MEAN REVERSION IN THE NIKKEI, STANDARD & POOR AND DOW JONES STOCK MARKET INDICES

Guglielmo Maria Caporale[a]
Luis A. Gil-Alana[b]

[a] **Brunel University, London**
[b] **University of Navarra**

February 2007

ABSTRACT

Three stock market indices (the Nikkei 225, the Standard and Poor's 500 and the Dow Jones EURO STOXX 50) are analysed in this paper using a parametric procedure for fractional integration. We find that the orders of integration of these three series range between 0.75 and 1.25. A model selection criterion suggests that they can be specified as fractional processes of order 0.75, with AR(1) disturbances. This indicates that the three series exhibit mean reversion.

Keywords: Fractional integration; Mean Reversion

JEL Classification: C22.

Corresponding author: Professor Guglielmo Maria Caporale, Brunel University, Uxbridge, Middlesex UB8 3PH, UK. Tel.: +44 (0)1895 266713. Fax: +44 (0)1895 269770. *Email:* *Guglielmo-Maria.Caporale@brunel.ac.uk*

The second-named author gratefully acknowledges financial support from the Ministerio de Ciencia y Tecnologia (SEJ2005-07657/ECON, Spain).

1. Introduction

The univariate behaviour of three major stock market indices is analysed in this paper by means of fractionally integrated techniques. In the existing literature, it is generally assumed that financial series are integrated of order 1, i.e. I(1), following either a random walk, in which case they are completely unpredictable, or an ARIMA model, therefore incorporating weakly autocorrelated disturbances. Most empirical applications are based on tests for unit roots which are embedded in autoregressive (AR) alternatives (e.g., Dickey and Fuller, 1979; Phillips and Perron, 1988; Kwiatkowski et al., 1992, or, for more recent developments, Elliot et al., 1996; Ng and Perron, 2001; etc.). Several studies, however, have shown that these tests have very low power not only when the alternatives are close to the unit root case (see, e.g. Christiano and Eichenbaum, 1990; Stock, 1991; DeJong et al., 1992; Rudebusch, 1992), but also if they are of a fractional form (Diebold and Rudebusch, 1991; Hassler and Wolters, 1994; Lee and Schmidt, 1996; etc.).

In this paper, we use instead the tests of Robinson (1994a), which allow us to consider the unit root hypothesis as a particular case within a fractionally integrated structure. These tests have a standard null limit distribution and are efficient against different fractional alternatives. Fractional models in the stock market were discussed first in Lo and MacKinlay (1988) and Lo (1991). The paper by Lee and Robinson (1996) contains a detailed review of these and other studies on fractional integration in the stock market. The layout of the present study is as follows: Section 2 briefly describes Robinson's (1994a) testing procedure. Section 3 applies the tests to three stock market indexes: the Nikkei 225; the Standard & Poor's 500; and the Dow Jones Euro Stoxx 50. In Section 4 a model selection criterion is applied to determine the best specification for each series. Section 5 contains some concluding comments.

2. Testing unit roots and other hypotheses

A simple way of testing a unit root is to consider the null hypothesis

$$H_o: \ \rho = 1 \tag{1}$$

in a model given by

$$(1 - \rho L)\, x_t \; = \; u_t, \quad t = 1, 2, \ldots \tag{2}$$

where L is the lag operator, ($Lx_t = x_{t-1}$) and u_t is an unobservable covariance stationary sequence that may include stationary AR and MA components. However, the AR class (2) is merely one of a number of mathematical forms that can be used for testing the unit root hypothesis. One of them allows for a "fractional" degree of integration. An I(0) process u_t, t = 1,2,…, is in this case defined as a covariance stationary process with spectral density which is positive and finite at zero frequency. Then, an I(d) process, x_t, t = 1,2,…, is given by:

$$(1 - L)^d\, x_t \; = \; u_t, \qquad t = 1, 2, \ldots \tag{3}$$

where $(1 – L)^d$ can be expressed for all real d in terms of the expansion

$$(1 - L)^d \; = \; 1 \; + \; \sum_{j=0}^{\infty} \frac{\Gamma(d+1)\; (-L)^j}{\Gamma(d-j+1)\; \Gamma(j+1)} \; = \; 1 - d\, L + \frac{d\,(d-1)}{2!} L^2 \; - \; \frac{d\,(d-1)\,(d-2)}{3!} L^3 \; + \ldots$$

where the gamma function is defined as

$$\Gamma(g) \; = \; \int_0^{\infty} x^{g-1}\, e^{-x}\, d\, x.$$

If d > 0 in (3), x_t is said to be long memory because of the strong association between observations widely separated in time, and, if d is not an integer, x_t is fractionally integrated. Granger (1980) and Robinson (1978) showed that fractional models can arise from aggregation of ARMA series with randomly varying coefficients. Thus, it makes sense to consider I(d) processes when analysing aggregate data. A unit root test can be specified then by testing the null:

$$H_o: \ d = 1 \tag{4}$$

in (3). However, fractional and AR departures from (4) and (1) have very different long-run implications. In (3), x_t is nonstationary but non-explosive for all $d \geq 0.5$, unlike in case of (2) around (1), where x_t becomes explosive for all $\rho > 1$. On the other hand, $\rho = 0$ in (2) or $d = 0$ in (3) implies that a weakly autocorrelated I(0) x_t is allowed for. Following the work of Bhargava (1986), and Schmidt and Phillips (1992) on the parameterisation of unit root models, we can consider x_t in (3) as the regression errors in the model

$$y_t \;=\; \beta' z_t \;+\; x_t, \qquad t \;=\; 1, 2, ... \tag{5}$$

where z_t is a (q x 1) vector of regressors, for example $z_t \equiv 1$ or $z_t = (1, t)'$ and β is a (q x 1) vector of unknown parameters. Robinson (1994a) proposes LM tests for testing unit roots and other fractionally integrated hypotheses in a model given by (3) and (5).

