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Multivariate FIAPARCH modelling of financial markets with dynamic correlations in times of crisis

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ABSTRACT

This paper applies the vector AR-DCC-FIAPARCH model to eight national stock market indices' daily returns from 1988 to 2010, taking into account the structural breaks of each time series linked to the Asian and the recent Global financial crisis. We find significant cross effects, as well as long range volatility dependence, asymmetric volatility response to positive and negative shocks, and the power of returns that best fits the volatility pattern. One of the main findings of the model analysis is the higher dynamic correlations of the stock markets after a crisis event, which means increased contagion effects between the markets. The fact that during the crisis the conditional correlations remain on a high level indicates a continuous herding behaviour during these periods of increased market volatility. Finally, during the recent Global financial crisis the correlations remain on a much higher level than during the Asian financial crisis.

1. Introduction

The intrinsic informational content that financial crises provide to the research community is certainly one of the key reasons they remain in the spotlight of the finance and broader economic literature long after they are resolved. The 1997 Asian financial crisis, the Global financial crisis of 2007–08 and the ongoing European sovereign-debt crisis are evidently among the most important events that stirred universal fear of a worldwide economic meltdown due to financial contagion among investors, financial market practitioners and policy makers alike. And inevitably, what our modelling tools can tell us about the period around those times is, among other things, the channel through which our existing risk management paradigms and decision-making processes will evolve to better address similar episodes in the future.

In this spirit, the availability of data and processing power capacity together with the recent developments in econometrics allow us to pinpoint better than ever before, properties of the underlying stochastic processes that are crucial albeit hard to uncover (i) in constructively challenging long-established assumptions of the financial practice such as the benefits of international portfolio diversification, especially during periods of economic turmoil or (ii) in shedding light on how

the properties of our modelling efforts of the underlying stochastic processes project the impact of these crises. Our paper introduces a unified approach and demonstrates how it can be used to determine key aspects of modelling around periods of economic turmoil, such as changes in the linkages between financial markets, in long memory and power effects among others. In particular, we focus on stock market volatilities and co-volatilities and how they have changed due to the Asian and the recent Global financial crises.

The study of the linkages between volatilities and co-volatilities of the financial markets is a critical issue in risk management practice. The multivariate GARCH framework provides the tools to understand how financial volatilities move together over time and across markets. For thorough surveys of the available Multivariate GARCH models and their use in various fields of risk management such as option pricing, hedging and portfolio selection see Bauwens, Laurent, and Rombouts (2006) and Silvennoinen and Teräsvirta (2009).

Conrad, Karanasos, and Zeng (2011) applied a multivariate fractionally integrated asymmetric power ARCH (FIAPARCH) model that combines long memory, power transformations of the conditional variances, and leverage effects with constant conditional correlations (CCC) on eight national stock market indices returns. The long-range volatility dependence, the power transformation of returns and the asymmetric response of volatility to positive and negative shocks are three features that improve the modelling of the volatility process of asset returns and its implications for the various risk management practices. We extend their model by allowing for cross effects between the markets

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in the mean of returns and by estimating time varying conditional correlations. We also study the effect of financial crisis events on the dynamic conditional correlations as well as on the three key features of the conditional variance nested in the model. Therefore, the contribution of the present study is that our model provides a complete framework for the analysis of financial markets' co-volatility processes.

The empirical analysis of our model applied to eight stock indices daily returns in a bivariate and trivariate framework provides evidence that confirms the importance of long memory in the conditional variance, of the power transformations of returns to best fit the volatility process and of the asymmetric response of volatility to positive and negative shocks. A Wald testing procedure strongly supports our results. We extend the existing empirical evidence on the dynamic conditional correlations (DCC) models by adding all cross effects in the mean equation, that is we estimate a full vector autoregressive (VAR) model, to reveal the relationship among the returns of each multivariate specification. In the previous studies the researchers have added as regressor in the mean for all stock market indices a prevailing global index return, such as S&P 500 or an index of particular interest for the region and the period investigated. Our cross effects are found significant in most cases.

