}A_k x_k \\ &\qquad + \Xi_{k+1}C_{k+1}(f(x_k,\vartheta_k) + D_k w_k) + E_{k+1}v_{k+1} \Big)y_i^T \big| \mathcal{I}_{k+1}^{N+1} \Big\} \\ &= 0. \end{aligned} \tag{13}$$

Similarly, for $i = k + 1$, one has

$$\begin{aligned} &\mathrm{E}\Big\{ (x_{k+1} - \hat{x}_{(k+1|k+1;N)})y_{k+1}^T \big| \mathcal{I}_{k+1}^{N+1} \Big\} \\ &= \mathrm{E}\Big\{ \Big(x_{k+1} - A_k\hat{x}_{(k|k;N)} \Big)y_{k+1}^T - K_{k+1}\Xi C_{k+1}\Big(x_{k+1} - A_k\hat{x}_{(k|k;N)} \Big)y_{k+1}^T \\ &\quad - K_{k+1}\Big((\Xi_{k+1} - \Xi)C_{k+1}x_{k+1} + E_{k+1}v_{k+1} \Big)y_{k+1}^T \big| \mathcal{I}_{k+1}^{N+1} \Big\} \\ &= (I - K_{k+1}\Xi C_{k+1})\mathrm{E}\Big\{ \Big(x_{k+1} - A_k\hat{x}_{(k|k;N)} \Big)y_{k+1}^T \big| \mathcal{I}_{k+1}^{N+1} \Big\} - K_{k+1}\mathcal{S}_{[k+1;N+1]}. \end{aligned} \tag{14}$$

On the other hand, similar to Kwon et al. (1989), over the interval $k_N \le i \le k_N + n$, one has

$$\mathcal{R}_{[k,n;N]} := \mathcal{P}_{[k_N+n,k_N+n;N]} - \sum_{s=k_N}^{k_N+n} H_{[k,s;N]}\Xi C_s \mathcal{P}_{[s,k_N+n;N]}. \tag{15}$$

Then, it follows from (14) and (15) that

$$
\begin{aligned}
&\mathrm{E}\Big\{\Big(x_{k+1} - A_k\hat{x}_{(k|k;N)}\Big)y_{k+1}^T\big|\mathcal{I}_{k+1}^{N+1}\Big\}\\
&= \mathrm{E}\Big\{x_{k+1}x_{k+1}^T C_{k+1}^T\Xi_{k+1}\big|\mathcal{I}_{k+1}^{N+1}\Big\} - \sum_{s=k_N}^{k} A_k H_{[k,s;N]}\Xi C_s \mathrm{E}\big\{x_s x_{k+1}^T\big|\mathcal{I}_{k+1}^{N+1}\big\}C_{k+1}^T\Xi\\
&= \Big(\mathcal{P}_{[k+1,k+1;N+1]} - A_k\sum_{s=k_N}^{k} H_{[k,s;N]}\Xi C_s\mathcal{P}_{[s,k;N+1]}A_k^T\Big)C_{k+1}^T\Xi\\
&= \Big(\mathcal{P}_{[k+1,k+1;N+1]} + A_k\big(\mathcal{R}_{[k,N;N]} - \mathcal{P}_{[k,k;N]}\big)A_k^T\Big)C_{k+1}^T\Xi\\
&\quad - A_k\sum_{s=k_N}^{k} H_{[k,s;N]}\Xi C_s\Delta_{[s,k]}^{\mathcal{P}}A_k^T C_{k+1}^T\Xi\\
&= \Big(A_k\mathcal{R}_{[k,N;N]}A_k^T + \sum_{l=1}^{m}\Pi_l\mathrm{tr}\big(\mathcal{P}_{[k,k;N]}\Gamma_l\big) + D_k Q D_k^T\Big)C_{k+1}^T\Xi\\
&\quad + \Big(\Delta_{[k+1,k+1]}^{\mathcal{P}} - A_k\sum_{s=k_N}^{k} H_{[k,s;N]}\Xi C_s\Delta_{[s,k+1]}^{\mathcal{P}}\Big)C_{k+1}^T\Xi\\
&= \big(\bar{\mathcal{R}}_{[k,N;N]} + \Delta_{k+1}^{\mathcal{H}}\big)C_{k+1}^T\Xi.
\end{aligned}
\tag{16}
$$

Therefore, in light of the orthogonality, substituting (16) into (14) results in (10).