The testing procedure is the following. We observe $\{(y_t, z_t), t = 1, 2, \ldots T\}$, and it is assumed that the I(0) u_t in (3) has parametric autocorrelation, such that u_t has spectral density f, which is a given function of frequency and of unknown parameters, specifically,

$$f(\lambda; \sigma^2; \tau) \;=\; \frac{\sigma^2}{2\pi} g(\lambda; \tau), \qquad -\pi \;<\; \lambda \;\leq\; \pi,$$

where the scalar σ^2 and the (kx1) vector τ are unknown but g is of known form. Thus, if u_t is white noise, $g \equiv 1$, and if u_t is AR, we have

$$g(\lambda; \tau) \;=\; \frac{1}{\left|\phi(e^{i\lambda})\right|^2},$$

where ϕ corresponds to the AR polynomial and therefore the AR coefficients are function of τ. In general, we are interested in testing the null hypothesis

$$H_o: \; d = d_o \tag{6}$$

for a given real number d_o. The test statistic takes the form:

$$\hat{s} \;=\; \left(\frac{T}{\hat{A}}\right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2}, \tag{7}$$

where

$$\hat{a} = \frac{-2\pi}{T}\sum_{j=1}^{T-1}\psi(\lambda_j)\, g(\lambda_j;\hat{\tau})^{-1}\, I(\lambda_j)$$

$$\hat{A} = \frac{2}{T}\left(\sum_{j=1}^{T-1}\psi(\lambda_j)^2 - \sum_{j=1}^{T-1}\psi(\lambda_j)\,\hat{\varepsilon}(\lambda_j)' \times \left(\sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\,\hat{\varepsilon}(\lambda_j)'\right)^{-1} \times \sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\psi(\lambda_j)\right)$$

$$\hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T}\sum_{j=1}^{T-1} g(\lambda_j;\hat{\tau})^{-1}\, I(\lambda_j); \quad \psi(\lambda_j) = \log\left|2\sin\frac{\lambda_j}{2}\right|; \quad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial\tau}\log g(\lambda_j;\hat{\tau});$$

$I(\lambda_j)$ is the periodogram of $\hat{u}_t = (1-L)^d\, y_t - \hat{\beta}'\, w_t$, evaluated at $\lambda_j = 2\pi j/T$; $w_t = (1-L)^d\, z_t$ and

$\hat{\beta} = \left(\sum_{t=1}^{T} w_t\, w_t'\right)^{-1}\sum_{t=1}^{T} w_t\,(1-L)^d\, z_t$. In general, we have to estimate the nuisance parameter τ,

for example by $\hat{\tau} = \arg\min_{\tau\in T^*}\sigma^2(\tau)$, where T^* is a suitable subset of R^k.

Robinson (1994a) showed that under regularity conditions:

$$\hat{s} \;\to_d\; N(0,1) \qquad as\;\; T \to \infty. \tag{8}$$

Thus, a one-sided 100α%-level test of (6) against the alternative

$$H_1: \quad d > d_o \tag{9}$$

is given by the rule:

$$\text{“Reject } H_o \text{ if } \hat{s} > z_\alpha\text{”}, \tag{10}$$

where the probability that a standard normal variate exceeds z_α is α, and, conversely, an approximate one-sided 100α%-level test of (6) against the alternative

$$H_1: \quad d < d_o \tag{11}$$

is given by the rule:

$$\text{“Reject } H_o \text{ if } \hat{s} < -z_\alpha\text{”}. \tag{12}$$

Furthermore, he shows that the above tests are efficient in the Pitman sense, i.e. that against local alternatives of form: H_a: $\theta = \delta\, T^{-1/2}$, for $\delta \neq 0$, the limit distribution is normal with variance 1 and mean which cannot (when u_t is Gaussian) be exceeded in absolute value by that of any rival regular statistic. The tests of Robinson (1994a) based on (7) were applied to several U.S. historical annual macroeconomic data in Gil-Alana and Robinson (1997), and other versions of his tests based on quarterly and monthly data can be found respectively in Gil-Alana and Robinson (1998) and Gil-Alana (1999a). Baillie (1996) provides a survey of fractional models for economic time series, and Willinger et al. (1999) discuss other recent empirical application of long-memory processes to the stock market. In the following section, we carry out the tests of Robinson (1994a) for three major stock market indices.

3. Empirical analysis

The three stock market indices analysed here are the Nikkei 225, the Standard & Poor’S 500 and the Dow Jones EURO STOXX 50 for the time period 1994.1 – 1999.12 monthly. Plots of the three series are shown in the upper panel of Figure 1. Visual inspection suggests that all of them might be nonstationarity, especially the S&P 500 and the Dow Jones indices. The first 25 sample autocorrelation values are also plotted in Figure 1. These are significant even at lags far away from zero, with some apparent decay and/or oscillation, which could be indicative of a fractional integration parameter greater than or less than unity. We also computed the periodogram of each series: although this is not a consistent estimate of the spectral density function, it can give us an indication about the possible monthly structure of the data. In all cases, the largest values are those around the zero frequency, suggesting that the monthly component is not important when modelling these series.

INSERT FIGURE 1 ABOUT HERE

Denoting each of the series in turn y_t, we employ throughout the model (3) and (5) with $z_t = (1,t)'$, $t \geq 1$, $z_t = (0, 0)'$ otherwise, so

$$y_t \;=\; \beta_1 \;+\; \beta_2 \, t \;+\; x_t\,, \qquad t \;=\; 1, 2, \ldots \qquad (13)$$

$$(1-L)^d x_t = u_t, \qquad t = 1, 2, \ldots, \tag{14}$$

treating separately the cases $\beta_1 = \beta_2 = 0$ a priori; β_1 unknown and $\beta_2 = 0$ a priori, and (β_1, β_2) unknown, i.e., we consider respectively the cases of no regressors in the undifferenced regression model (13); an intercept; and an intercept and a time trend, and model the I(0) disturbances u_t as white noise and weakly autocorrelated processes in turn.