Moreover, another of our main findings regards the DCC analysis with structural breaks. In line with the literature, our model estimates always highly persistent conditional correlations. The correlations increase during crisis events, indicating contagion effects between the markets, and remain on a high level after the crisis break, showing the investors' herding behaviour. Finally, we contribute to the existing literature findings by comparing two different financial crises, the Asian (1997) and the recent Global (2007–08) crisis, in terms of their effects on the correlations, where we observe much more heightened conditional correlation estimates for the recent Global crisis than for the Asian crisis. This is reasonable since the international financial integration followed by the financial liberalisation and deregulation in capital controls has reached its peak nowadays compared to its evolution during the Asian financial crisis in 1997.

The remainder of the paper is structured as follows. Section 2 discusses the existing empirical literature on the financial crises, the contagion effects among the financial markets and the investors' herding behaviour. In Section 3 we detail the multivariate FIAPARCH model with DCC and the methodology for detecting structural breaks. Section 4 discusses the data and presents the empirical results. Quasi Maximum likelihood parameter estimates for the various specifications and results of the Wald testing procedures are presented. We also evaluate the different specifications, taking into account the structural breaks of each time series linked with two financial crisis events. Each multivariate specification is re-estimated under three subsamples defined by the break dates detected for each country combination. In addition, two contagion tests are performed in Section 6. The final Section concludes the analysis.

2. Literature review

2.1. Financial crises and the DCC model

There are several studies that investigate the two crises (the Asian and the recent Global one) using the DCC model. Cho and Parhizgari (2008) study the Asian financial crisis effects on correlations between eight East Asian stock markets. Using the AR(1)-DCC-GARCH(1,1) model on daily returns they find an upward trend in DCCs after the break date of the crisis. They observe a shift in the mean and the median of the DCCs computed by the model. Chiang, Jeon, and Li (2007) also use an AR(1)-DCC-GARCH(1,1) on nine Asian stock markets plus the US market (as explanatory variable in the mean equation) to investigate the effects of the Asian crisis. They conclude that there are

higher correlations during the crisis, where volatility is also increased. They also observe two phases in the crisis period. In the first phase the correlations increase, which means contagion effect, and in the second phase the correlations remain high, which means investors' herding behaviour.

Syllignakis and Kouretas (2011) use the AR(1)-DCC-GARCH(1,1) model to investigate the correlation pattern (before and after the current financial crisis) between the US, the Russian and seven emerging markets of Central and East Europe. They consider cross effects in the mean caused only by either the US, the German or the Russian index returns but not by the other dependent variables of each multivariate model. They find an increase in conditional correlations between the stock market returns during the crisis (2007–2009). They use weekly returns and then dummy variables for the crisis periods as regressors in a separate regression of the generated DCC. Kenourgios and Samitas (2011) apply the asymmetric generalized (AG) DCC-GARCH(1,1) model of Cappiello, Engle, and Sheppard (2006) to confirm the increased dynamic correlations between five emerging Balkan stock markets, the US and three developed European markets during the current financial crisis, also considering asymmetries in correlation dynamics. They conclude that the higher stock market interdependence is due to herding behaviour during the crisis period. Kenourgios, Samitas, and Paltalidis (2011) extend their paper to investigate the conditional correlations over five financial crisis events from 1995 to 2006 for the BRICs, the US and the UK using various DCC models like the original one of Engle (2002) and the AG-DCC as well. More recently, Kenourgios and Padhi (2012) again estimated AG-DCC models to study correlations during crisis periods between 1994 and 2008 on nine emerging markets and the US.