Finally, we can calculate the filtering error covariance:

$$
\begin{aligned}
&\mathcal{R}_{[k+1,N+1;N+1]}\\
&= \mathrm{E}\Big\{\big(x_{k+1} - \hat{x}_{(k+1|k+1;N+1)}\big)\big(x_{k+1} - \hat{x}_{(k+1|k+1;N+1)}\big)^T\big|\mathcal{I}_{k+1}^{N+1}\Big\}\\
&= \frac{1}{2}\mathrm{E}\Big\{\big(x_{k+1} - \hat{x}_{(k+1|k+1;N+1)}\big)x_{k+1}^T + x_{k+1}\big(x_{k+1} - \hat{x}_{(k+1|k+1;N+1)}\big)^T\big|\mathcal{I}_{k+1}^{N+1}\Big\}\\
&= \frac{1}{2}\mathrm{E}\Big\{\Big[\big(I - K_{k+1}\Xi C_{k+1}\big)A_k\big(x_k - \hat{x}_{(k|k;N)}\big) + \big(I - K_{k+1}\Xi_{k+1}C_{k+1}\big)\big(f(x_k,\vartheta_k) + D_k w_k\big)\\
&\quad - K_{k+1}(\Xi_{k+1} - \Xi)C_{k+1}A_k x_k - K_{k+1}E_{k+1}v_{k+1}\Big]\Big[A_k x_k + f(x_k,\vartheta_k) + D_k w_k\Big]^T\\
&\quad + \Big[A_k x_k + f(x_k,\vartheta_k) + D_k w_k\Big]\Big[\big(I - K_{k+1}\Xi C_{k+1}\big)A_k\big(x_k - \hat{x}_{(k|k;N)}\big)\\
&\quad + \big(I - K_{k+1}\Xi_{k+1}C_{k+1}\big)\big(f(x_k,\vartheta_k) + D_k w_k\big)\\
&\quad - K_{k+1}(\Xi_{k+1} - \Xi)C_{k+1}A_k x_k - K_{k+1}E_{k+1}v_{k+1}\Big]^T\big|\mathcal{I}_{k+1}^{N+1}\Big\}\\
&= \frac{1}{2}\Big\{\big(I - K_{k+1}\Xi C_{k+1}\big)\Delta_{\mathcal{R}}^{\mathcal{H}} + \big(\Delta_{\mathcal{R}}^{\mathcal{H}}\big)^T\big(I - K_{k+1}\Xi C_{k+1}\big)^T\Big\},
\end{aligned}
\tag{17}
$$

which completes the proof. $\square$

3.2   *The estimation of state covariance matrix*

As discussed in Introduction, the state covariance matrix $\mathcal{P}_{[k_N-1,k_N-1;N]}$ has to be estimated by means of the known measurement set $\mathcal{I}_k^{N+1}$. Obtaining such an estimation can be divided into two steps, that is, 1) obtaining the unbiased estimate $\hat{x}_{(k_N-1|k)}$ and 2) calculating the state covariance matrix. Now, let us introduce the following assumption. It should be pointed out that, for the case of linear time-invariant systems, such an assumption is equivalent to the observability condition which is a basic requirement for state estimation problems.