We start with the assumption that u_t in (14) is white noise. In this case, when d = 1, for example, the differences $(1 – L)\, y_t$ behave, for t > 1, like a random walk when $\beta_2 = 0$, and a random walk with a drift when $\beta_2 \neq 0$. However, we report tests statistics not merely for d = 1 in (14) but also for d = 0, 0.25, …(0.25), …, 1.75 and 2, hence including also a test for stationarity (d = 0.50) as well as allowing for other possibilities.

The test statistic reported in Tables 1 – 3 is the one-sided one given by $\hat{s}$ in (7), which means that significantly positive values are consistent with $d > d_o$, implying a higher order of integration, whereas significant negative ones imply smaller values of d_o. Thus, we should expect a monotonic decrease in the value of $\hat{s}$ as d_o increases, because, for example, if (6) is rejected against (9) when $d_o = 0.75$, an even more significant result in this direction should be expected when $d_o = 0.50$ or 0.25 are tested. Starting with white noise disturbances, in Table 1, one can see that $\hat{s}$ is always monotonic with d_o. The null (6) is not rejected when d = 1 for the Nikkei 225, this being the only non-rejection value for this series whether or not we include deterministic regressors in z_t. Considering now the S&P 500, the unit root null cannot be rejected along with d = 1.25, while d = 0.75 and 1 are the non-rejection values for the Dow Jones. Therefore, the unit root null hypothesis cannot be rejected for any series whether or not an intercept and/or a time trend are included in the regression model (13). Finally, the null H_o (6) is always rejected for values of d smaller than 0.75 and higher than 1.25, suggesting that the optimal power properties of Robinson's (1994a) tests hold, not only against local but also against non-local departures from the null.

INSERT TABLE 1 ABOUT HERE

Tables 2 and 3 report values of the same statistic as in Table 1 but imposing respectively AR(1) and AR(2) disturbances. Higher order autoregressions were also considered, obtaining results very similar to those reported here. A notable feature of our findings is the lack of monotonicity in $\hat{s}$ with respect to d_o, particularly when d_o takes small values. This might suggest that these models are misspecified. Note that in the event of misspecification, monotonicity is not necessarily to be expected: frequently misspecification inflates both numerator and denominator of $\hat{s}$, to varying degrees, and thus affects $\hat{s}$ in a complicated way. However, this lack of monotonicity can also be due to the fact that the AR parameters have been obtained using the Yule-Walker method, implying roots that are automatically less than one in absolute value, though they can be arbitrarily close to one. Thus, it might be the case, for example, that for an I(1) process the tests of Robinson (1994a) with AR(1) u_t do not reject H_o (6), neither with $d_o = 1$ and an estimate of the AR parameter close to 0, nor with $d_o = 0$ and an AR estimate close to 1, but reject it instead for values of d_o between 0 and 1. Tables 2 and 3 also report the estimated values of the AR parameters, along with the results of the Dickey-Fuller procedure for testing the I(1) null on the estimated residuals $\hat{u}_t$. In those cases where the unit root cannot be rejected, we do not consider the values of the test statistics, since the I(0) assumption for u_t is then violated.

The results for $\hat{s}$ based on AR(1) disturbances are given in Table 2. Starting with the Nikkei 225, it can be seen that, if we do not include regressors, H_o (6) cannot be rejected when d = 1, 1.25 and 1.50. However, when including an intercept and a linear time trend, the orders of integration seem to be slightly smaller, and the non-rejection values correspond now to d = 0.75 and 1. Looking at the results for the S&P 500, one can see that the non-rejection values of d also range between 1 and 1.50 with no regressors, while d = 0.75 appears as the only non-rejection value when an intercept and a linear trend are included. Similarly, for the Dow Jones, if we do not include regressors, H_o (6) cannot be rejected if d = 1, 1.25 and 1.50; the null is always rejected if we include an intercept; and d = 0.75 is the only non-rejection value with a linear time trend.

INSERT TABLES 2 AND 3 ABOUT HERE

On the whole, the results for the three series are very similar. To summarise them by looking at the lowest statistics for the different d_o's, when we do not include regressors in (5), the lowest statistics correspond in all cases to d = 1.25. However, when including an intercept or a linear time trend, the lowest values occur at d = 0.75.

Table 3 reports the values of $\hat{s}$ when imposing AR(2) disturbances. Comparing the results here with those in Table 2, one can see a greater proportion of non-rejection values, and a slightly higher degree of integration in some cases. Specifically, if z_t = 0, the non-rejection values occur at d = 1.25, 1.50 and 1.75 for the three series, i.e., they are greater by about 0.25 than those in Table 2; when including an intercept and a linear time trend, again the results are similar for the three series, the null not being rejected when d = 0.75 and 1 in the former model , and when d = 0.50, 0.75 and 1 in the latter.

Tables 1 – 3 report a great variety of potential model specifications for each of the series of interest. Most of these models exhibit orders of integration ranging between 0.75 and 1.25. In the following section, we are concerned with selecting the best model specification for each series, and also with its economic implications in each case.

4. Selecting a model specification and economic implications

From an economic point of view, it is crucial to determine the correct order of integration of a given time series. This is because, if a series is I(d) with d $\in$ [0,5, 1), it will be nonstationary but mean-reverting, since shocks will have only transitory effects, and therefore the series will return to its original path some time in the future. On the other hand, if a series is I(d) with d ≥ 1, it will be nonstationary and non-mean-reverting, with shocks having a permanent effect on its level. This also has implications for the predictability of the series: if it is I(d) with d smaller than one, it will be predictable, while if d is equal to or greater than one it will be unpredictable except for the autocorrelated structure that can be imposed on the disturbances.