Kazi, Guesmi, and Kaabia (2011) use a multivariate DCC-GARCH(1,1) model to investigate the correlations between seventeen OECD stock market returns before and during the current Global financial crisis. They use the Bai and Perron (2003) structural break test and apply the DCC model for the whole period (2002–2009) and the two subperiods, defined by the structural break detected (1–10–2007), which corresponds to the beginning of the crisis. They observe a significant increase in DCC during the crisis (after October 2007) compared to the pre-crisis period (before October 2007), which confirms the finding of previous studies of a higher contagion effect during financial crisis periods. Kotkatvuori-Örnberg, Nikkinen, and Äijö (2013) also focus on the current financial crisis with data from fifty stock market indices for the period 2007 to 2009, accounting for two major events: JP Morgan's acquisition of Bear Stearns and the Lehman Brothers' collapse with dummy variables for the unconditional variance in the multivariate GARCH(1,1) equation. Then the DCC model is applied in six multivariate specifications for each region and the correlations generated are further used to run multivariate GARCH(1,1) with the same intercept dummies in the mean and the variance. The impact of the crisis is found significant on stock markets' comovements and especially the effect of the Lehman Brothers' collapse is prominent across all regions.

The advantage of our analysis in comparison with the above studies is the FIAPARCH specification of the conditional variance, while the existing studies use the simple GARCH model. We also assume t-distributed innovations, since daily financial data exhibit excess kurtosis, while all the above mentioned papers assume Gaussian innovations. Moreover, we add in the mean equation the cross effects between all the dependent variables and not a common regressor for all the returns, such as the US stock index in Chiang et al. (2007), and thus we estimate a full VAR model. We also apply the complete methodology of Karoglou (2010) to identify the structural breaks in the mean and the volatility dynamics of the stock returns, using a comprehensive set of data-driven methods of structural change detection and not only a single statistical test. We finally use a very large sample period from 1988 to 2010 of daily stock returns, the widest among the studies considered under our literature review.

2.2. Long memory and power transformed returns

There are some recent studies that use the DCC models of either Engle (2002) or Tse and Tsui (2002) with the FIAPARCH specification in the variance equation. Aloui (2011) uses daily stock index returns from Latin American markets for the period 1995–2009 and runs the multivariate FIAPARCH with Engle's DCC, assuming t-distributed innovations following Conrad et al. (2011). The DCCs generated are modelled separately with an AR(p)-GARCH(1,1) with intercept dummies for the crisis events in the mean and the variance equation. The breaks are defined from the economic approach of each crisis timing, the Asian financial crisis (AFC), the Global financial crisis (GFC) and the regional Latin American crises. They prove that the correlations are much higher during periods of financial crises and especially the regional crises and the GFC. Ho and Zhang (2012) apply among other models the multivariate FIAPARCH framework with the DCC of Tse and Tsui (2002) with the normality assumption for the errors on daily Chinese stock index returns from 1992 to 2006. They focus on the key features of the variance specification, the asymmetries and the long memory and on the time varying behaviour of the conditional correlations. They do not use breaks and do not investigate the effect of crisis events.

Dimitriou and Kenourgios (2013) apply the multivariate FIAPARCH framework of Tse (1998) with the DCC of Engle (2002) on foreign exchange rates daily data from 2004 to 2011 with t-distributed errors, in order to identify the effect of the recent financial crisis. They detect the structural breaks according to an economic approach defining the exact timing of the major crisis events and a statistical approach applying the Markov Switching Dynamic Regression model. They run the multivariate DCC-FIAPARCH on the whole sample without cross effects for the five currency series and with the DCCs generated they run an AR(p)-GJR-GARCH(1,1) with intercept dummies for the crisis breaks in the mean and the variance equation of the DCCs to measure the crisis effects. They conclude that there are lower exchange rate correlations during turbulent times. Dimitriou, Kenourgios, and Simos (2013) also use the same FIAPARCH specification in a bivariate framework for stock returns of the US and the BRICs markets pairwise for the period 1997–2012. They assume again t-distributed errors but they use the DCC of Tse and Tsui (2002) instead of Engle's (2002) specification. They model the DCCs extracted from the whole sample and detect the breaks in the same way in order to investigate the correlation dynamics during the several phases of the recent financial crisis. Stock market correlations are found to be increased after early 2009.

In the light of the more recent DCC-FIAPARCH studies, our modelling still provides a comprehensive analysis of the volatility and correlation processes for three main reasons: we use an outstanding breaks methodology, we apply the mean cross effects (that is a full VAR model) and our data cover the longest sample period, which is split into subsamples for the crisis periods in order to re-estimate the same model specifications and analyse the time varying behaviour of the parameters and the effects of the financial crises.