Assumption 1. *For all $k \geq 0$, $r(E_k R E_k^T) = n_y$ and $r(\chi_k) = n_x$, where*

$$\chi_k := \begin{bmatrix} C_{k_n-1}^T & A_{k-1}^T C_{k_n}^T & \cdots & (A_{k-1}A_{k-2}\cdots A_{k_n+n-1})^T C_k^T \end{bmatrix}^T, \quad n \geq n_x.$$

Firstly, let us obtain the unbiased estimate $\hat{x}_{(k_N-1|k)}$ of the initial state x_{k_N-1} on each sliding-window. For this purpose, denoting

$$Y_k = [y_k^T,\ y_{k-1}^T,\ \cdots,\ y_{k_N-1}^T]^T, \quad \tilde{w}_k = [w_{k-1}^T,\ w_{k-2}^T,\ \cdots,\ w_{k_N-1}^T]^T,$$

$$\tilde{v}_k = [v_k^T,\ v_{k-1}^T,\ \cdots,\ v_{k_N-1}^T]^T, \quad \Theta_k = \mathrm{diag}\{\Xi_k, \Xi_{k-1}, \cdots, \Xi_{k_N-1}\},$$

$$\tilde{f}_k = [\, f^T(x_{k-1}, \vartheta_{k-1})\ f^T(x_{k-2}, \vartheta_{k-2}) \cdots f^T(x_{k_N-1}, \vartheta_{k_N-1})\,]^T,$$

one has

$$Y_k = \Theta_k \mathcal{C}_k x_{k_N-1} + \Theta_k \mathcal{F}_k \tilde{f}_k + \Theta_k \mathcal{D}_k \tilde{w}_k + \mathcal{E}_k \tilde{v}_k, \tag{18}$$

where

$$\mathcal{C}_k = \begin{bmatrix} \mathcal{A}_{(k-1:k_N-1)}^T C_k^T, & \mathcal{A}_{(k-2:k_N-1)}^T C_{k-1}^T, & \cdots \mathcal{A}_{(k_N-2:k_N-1)}^T C_{k_N-1}^T \end{bmatrix}^T,$$

$$\mathcal{E}_k = \mathrm{diag}\{\ E_k,\ E_{k-1}, \cdots, E_{k_N}, E_{k_N-1}\ \}, \quad \bar{\mathcal{C}}_{(m:n)} = C_m \mathcal{A}_{(m-1:n)},$$

$$\mathcal{D}_k = \mathcal{F}_k \bar{\mathcal{D}}_k, \quad \bar{\mathcal{D}}_k = \mathrm{diag}\{D_{k-1}, D_{k-2}, \cdots, D_{k_N-1}\},$$

$$\mathcal{F}_k = \begin{bmatrix} \bar{\mathcal{C}}_{(k:k)} & \bar{\mathcal{C}}_{(k:k-1)} & \cdots & \bar{\mathcal{C}}_{(k:k_N+1)} & \bar{\mathcal{C}}_{(k:k_N)} \\ 0 & \bar{\mathcal{C}}_{(k-1:k-1)} & \cdots & \bar{\mathcal{C}}_{(k-1:k_N+1)} & \bar{\mathcal{C}}_{(k-1:k_N)} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \bar{\mathcal{C}}_{(k_N+1:k_N+1)} & \bar{\mathcal{C}}_{(k_N+1:k_N)} \\ 0 & 0 & \cdots & 0 & \bar{\mathcal{C}}_{(k_N:k_N)} \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}.$$

Define $\hbar_k := \Theta_k \mathcal{C}_k x_{k_N-1} + \Theta_k \mathcal{F}_k \tilde{f}_k + \Theta_k \mathcal{D}_k \tilde{w}_k + \mathcal{E}_k \tilde{v}_k - Y_k$ and $\bar{\Theta} = I \otimes \Xi$. It follows that