In the previous section we have considered several models which may be suitable for modelling the three stock market indices. We are now concerned with selecting the best model specification for each series. We proceed as follows. First, we choose in each case the model specification with the value of d_o producing the lowest statistic $\hat{s}$. In other words, for each set of regressors for z_t in (5) and for each parametric specification for u_t in (3), we choose the value of d_o with the lowest $|\hat{s}|$. The intuition here is that, given a parametric specification, the lowest $|\hat{s}|$ will correspond to the 'd_o' with the estimated residuals closest to a white noise process, and therefore it should be preferred to another 'd_o' with higher $|\hat{s}|$. Table 4 summarises the best nine model specifications for the different cases of no regressors, an intercept, and a linear time trend, combined with white noise, AR(1) and AR(2) disturbances.

INSERT TABLE 4 ABOUT HERE

It can be seen in Table 4 (4^{th} column) that, if u_t is white noise, the unit root hypothesis (d_o = 1) appears to be the best specification for the three types of regressors for the three series. However, when imposing autoregressive disturbances, if z_t = 0, the orders of integration are higher than 1 for all series, and when including an intercept and/or a linear time trend they are equal to 0.75 or 1. In order to choose next the best specification across the different models for each series, we report, in the last column of Table 4, several diagnostic tests carried out on the residuals of the estimated models. Note that all them are based on the differenced regression, and therefore have short memory under the null hypothesis (6). In particular, we perform tests of homocedasticity, no serial correlation, functional form and normality.

Starting with the Nikkei 225, one can see that five models pass all the diagnostic tests on the residuals. They are models 3, 5, 6, 8 and 9, corresponding to $z_t = (1, t)'$ with white noise u_t (model 3); with AR(1) u_t (model 6); and with AR(2) u_t (model 9), and $z_t \equiv 1$ with AR(1) and AR(2) disturbances (models 5 and 8). All the remaining models fail to pass at least one of the diagnostic tests, and therefore we do not consider them. Looking at models 3, 6 and 9 (when a linear time trend is included in the regression model), one can see that the coefficients of t are not significantly different from zero, and consequently these models can also be discarded. Finally, the second AR coefficient in model 8 is insignificant. Therefore, model 5 appears as the best specification for this series, implying an order of integration of 0.75. The resulting model is:

$$\begin{aligned} y_t &= \underset{(888.45)}{19965.23} + x_t \\ (1-L)^{0.75} x_t = u_t; \quad u_t &= \underset{(0.11)}{0.33} u_{t-1} + \varepsilon_t . \end{aligned} \qquad (15)$$

Next we look at the S&P's 500. The results from the last column in Table 4 indicate that only two models (5 and 8) pass all the diagnostic tests. They are the ones with an intercept and AR(1) (model 5) and AR(2) (model 8) disturbances, and in both cases the order of integration is 0.75. In the latter model, the second AR coefficient appears to be insignificant, and therefore we model this series as:

$$y_t = 508.29 + x_t$$
$$(43.60)$$
$$(1 - L)^{0.75} x_t = u_t; \quad u_t = 0.40\, u_{t-1} + \varepsilon_t . \qquad (16)$$
$$(0.09)$$

Finally, looking at the Dow Jones, again we find that only two models pass all the diagnostic tests at the 5% significance level. They are models 6 and 9, including a linear time trend, and AR(1) and AR(2) disturbances. When imposing an AR(2) structure on u_t, the second AR coefficient is once more insignificant. Therefore, we can conclude that the best specification for this series is

$$y_t = 1316.91 + 40.69t + x_t$$
$$(143.53) \quad (6.94)$$
$$(1 - L)^{0.75} x_t = u_t; \quad u_t = 0.36 u_{t-1} + \varepsilon_t . \qquad (17)$$
$$(0.11)$$

Overall, all three series can be modelled as I(d) processes with AR disturbances. This fractional structure results in greater flexibility in modelling the dynamics of the series compared with the restrictiveness of ARIMA specifications. The order of integration of the three series is around 0.75, i.e., they exhibit mean reversion. Furthermore, the fact that the models incorporate an autocorrelated structure enables us to separate the short-run components (which are determined by the AR parameters) from the long-run ones (which are determined by the fractional differencing parameter).[1] Also, predictions can be evaluated according to these estimated models using the AR(∞) representation of the fractional polynomial (see equation below (3) and the short-run structure).

[1] Note that the results reported in Tables 1 – 3 were also recomputed using the finite sample critical values obtained in Gil-Alana (1999b), where samples of approximately the same size are used. The findings did not differ much from those reported here, the non-rejection values essentially corresponding to the same (z_t, d_o) combinations as when the asymptotic values are used.

5. Conclusions

Three stock market series (the Nikkei 225, the Standard and Poor's 500 and the Dow Jones EURO STOXX 50) have been analysed in this paper by means of fractionally integrated techniques. Specifically, we have used the tests of Robinson (1994a), which are efficient against fractional alternatives, and have a standard null asymptotic distribution. The results indicate that the orders of integration of the three series range in all cases between 0.75 and 1.25. A model selection criterion was adopted to determine the best model specification for each series – this suggests that all of them can be specified as fractional processes of order 0.75, with AR(1) disturbances. Therefore, they exhibit mean reversion.