2.3. Contagion effects

Our empirical results below (see Section 6) are in line with the existing empirical evidence that supports the increase in conditional correlations during crisis and justifies the contagion effects among the financial markets and the investors' herding behaviour. As a brief review of the studies on the markets' interdependence during crisis events we first refer to Lin, Engle, and Ito (1994), who report the link between higher correlations and higher volatility periods in equity market returns as an 'empirical regularity' to start their research on intradaily stock prices across markets. Ang and Bekaert (1999) and Longin and Solnik (2001) observe higher volatility periods associated with higher correlations between different stock index returns in bear markets.

Bartram and Wang (2005) provide evidence that contagion effects exist during crises with higher correlation estimates. Boyer, Kumagai, and Yuan (2006) show that correlation estimates increase during crisis periods and investigate the transmission mechanisms across different markets. Increased herding behaviour during crisis is proved in Chiang and Zheng (2010) for some of the countries under study. Sandoval and Franca (2012) use various techniques to measure the correlation between the markets during crises and find that in turbulent times markets exhibit higher degrees of comovement.

Corsetti, Pericoli, and Sbracia (2001) show that although the stock markets' volatilities and covariances increase during crises, the correlations are not necessarily higher. Forbes and Rigobon (2002) give the definition of contagion as "the significant increase in cross-market linkages after a shock to one country". They develop tests on the contagion effect during a crisis and show that the correlation coefficients are conditional on market volatilities. During a crisis the market volatilities are higher, so the correlations are biased upwards. They find no contagion effect during crises by estimating the unconditional correlations, but they accept that there is interdependence (high level of market comovement) across the markets in any state of the economy. Billio and Pelizzon (2003) investigate the tests proposed by the two above mentioned studies to detect contagion or interdependence across markets during financial crisis events. Chakrabarti and Roll (2002) observe higher covariances, correlations and volatilities, after the Asian financial crisis arose, in both Asian and European markets. Yang and Lim (2004) find that during the Asian financial crisis a contagion effect is apparent across the stock markets with a higher degree of interdependence in the whole region. Khan and Park (2009) find herding contagion across Asian markets during the Asian financial crisis, measuring the cross-country correlations. Finally, Moldovan (2011) proves that correlations between the three major financial markets (US, UK and Japan) are higher after the recent financial crash of 2007 than before.

In the financial crisis literature review we find no study that compares the AFC and the GFC, except for Aloui (2011), who investigates the effects of the two crises only on Latin American markets with a narrower sample. Our extended sample from 1988 to 2010 gave us the chance to compare the conditional correlations after each crisis. We find higher correlation estimates after the GFC break than after the AFC. This is absolutely expected since the international financial integration is more apparent in recent years. The evident risk transmission across markets as well as the key characteristics of volatility (co-persistence and asymmetry) during crises should be of primary interest for the market players (all sorts of investors and risk managers) and the regulators. The market participants must take into account the market's stylized facts captured by our model. For example, the volatility persistence affects the investment horizon and the higher correlations reduce the portfolio diversification gains. The financial authorities have to consider such findings in order to establish the appropriate market control measures and protect the investors from extreme risk exposures.

3. Methodology

3.1. Multivariate FIAPARCH-DCC model

The most common model in finance to describe a time series of daily stock index returns is the VAR of order 1 process. Let us define the N-dimensional column vector of the returns $\mathbf{r}_t$ as $\mathbf{r}_t = [r_{it}]_{i=1,...,N}$ and the corresponding residual vector ε_t as $\varepsilon_t = [\varepsilon_{it}]_{i=1,...,N}$. The structure of the VAR (1) mean equation with cross effects is given by