$$\mathrm{E}\{\hbar_k^T \hbar_k | x_{k_N-1}\}$$
$$= \mathrm{E}\Big\{ x_{k_N-1}^T \mathcal{C}_k^T \Theta_k^T \Theta_k \mathcal{C}_k x_{k_N-1} + 2 x_{k_N-1}^T \mathcal{C}_k^T \Theta_k^T (\Theta_k \mathcal{F}_k \tilde{f}_k + \Theta_k \mathcal{D}_k \tilde{w}_k + \mathcal{E}_k \tilde{v}_k)$$
$$- 2 x_{k_N-1}^T \mathcal{C}_k^T \Theta_k^T Y_k + \tilde{f}_k^T \mathcal{F}_k^T \Theta_k^T \Theta_k \mathcal{F}_k \tilde{f}_k + 2 \tilde{f}_k^T \mathcal{F}_k^T \Theta_k^T (\Theta_k \mathcal{D}_k \tilde{w}_k + \mathcal{E}_k \tilde{v}_k - Y_k)$$
$$+ (\Theta_k \mathcal{D}_k \tilde{w}_k + \mathcal{E}_k \tilde{v}_k)^T (\Theta_k \mathcal{D}_k \tilde{w}_k + \mathcal{E}_k \tilde{v}_k) - 2(\Theta_k \mathcal{D}_k \tilde{w}_k + \mathcal{E}_k \tilde{v}_k)^T Y_k + Y_k^T Y_k \Big| x_{k_N-1} \Big\}$$

which results in

$$
\begin{aligned}
&\mathrm{E}\big\{\hbar_k^T \hbar_k | x_{k_N-1}\big\} \\
&= x_{k_N-1}^T \mathcal{C}_k^T \bar{\Theta} \mathcal{C}_k x_{k_N-1} - 2x_{k_N-1}^T \mathcal{C}_k^T \bar{\Theta} Y_k + \mathrm{E}\Big\{\tilde{f}_k^T \mathcal{F}_k^T \bar{\Theta} \mathcal{F}_k \tilde{f}_k \big| x_{k_N-1}\Big\} \\
&\quad + \mathrm{E}\Big\{(\Theta_k \mathcal{D}_k \tilde{w}_k + \mathcal{E}_k \tilde{v}_k)^T (\Theta_k \mathcal{D}_k \tilde{w}_k + \mathcal{E}_k \tilde{v}_k)\Big\} + Y_k^T Y_k.
\end{aligned} \tag{19}
$$

Furthermore, denote

$$
\begin{aligned}
\varphi_k :=\ & x_{k_N-1}^T \mathcal{C}_k^T \bar{\Theta} \mathcal{C}_k x_{k_N-1} - 2x_{k_N-1}^T \mathcal{C}_k^T \bar{\Theta} Y_k \\
& + \mathrm{E}\big\{(\Theta_k \mathcal{D}_k \tilde{w}_k + \mathcal{E}_k \tilde{v}_k)^T (\Theta_k \mathcal{D}_k \tilde{w}_k + \mathcal{E}_k \tilde{v}_k)\big\} + Y_k^T Y_k
\end{aligned}
$$

and setting the derivative $d\varphi_k/dx_{k_N-1}$ to zero yields

$$
\tilde{x}_{(k_N-1|k)} := (\mathcal{C}_k^T \bar{\Theta} \mathcal{C}_k)^{-1} \mathcal{C}_k^T \bar{\Theta} Y_k = \mathcal{N}_k Y_k
$$

with $\mathcal{N}_k := (\mathcal{C}_k^T \bar{\Theta} \mathcal{C}_k)^{-1} \mathcal{C}_k^T \bar{\Theta}$.

On the other hand, taking the above equation into consideration, one has

$$
\begin{aligned}
&\mathrm{E}\big\{x_{k_N-1} - \tilde{x}_{(k_N-1|k)} \big| \mathcal{I}_k^{N+1}\big\} \\
&= \mathrm{E}\big\{x_{k_N-1} - \mathcal{N}_k Y_k \big| \mathcal{I}_k^{N+1}\big\} \\
&= \mathrm{E}\big\{x_{k_N-1} - \mathcal{N}_k(\Theta_k \mathcal{C}_k x_{k_N-1} + \Theta_k \mathcal{F}_k \tilde{f}_k + \Theta_k \mathcal{D}_k \tilde{w}_k + \mathcal{E}_k \tilde{v}_k) \big| \mathcal{I}_k^{N+1}\big\} \\
&= \mathrm{E}\big\{x_{k_N-1} - \mathcal{N}_k \Theta_k \mathcal{C}_k x_{k_N-1} \big| \mathcal{I}_k^{N+1}\big\} \\
&= \mathrm{E}\big\{x_{k_N-1} - \mathcal{N}_k \bar{\Theta} \mathcal{C}_k x_{k_N-1} - \mathcal{N}_k(\Theta_k - I)\mathcal{C}_k x_{k_N-1} \big| \mathcal{I}_k^{N+1}\big\} \\
&= \mathrm{E}\big\{(I - \mathcal{N}_k \bar{\Theta} \mathcal{C}_k)x_{k_N-1} \big| \mathcal{I}_k^{N+1}\big\}.
\end{aligned}
$$