Note that the approach used in this paper generates simply diagnostics for departures from any real d. It is not at all surprising then that, when fractional hypotheses are considered, some evidence supporting them is found, because this can happen even when the unit root model is highly appropriate. Therefore, it would be of interest to obtain point estimates of d. This can be done either with parametric approaches (e.g., Sowell's (1992) procedure of estimating by maximum likelihood), or with semi-parametric ones (e.g., Robinson, 1994b; 1995a and 1995b), the latter methods being more appropriate if the focus is on the long run properties of the series.

References

Baillie, R.T. (1996) Long memory processes and fractional integration in econometrics, Journal of Econometrics, 73, 5--59.

Bhargava, A. (1986) On the theory for testing for unit roots in observed time series, Review of Economic Studies, 53, 369—384.

Christiano, L.J. and M. Eichenbaum (1990) Unit roots in real GNP: Do we know and do we care?. In Karl Brunner and Allern Meltzer eds., Carnegie-Rochester Series on Public Policy, 32, 7--62.

DeJong, D.N., J.C. Nankervis, N.E. Savin and C.H. Whiteman (1992) The power problems of unit root tests in time series with autoregressive errors, Journal of Econometrics, 53, 323--343.

Dickey, D.A. and W.A. Fuller (1979) Distribution of the estimators for autoregressive time series with a unit root, Journal of the American Statistical Association, 74, 427--431.

Diebold, F.X. and G. Rudebusch, 1991, On the power of Dickey-Fuller tests against fractional alternatives, Economic Letters 35, 155-160.

Elliot, G., Rothenberg, T.J., Stock, J.H., (1996). Efficient tests for an autoregressive unit root. Econometrica 64, 813-836.

Gil-Alana, L.A. (1999a) Testing fractional integration with monthly data, Economic Modelling, 16, 613—629.

Gil-Alana, L.A. (1999b) Evaluation of Robinson's (1994) tests in finite samples, Centre for Economic Forecasting, London Business School, Discussion Paper 01-99.

Gil-Alana, L.A. and P.M. Robinson (1997) Testing of unit roots and other nonstationary hypotheses in macroeconomic time series, Journal of Econometrics, 80, 241--268.

Gil-Alana, L.A. and P.M. Robinson (1998) Testing of seasonal fractional integration in U.K. and Japanese consumption and income, European University Institute, Working Paper ECO 98/20.

Granger, C.W.J. (1980) Long memory relationships and the aggregation of dynamic models, Journal of Econometrics, 14, 227--238.

Hassler, U. and J. Wolters, 1994, On the power of unit root tests against fractional alternatives, Economic Letters 45, 1-5.

Kwiatkowski, D., P.C.B. Phillips, P. Schmidt and Y. Shin (1992) Testing the null hypothesis of stationarity against the alternative of a unit root, Journal of Econometrics, 54,159—178.

Lee, D. and P.M. Robinson (1996) Semiparametric exploration of long memory in stock prices, Journal of Statistical Planning and Inference, 50, 155--174.

Lee, D. and P. Schmidt, 1996, On the power of the KPSS test of stationarity against fractionally integrated alternatives, Journal of Econometrics 73, 285-302.

Lo, A.W. (1991) Long term memory in stock prices, Econometrica, 59, 1279--1314.

Lo, A.W. and A.C. MacKinlay (1988) Stock market prices do not follow random walks: Evidence from a simple specification test, The Review of Financial Studies, 1, 41—66.

Ng, S., Perron, P., (2001). Lag length selection and the construction of unit root test with good size and power. Econometrica 69, 1519-1554.

Phillips, P.C.B. and P. Perron (1988) Testing for a unit root in time series regression, Biometrika, 75, 335--346.

Robinson, P.M. (1978) Statistical inference for a random coefficient autoregressive model, Scandinavian Journal of Statistics, **5**, 163--168.

Robinson, P.M. (1994a) Efficient tests of nonstationary hypotheses, Journal of the American Statistical Association, 89, 1420--1437.

Robinson, P.M. (1994b) Semiparametric analysis of long memory time series, Annals of Statistics, 22, 515--539.

Robinson, P.M. (1995a) Log-periodogram regression of time series with long range dependence, Annals of Statistics, 23, 1048--1072.

Robinson, P.M. (1995b) Gaussian semiparametric estimation of long ran dependence, Annals of Statistics, 23, 1630--1661.

Rudebusch, G.D. (1992) Trends and random walks in macroeconomic time series: A re-examination, International Economic Review, 33, 661--680.

Schmidt, P. and P.C.B. Phillips (1992) LM tests for a unit root in the presence of deterministic trends, Oxford Bulletin of Economics and Statistics, 54, 257--287.