$$\mathbf{r}_t = \phi + \mathbf{\Phi}\mathbf{r}_{t-1} + \varepsilon_t \tag{1}$$

where $\phi = [\phi_i]_{i=1,...,N}$ is an $N \times 1$ vector of constants; the $N \times N$ coefficient matrix $\mathbf{\Phi} = [\phi_{ij}]_{i,j=1,...,N}$ can be expressed as $\mathbf{\Phi} = \mathbf{\Phi}^{(d)} + \mathbf{\Phi}^{(od)}$,

with $\Phi^{(d)} = \mathrm{diag}(\phi_{11}, \ldots, \phi_{NN})$, that is to allow for cross effects we allow $\Phi^{(od)} \neq 0$ (matrices and vectors are denoted by upper and lower case boldface symbols, respectively). For example, the bivariate AR(1) model is given by

$$\begin{bmatrix} r_{1t} \\ r_{2t} \end{bmatrix} = \begin{bmatrix} \phi_{01} \\ \phi_{02} \end{bmatrix} + \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} r_{1,t-1} \\ r_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \text{ or}$$

$$\begin{bmatrix} r_{1t} \\ r_{2t} \end{bmatrix} = \begin{bmatrix} \phi_{01} \\ \phi_{02} \end{bmatrix} + \left\{ \begin{bmatrix} \phi_{11} & 0 \\ 0 & \phi_{22} \end{bmatrix} + \begin{bmatrix} 0 & \phi_{12} \\ \phi_{21} & 0 \end{bmatrix} \right\} \begin{bmatrix} r_{1,t-1} \\ r_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}.$$

Regarding ε_t we assume that it is conditionally student-t distributed with mean vector $\mathbf{0}$, covariance matrix $\Sigma_t = \mathbb{E}(\varepsilon_t \varepsilon_t' | \mathcal{F}_{t-1}) = [\sigma_{ij,t}]_{i,j=1,\ldots,N}$, and variance vector $\sigma_t = \mathbb{E}(\varepsilon_t^{\wedge 2} | \mathcal{F}_{t-1}) = [\sigma_{ii,t}]_{i=1,\ldots,N}$ or $\sigma_t = (\mathbf{I}_N \odot \Sigma_t)\mathbf{i}$ with $\mathbf{i}$ being an $N \times 1$ vector of ones (the symbol $\odot$ denotes element wise multiplication); σ_t follows a multivariate FIAPARCH(1,d,1) model (see below).

Notice that ε_t can be written as $(\mathbf{e}_t \odot \mathbf{q}_t^{\wedge-1/2}) \odot \sigma_t^{\wedge 1/2}$ (the symbol $\wedge$ denotes element wise exponentiation) where $\mathbf{e}_t = [e_{it}]_{i=1,\ldots N}$ is conditionally student-t distributed with mean vector $\mathbf{0}$, time varying covariance (symmetric positive definite) matrix $\mathbf{Q}_t = [q_{ij,t}]_{i,j=1,\ldots,N}$ (the so called quasi-correlations, see Engle, 2009) and variance vector $\mathbf{q}_t = (\mathbf{I}_N \odot \mathbf{Q}_t)\mathbf{i}$. It follows that

$$\sigma_{ij,t} = \mathbb{E}(\varepsilon_{it}\varepsilon_{jt} | \mathcal{F}_{t-1}) = \mathbb{E}\left(\frac{e_{it}e_{jt}}{\sqrt{q_{ii,t}q_{jj,t}}} \sqrt{\sigma_{ii,t}\sigma_{jj,t}} | \mathcal{F}_{t-1} \right)$$
$$= \frac{\sqrt{\sigma_{ii,t}\sigma_{jj,t}}}{\sqrt{q_{ii,t}q_{jj,t}}} \mathbb{E}\left(e_{it}e_{jt} | \mathcal{F}_{t-1} \right) = \sqrt{\sigma_{ii,t}\sigma_{jj,t}} \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}} = \sqrt{\sigma_{ii,t}\sigma_{jj,t}}\rho_{ij,t}.$$

Most importantly, we allow for DCC, $\rho_{ij,t} = \sigma_{ij,t}/\sqrt{\sigma_{ii,t}\sigma_{jj,t}}$, $|\rho_{ij,t}| \leq 1$ $(i,j = 1,\ldots,N) \forall t$, instead of the constant ones, ρ_{ij}, used by Conrad et al. (2011) (see below).