which means that the unbiased estimate $\hat{x}_{(k_N-1|k)}$ is

$$
\mathcal{N}_k \bar{\Theta} \mathcal{C}_k \hat{x}_{(k_N-1|k)} = \tilde{x}_{(k_N-1|k)}, \quad \text{or} \quad \hat{x}_{(k_N-1|k)} = (\mathcal{C}_k^T \bar{\Theta}^2 \mathcal{C}_k)^{-1} \mathcal{C}_k^T \bar{\Theta} Y_k. \tag{20}
$$

Now, we are in a position to obtain the estimate $\hat{\mathcal{P}}_{[k_N-1,k_N-1;N]}$ of the state covariance matrix $\mathcal{P}_{[k_N-1,k_N-1;N]}$. It is easily found that

$$
\begin{aligned}
&\mathrm{E}\Big\{(\mathcal{N}_k \bar{\Theta} \mathcal{C}_k x_{k_N-1} - \tilde{x}_{(k_N-1|k)})(\mathcal{N}_k \bar{\Theta} \mathcal{C}_k x_{k_N-1} - \tilde{x}_{(k_N-1|k)})^T \big| \mathcal{I}_k^{N+1}\Big\} \\
&= \mathrm{E}\Big\{\Big(\mathcal{N}_k(\bar{\Theta} - \Theta_k)\mathcal{C}_k x_{k_N-1} - \mathcal{N}_k(\Theta_k \mathcal{F}_k \tilde{f}_k + \Theta_k \mathcal{D}_k \tilde{w}_k + \mathcal{E}_k \tilde{v}_k)\Big) \\
&\qquad \times \Big(\mathcal{N}_k(\bar{\Theta} - \Theta_k)\mathcal{C}_k x_{k_N-1} - \mathcal{N}_k(\Theta_k \mathcal{F}_k \tilde{f}_k + \Theta_k \mathcal{D}_k \tilde{w}_k + \mathcal{E}_k \tilde{v}_k)\Big)^T \big| \mathcal{I}_k^{N+1}\Big\} \\
&= \mathcal{N}_k\Big(\mathcal{E}_k(I \otimes R)\mathcal{E}_k^T + \tilde{\Theta} \circ (\mathcal{C}_k \mathcal{P}_{[k_N-1,k_N-1;N]} \mathcal{C}_k^T) + \bar{\Theta} \mathcal{F}_k \mathrm{E}\{\tilde{f}_k \tilde{f}_k^T\} \mathcal{F}_k^T \bar{\Theta} \\
&\quad + \tilde{\Theta} \circ (\mathcal{F}_k \mathrm{E}\{\tilde{f}_k \tilde{f}_k^T\} \mathcal{F}_k^T) + \bar{\Theta} \mathcal{D}_k(I \otimes Q)\mathcal{D}_k^T \bar{\Theta} + \tilde{\Theta} \circ (\mathcal{D}_k(I \otimes Q)\mathcal{D}_k^T)\Big)\mathcal{N}_k^T \\
&= \mathcal{N}_k\Big(\mathcal{E}_k(I \otimes R)\mathcal{E}_k^T + \tilde{\Theta} \circ (\mathcal{C}_k \mathcal{P}_{[k_N-1,k_N-1;N]} \mathcal{C}_k^T) + \bar{\Theta} \mathcal{F}_k \Omega_f(\mathcal{P}) \mathcal{F}_k^T \bar{\Theta} \\
&\quad + \tilde{\Theta} \circ (\mathcal{F}_k \Omega_f(\mathcal{P}) \mathcal{F}_k^T) + \bar{\Theta} \mathcal{D}_k(I \otimes Q)\mathcal{D}_k^T \bar{\Theta} + \tilde{\Theta} \circ (\mathcal{D}_k(I \otimes Q)\mathcal{D}_k^T)\Big)\mathcal{N}_k^T
\end{aligned} \tag{21}
$$