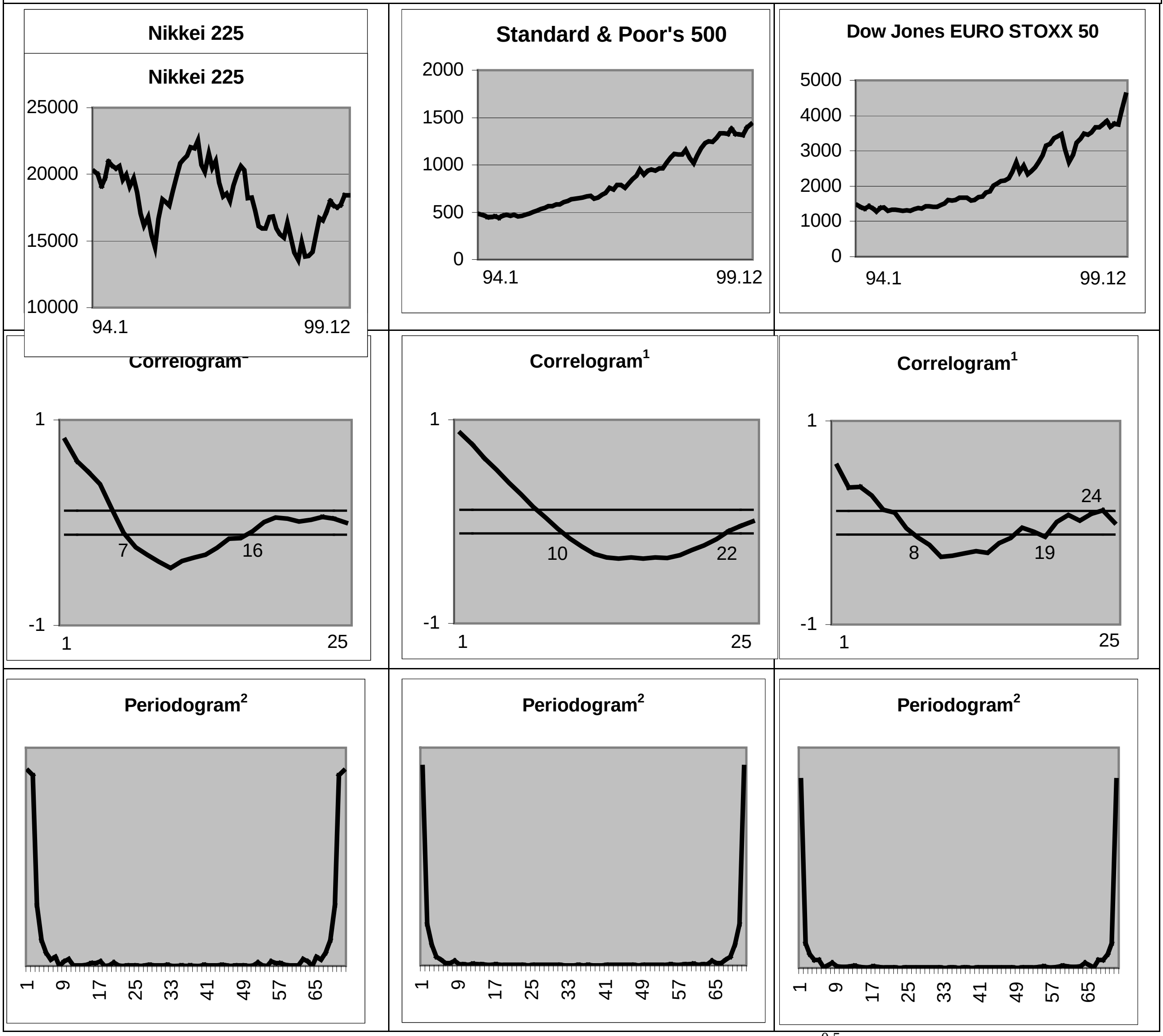
Sowell, F. (1992) Maximum likelihood estimation of stationary univariate time series models, Journal of Econometrics, 53, 165--188.

Stock, J.H. (1991) Confidence intervals for the largest autoregressive root in U.S. macroeconomic time series, Journal of Monetary Economics, 28, 435--459.

Willinger, W., M.S. Taqqu and V. Teverovsky (1999) Stock market prices and long range dependence, Finance Stochastic, 3, 1--13.

FIGURE 1

Plots of the original series with their corresponding correlograms and periodograms



1: The large sample standard error under the null hypotheses of no autocorrelation is $T^{0.5}$ or roughly 0.11 for series of length considered here. 2: The periodogram was calculated based on the discrete Fourier frequencies $\lambda_j = 2\pi j/T$.

TABLE 1										
Testing (6) in (3) and (5) with white noise disturbances										
Series	z_t / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Nikkei 225	z_t = 0	13.84	11.29	7.26	2.93	**-0.29'**	-2.14	-3.12	-3.67	-4.00
	$z_t \equiv 1$	13.84	10.90	6.85	2.99	**0.13'**	-1.69	-2.74	-3.36	-3.74
	z_t = (1, t)'	11.50	9.34	6.33	2.94	**0.13'**	-1.69	-2.75	3.36	-3.75
S & P 500	z_t = 0	15.75	14.95	8.29	3.44	**0.80'**	**-0.96'**	-2.23	-3.12	-3.73
	$z_t \equiv 1$	15.75	13.60	9.26	3.21	**-0.01'**	**-1.34'**	-2.18	-2.88	-3.46
	z_t = (1, t)'	14.36	10.51	6.21	2.43	**0.06'**	**-1.29'**	-2.19	-2.86	-3.43
Dow Jones EURO STOXX 50	z_t = 0	15.96	14.29	6.85	**1.40'**	**-0.78'**	-2.13	-3.01	-3.59	-3.99
	$z_t \equiv 1$	15.96	14.37	11.32	4.15	**-0.90'**	-2.28	-2.91	-3.37	-3.72
	z_t = (1, t)'	14.00	9.59	5.22	**1.48'**	**-0.93'**	-2.20	-2.91	-3.37	-3.73

TABLE 2										
Testing (6) in (3) and (5) with AR(1) disturbances										
Series	z_t / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Nikkei 225	z_t = 0	1.16	-3.57	-6.31	16.26	**-0.61'**	**0.05'**	**-1.24'**	-2.20	-2.85
	$(\hat{\tau})$	(0.92)	(0.92)	(0.84)	(0.59)	(0.003)	(-0.43)	(-0.57)	(-0.61)	(-0.63)
	H_o: α = 1	NR	NR	NR	R	R	R	R	R	R
	z_t ≡ 1	1.16	-1.01	-1.68	**-0.44'**	**-0.92'**	-1.84	-2.64	-3.19	-3.54
	$(\hat{\tau})$	(0.92)	(0.80)	(0.59)	(0.33)	(0.08)	(-0.11)	(-0.24)	(-0.34)	(-0.41)
	H_o: α = 1	NR	NR	R	R	R	R	R	R	R
	z_t = (1, t)'	1.12	-0.88	--2.08	**-0.55'**	**-0.93'**	-1.84	-2.64	-3.19	-3.55
	$(\hat{\tau})$	(0.89)	(0.77)	(0.58)	(0.32)	(0.08)	(-0.11)	(-0.24)	(-0.34)	(-0.41)
	H_o: α = 1	NR	NR	R	R	R	R	R	R	R
S & P 500	z_t = 0	-0.40	0.58	-5.80	-3.08	**0.49'**	**-0.45'**	**-1.20'**	-1.99	-2.64
	$(\hat{\tau})$	(0.99)	(0.92)	(0.61)	(0.29)	(-0.07)	(-0.35)	(-0.49)	(-0.56)	(-0.60)
	H_o: α = 1	NR	NR	NR	R	R	R	R	R	R
	z_t ≡ 1	-0.40	-2.01	-4.15	**-0.73'**	-2.14	-2.89	-3.08	-3.25	-3.45
	$(\hat{\tau})$	(0.99)	(0.97)	(0.86)	(0.40)	(0.03)	(-0.13)	(-0.24)	(-0.34)	(-0.41)
	H_o: α = 1	NR	NR	NR	R	R	R	R	R	R
	z_t = (1, t)'	0.01	0.23	1.94	**-0.64'**	-2.11	-2.80	-3.09	-3.23	-3.44
	$(\hat{\tau})$	(0.86)	(0.68)	(0.45)	(0.22)	(0.03)	(-0.12)	(-0.24)	(-0.34)	(-0.42)
	H_o: α = 1	NR	NR	R	R	R	R	R	R	R
Dow Jones EURO STOXX 50	z_t = 0	-1.23	0.17	3.36	1.97	**0.96'**	**0.56'**	**-0.23'**	-1.90	-1.91
	$(\hat{\tau})$	(0.98)	(0.90)	(0.63)	(0.34)	(0.04)	(-0.21)	(-0.39)	(-0.50)	(-0.57)
	H_o: α = 1	NR	NR	R	R	R	R	R	R	R
	z_t ≡ 1	-1.23	-2.84	-5.01	-3.92	-2.60	-2.46	-2.29	-2.28	-2.47
	$(\hat{\tau})$	(0.98)	(0.95)	(0.81)	(0.45)	(0.18)	(-0.002)	(-0.15)	(-0.29)	(-0.41)
	H_o: α = 1	NR	NR	NR	R	R	R	R	R	R
	z_t = (1, t)'	1.52	2.40	2.20	**-1.39'**	-2.37	-2.46	-2.33	-2.21	-2.36
	$(\hat{\tau})$	(0.88)	(0.74)	(0.55)	(0.36)	(0.17)	(0.004)	(-0.15)	(-0.29)	(-0.41)
	H_o: α = 1	NR	NR	R	R	R	R	R	R	R

TABLE 3

Testing (6) in (3) and (5) with AR(2) disturbances

Series	z_t / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Nikkei 225	$z_t = 0$	-0.21	-3.40	-4.21	-3.78	3.19	**1.60’**	**1.29’**	**0.22’**	-2.53
	$(\hat{\tau}_1 + \hat{\tau}_2)$	(0.91)	(0.92)	(0.84)	(0.58)	(-0.12)	(-1.36)	(-2.14)	(-2.36)	(-2.27)
	H_o: Unit root	NR	NR	NR	R	R	R	R	R	R
	$z_t \equiv 1$	-0.21	-0.53	-0.40	**-0.21’**	**-0.23’**	-1.68	-1.71	-1.74	-2.12
	$(\hat{\tau}_1 + \hat{\tau}_2)$	(0.91)	(0.80)	(0.60)	(0.33)	(0.007)	(-0.31)	(-0.58)	(-0.79)	(-0.96)
	H_o: Unit root	NR	NR	NR	R	R	R	R	R	R
	$z_t = (1, t)'$	-0.45	-0.46	-0.47	**0.25’**	**-0.23’**	-1.66	-1.68	-1.71	-2.12
	$(\hat{\tau}_1 + \hat{\tau}_2)$	(0.87)	(0.77)	(0.60)	(0.33)	(0.007)	(-0.31)	(-0.58)	(-0.79)	(-0.96)
	H_o: Unit root	NR	NR	NR	R	R	R	R	R	R
S & P 500	$z_t = 0$	-0.80	0.69	-1.46	2.30	2.31	**1.20’**	**0.90’**	**0.26’**	-0.39
	$(\hat{\tau}_1 + \hat{\tau}_2)$	(0.99)	(0.93)	(0.58)	(0.21)	(-0.29)	(-0.95)	(-1.47)	(-1.74)	(-1.83)
	H_o: Unit root	NR	NR	NR	R	R	R	R	R	R
	$z_t \equiv 1$	-0.80	-1.25	-2.04	**-0.34’**	**-0.99’**	-1.68	-1.84	-1.93	-2.06
	$(\hat{\tau}_1 + \hat{\tau}_2)$	(0.99)	(0.97)	(0.88)	(0.73)	(-0.11)	(-0.38)	(-0.58)	(-0.76)	(-0.94)
	H_o: Unit root	NR	NR	NR	R	R	R	R	R	R
	$z_t = (1, t)'$	-0.73	-0.02	**0.60’**	**-0.01’**	**-0.97’**	-1.78	-1.85	-1.89	-2.01
	$(\hat{\tau}_1 + \hat{\tau}_2)$	(0.84)	(0.66)	(0.41)	(0.14)	(-0.12)	(-0.36)	(-0.57)	(-0.76)	(-0.94)
	H_o: Unit root	NR	NR	R	R	R	R	R	R	R
Dow Jones EURO STOXX 50	$z_t = 0$	-2.59	-0.18	2.78	2.48	2.83	**1.10’**	**0.95’**	**0.57’**	-2.16
	$(\hat{\tau}_1 + \hat{\tau}_2)$	(0.98)	(0.89)	(0.60)	(0.31)	(-0.02)	(-0.42)	(-0.82)	(-1.13)	(-1.34)
	H_o: Unit root	NR	NR	NR	R	R	R	R	R	R
	$z_t \equiv 1$	-2.59	-2.83	-2.66	**-1.17’**	**-1.39’**	-1.66	-1.68	-1.73	1-.85
	$(\hat{\tau}_1 + \hat{\tau}_2)$	(0.98)	(0.95)	(0.81)	(0.44)	(0.13)	(-0.05)	(-0.22)	(-0.39)	(-0.58)
	H_o: Unit root	NR	NR	NR	R	R	R	R	R	R
	$z_t = (1, t)'$	-0.12	1.66	**0.52’**	**-0.50’**	**-1.27’**	-1.69	-1.72	-1.77	-1.82
	$(\hat{\tau}_1 + \hat{\tau}_2)$	(0.87)	(0.70)	(0.50)	(0.31)	(0.12)	(-0.04)	(-0.21)	(-0.39)	(-0.58)
	H_o: Unit root	NR	R	R	R	R	R	R	R	R