where $\tilde{\Theta} = I \otimes \tilde{\Xi}$ and

$$\Omega_f(\mathcal{P}) := \mathrm{E}\{\tilde{f}_k \tilde{f}_k^T\} = \sum_l^m (I \otimes \Pi_l)\mathrm{diag}\Big\{\mathrm{tr}(\mathcal{P}_{[k-1,k-1;N]}\Gamma_l), \cdots, \mathrm{tr}(\mathcal{P}_{[k_N-1,k_N-1;N]}\Gamma_l)\Big\}.$$

Considering $\mathrm{E}\{\mathcal{N}_k \bar{\Theta} \mathcal{C}_k x_{k_N-1} \big| \mathcal{I}_k^{N+1}\} = \mathcal{N}_k Y_k$, one has

$$\mathrm{E}\Big\{(\mathcal{N}_k \bar{\Theta} \mathcal{C}_k x_{k_N-1} - \tilde{x}_{(k_N-1|k)})(\mathcal{N}_k \bar{\Theta} \mathcal{C}_k x_{k_N-1} - \tilde{x}_{(k_N-1|k)})^T \big| \mathcal{I}_k^{N+1}\Big\}$$
$$= \mathcal{N}_k \bar{\Theta} \mathcal{C}_k \mathcal{P}_{[k_N-1,k_N-1;N]} \mathcal{C}_k^T \bar{\Theta} \mathcal{N}_k^T - \mathcal{N}_k Y_k Y_k^T \mathcal{N}_k^T. \tag{22}$$

Therefore, according to (21) and (22), we have the following theorem.

Theorem 2. *For the given information set $\mathcal{I}_k^{N+1}$, the estimate $\hat{\mathcal{P}}_{[k_N-1,k_N-1;N]}$ of the state covariance matrix $\mathcal{P}_{[k_N-1,k_N-1;N]}$ satisfies the following nonlinear matrix equation:*

$$\hat{\mathcal{P}}_{[k_N-1,k_N-1;N]} = (\mathcal{C}_k^T \bar{\Theta}^2 \mathcal{C}_k)^{-1}(\mathcal{L}(\hat{\mathcal{P}}) + \mathcal{J}(Y))(\mathcal{C}_k^T \bar{\Theta}^2 \mathcal{C}_k)^{-1} \tag{23}$$

with

$$\begin{cases} \mathcal{L}(\hat{\mathcal{P}}) := \mathcal{C}_k^T \bar{\Theta}\Big(\tilde{\Theta} \circ (\mathcal{F}_k \Omega_f(\hat{\mathcal{P}})\mathcal{F}_k^T) \\ \qquad\qquad + \tilde{\Theta} \circ (\mathcal{C}_k \hat{\mathcal{P}}_{[k_N-1,k_N-1;N]}\mathcal{C}_k^T) + \bar{\Theta}\mathcal{F}_k \Omega_f(\hat{\mathcal{P}})\mathcal{F}_k^T \bar{\Theta}\Big)\bar{\Theta}\mathcal{C}_k, \\ \mathcal{J}(Y) := \mathcal{C}_k^T \bar{\Theta}\Big(\mathcal{E}_k(I \otimes R)\mathcal{E}_k^T + \bar{\Theta}\mathcal{D}_k(I \otimes Q)\mathcal{D}_k^T \bar{\Theta} \\ \qquad\qquad + \tilde{\Theta} \circ (\mathcal{D}_k(I \otimes Q)\mathcal{D}_k^T) + Y_k Y_k^T\Big)\bar{\Theta}\mathcal{C}_k. \end{cases}$$

Remark 1. *Note that the nonlinear matrix equation (23) is solvable if there exists a scalar $\lambda > 0$ satisfying $(\mathcal{C}_k^T \bar{\Theta}^2 \mathcal{C}_k)^2 - \lambda^{-1}(\mathcal{L}(\lambda I) + \mathcal{N}(Y)) > 0$. Furthermore, its approximate solution can be acquired by using some basic iterative methods, such as Aitken's multi-point iterative algorithm, Newton iterative method, and so forth.*

Remark 2. *In most existing literature, three arguably theoretical frameworks, namely, the setting values, the least-squares technique and the maximum-likelihood method, have been widely exploited to estimate the covariance matrix. Unfortunately, all of these methods are no longer valid because of the nonlinear operation (i.e. the "trace" and Hadamard-product operations) included in (21) and the unknown probability density function of the augmented measurement Y_k with missing measurements. As such, in this paper, a novel two-stage scheme is provided to overcome such a challenge. It is worth mentioning that the state covariance $\mathcal{R}_{[k,N;N]}$ are affected by both missing measurements and finite measurement data. An important research topic in future is how to reduce such an effect from these two aspects.*

4.   A Numerical Example

In this section, a numerical example is utilized to illustrate the effectiveness of the established filtering algorithm for discrete time-varying systems (1) with missing

measurements. Consider the system with

$$A_k = \begin{bmatrix} 0.64 + 0.22\sin(0.12k) & 0.40 & 0 \\ 0.25 & 0.32 + 0.10\cos(2k) & 0.10 \\ 0.20 & -0.30 & 0.45 \end{bmatrix},$$

$$C_k = \begin{bmatrix} 0.82 & 0.15 \\ 0.75 & 0 \\ 0 & -0.80 \end{bmatrix}^T, \quad D_k = \begin{bmatrix} 0.10 \\ 0 \\ -0.05 \end{bmatrix}, \quad E_k = \begin{bmatrix} -0.05 & 0.20 \\ 0.10 & -0.15 \end{bmatrix}.$$

Then, the nonlinear vector-valued function $f_k(x_k, \vartheta_k)$ is chosen as

$$f_k(x_k, \vartheta_k) = (0.1\text{sign}(x_k^1)x_k^1\vartheta_k^1 + 0.2\text{sign}(x_k^2)x_k^2\vartheta_k^2) \begin{bmatrix} 0.06 \\ 0.09 \\ 0.12 \end{bmatrix}$$

where x_k^i and ϑ_k^i are, respectively, the i-th element of the system state x_k and the stochastic variable ϑ_k. Furthermore, ϑ_k^1 and ϑ_k^2 are zero mean, uncorrelated Gaussian white noise sequences with unity covariances. The probabilities of multiple missing measurements are taken as $\bar{\alpha}_1 = 0.98$ and $\bar{\alpha}_2 = 0.96$. The N is selected as 3. It is not difficult to verify that the above stochastic nonlinear function satisfies

$$\text{E}\{f(x_k, \vartheta_k)|x_k\} = 0,$$

$$\text{E}\{f(x_k, \vartheta_k)f^T(x_k, \vartheta_k)|x_k\} = \begin{bmatrix} 0.06 \\ 0.09 \\ 0.12 \end{bmatrix} \begin{bmatrix} 0.06 \\ 0.09 \\ 0.12 \end{bmatrix}^T x_k^T \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0 & 0 \end{bmatrix} x_k.$$

The simulation results are shown in Fig. 1, where Figs. 1(a)∼1(c) plot the system state and its estimation, and Figs. 1(d)∼1(f) depict the absolute values of filtering errors for the addressed system. The simulation results have confirmed that the designed filter performance is very well.