TABLE 4									
Best model specifications according to Tables 1 – 3									
Model: $y_t = \beta_1 + \beta_2 t + x_t ; \quad (1 - L)^d x_t = u_t ; \quad u_t = \tau_1 u_{t-1} + \tau_2 u_{t-2} + \varepsilon_t$									
Series		Model	$\hat{s}$	d	β_1	β_2	τ_1	τ_2	Diagnostic
Nikkei 225	1	$z_t = 0$; WN u_t	-0.29	1.00	---	---	---	---	A; B
	2	$z_t \equiv 1$; WN u_t	0.13	1.00	20229.12 (910.06)	---	---	---	A; C; D
	3	$z_t = (1,t)$; WN u_t	0.13	1.00	20254.45 (922.61)	-25.33 (108.73)	---	---	A; B; C; D
	4	$z_t = 0$; AR1 u_t	0.05	1.25	---	---	-0.43 (0.07)	---	A; C; D
	5	$z_t \equiv 1$; AR1 u_t	-0.44	0.75	19965.23 (888.45)	---	0.33 (0.11)	---	A; B; C; D
	6	$z_t = (1,t)$; AR1 u_t	-0.55	0.75	20148.31 (916.82)	-36.91 (44.33)	0.32 (0.11)	---	A; B; C; D
	7	$z_t = 0$; AR2 u_t	0.22	1.75	---	---	-1.28 (0.07)	-1.08 (0.05)	C
	8	$z_t \equiv 1$; AR2 u_t	-0.21	0.75	19965.23 (888.45)	---	0.32 (0.12)	0.006 (0.12)	A; B; C; D
	9	$z_t = (1,t)$; AR2 u_t	-0.23	1.00	20254.45 (922.61)	-25.33 (108.73)	0.087 (0.12)	-0.080 (0.12)	A; B; C; D
S&P's 500	1	$z_t = 0$; WN u_t	0.80	1.00	---	---	---	---	A
	2	$z_t \equiv 1$; WN u_t	-0.01	1.00	481.61 (34.14)	---	---	---	A; C; D
	3	$z_t = (1,t)$; WN u_t	0.06	1.00	468.26 (31.87)	13.34 (3.75)	---	---	A; C; D
	4	$z_t = 0$; AR1 u_t	-0.45	1.25	---	---	-0.35 (0.06)	---	A; C; D
	5	$z_t \equiv 1$; AR1 u_t	-0.73	0.75	508.29 (43.60)	---	0.40 (0.09)	---	A; B; C; D
	6	$z_t = (1,t)$; AR1 u_t	-0.64	0.75	442.06 (30.69)	13.43 (1.48)	0.22 (0.11)	---	A; B; C
	7	$z_t = 0$; AR2 u_t	0.26	1.75	---	---	-0.99 (0.09)	-0.75 (0.06)	D
	8	$z_t \equiv 1$; AR2 u_t	-0.34	0.75	508.29 (43.60)	---	0.53 (0.11)	0.20 (0.12)	A; B; C; D
	9	$z_t = (1,t)$; AR2 u_t	-0.01	0.75	442.06 (30.69)	13.43 (1.48)	0.25 (0.12)	-0.09 (0.12)	A; B; C
Dow Jones EURO ST.50	1	$z_t = 0$; WN u_t	-0.78	1.00	---	---	---	---	A; B
	2	$z_t \equiv 1$; WN u_t	-0.90	1.00	1456.91 (148.28)	---	---	---	D
	3	$z_t = (1,t)$; WN u_t	-0.93	1.00	1412.80 (143.57)	44.13 (16.92)	---	---	A; D
	4	$z_t = 0$; AR1 u_t	-0.23	1.50	---	---	-0.39 (0.07)	---	---
	5	$z_t \equiv 1$; AR1 u_t	---	---	---	---	---	---	---
	6	$z_t = (1,t)$; AR1 u_t	-1.39	0.75	1316.91 (143.53)	40.69 (6.94)	0.36 (0.11)	---	A; B; C; D
	7	$z_t = 0$; AR2 u_t	0.57	1.75	---	---	-0.71 (0.10)	-0.41 (0.08)	A; B; D
	8	$z_t \equiv 1$; AR2 u_t	-1.17	0.75	1517.63 (169.02)	---	0.57 (0.12)	-0.12 (0.13)	A; B
	9	$z_t = (1,t)$; AR2 u_t	-0.50	0.75	1316.91 (143.53)	40.69 (6.94)	0.41 (0.12)	-0.09 (0.13)	A; B; C; D