5.   Conclusions

In this paper, we have investigated the receding horizon filtering problem for a class of discrete time-varying nonlinear systems. In order to reflect the reality more closely, on one hand, the phenomenon of multiple missing measurements is addressed, where the missing probability of each sensor is different. On the other hand, the stochastic nonlinearity which covers several classes of well-studied non-linearities as special cases, has been taken into account. In terms of the projection theory combined with stochastic analysis techniques, a Kalman-type receding horizon filter has been designed. Furthermore, by utilizing the conditional expectation, a novel estimation scheme has been proposed to obtain the estimation of the state covariance matrix. Further research topics include the extension of our results to more general networked systems with various network-induced phenomena, such as stochastic occurring nonlinearities, time-delays, quantization effect and so forth.
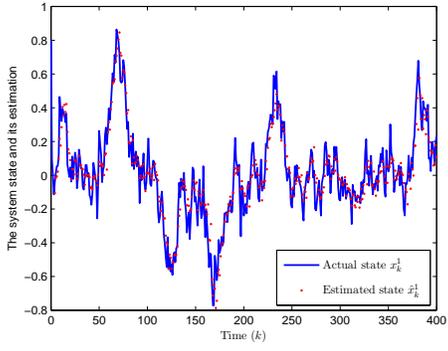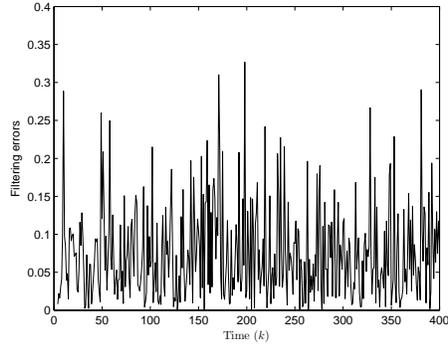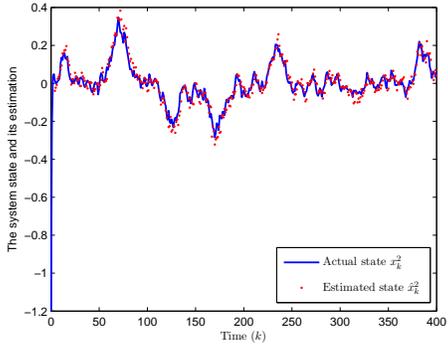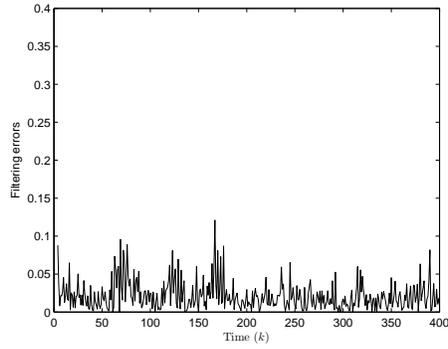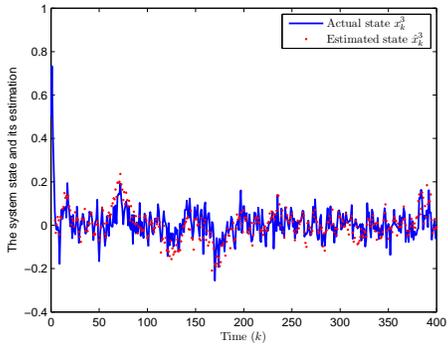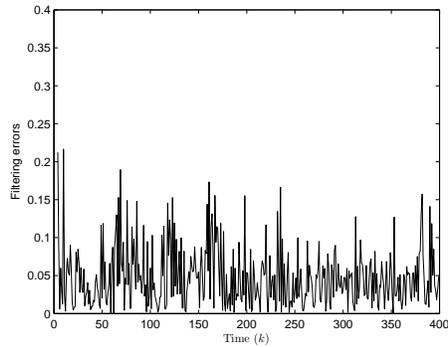
(a)   The actual state x_1^k and its estimation $\hat{x}_1^k$.

(d)   Filtering errors of x_k^1.

(b)   The actual state x_2^k and its estimation $\hat{x}_2^k$.

(e)   Filtering errors of x_k^2.

(c)   The actual state x_3^k and its estimation $\hat{x}_3^k$.

(f)   Filtering errors of x_k^3.

Figure 1. The state estimation and filtering errors (absolute values).

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many.

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