The covariance matrix Σ_t can be expressed as

$$\Sigma_t = \left(\mathbf{I}_N \odot \Sigma_t^{\wedge 1/2} \right) \mathbf{R}_t \left(\mathbf{I}_N \odot \Sigma_t^{\wedge 1/2} \right), \tag{2}$$

where $\mathbf{R}_t = [\rho_{ij,t}]_{i,j=1,\ldots,N}$ is the $N \times N$ symmetric positive semi-definite time varying correlation matrix with ones on the diagonal ($\rho_{ii,t} = 1$) and the off-diagonal elements less than one in absolute value.

Next, the structure of the conditional variance is specified as in Tse (1998), who combines the FIGARCH formulation of Baillie, Bollerslev, and Mikkelsen (1996) with the APARCH model of Ding, Granger, and Engle (1993). The multivariate FIAPARCH(1,d,1) we estimate is specified as follows:

$$\beta(\mathbf{L}) \odot \sigma_t^{\wedge \delta_i/2} = \omega + [\beta(L) - \mathbf{c}(L) \odot \mathbf{d}(L)] \odot f(\varepsilon_t),$$
$$\mathbf{f}(\varepsilon_\mathbf{t}) = (|\varepsilon_\mathbf{t}| - \gamma \varepsilon_\mathbf{t})^{\wedge \delta_i}, \tag{3}$$

where $\beta(L) = [1 - \beta_i L]_{i=1,\ldots,N}$, $\omega = [\omega_i]_{i=1,\ldots,N}$, $\omega_i \in (0, \infty)$; $\mathbf{c}(L) = [1 - c_i L]_{i=1,\ldots,N}$, $|c_i| < 1$, and $\mathbf{d}(L) = \left[(1-L)^{d_i} \right]_{i=1,\ldots N}$, $0 \leq d_i \leq 1$ are all $N \times 1$ vectors; $|\varepsilon_t|$ is the vector ε_t with elements stripped of negative values and $\gamma = [\gamma_i]_{i=1,\ldots,N}$ is the vector of the leverage coefficients, $|\gamma_i| < 1$; the power terms, δ_i, take finite positive values and are used in element wise exponentiation, that is $\sigma_t^{\wedge \delta_i/2}$ raises the ith standard deviation to the power of δ_i. In other words, each conditional variance follows a FIAPARCH(1,d,1) model:

$$\left(1 - \beta_i L \right) \sigma_{ii,t}^{\delta_i/2} = \omega_i + \left[(1-\beta_i L) - (1-c_i L)(1-L)^{d_i} \right] (|\varepsilon_{it}| - \gamma_i \varepsilon_{it})^{\delta_i}, i = 1, \ldots, N. \tag{4}$$

The sufficient conditions of Bollerslev and Mikkelsen (1996) for the positivity of the conditional variance of a FIGARCH (1,d,1) model: $\omega_i > 0$, $\beta_i - d_i \leq c_i \leq \frac{2-d_i}{3}$ and $d_i(c_i - \frac{1-d_i}{2}) \leq \beta_i(c_i - \beta_i + d_i)$, should be satisfied $\forall i$ (see also Conrad and Haag (2006) and Conrad (2010)). Of course when

$d_i = 0$ the model reduces to the APARCH(1,1): $(1 - \beta_i L)\sigma_{ii,t}^{\delta_i/2} = \omega_i + \alpha_i L (|\varepsilon_{it}| - \gamma_i \varepsilon_{it})^{\delta_i}$, $\alpha_i = c_i - \beta_i$; in addition, when $\delta_i = 2$, $\gamma_i = 0$ it reduces to the GARCH(1,1): $(1 - \beta_i L)\sigma_{ii,t} = \omega_i + \alpha_i L \varepsilon_{it}^2$.

Finally, the structure of $\mathbf{R}_t$ according to Engle (2002) is given by

$$\mathbf{R}_t = \left(I_N \odot \mathbf{Q}_t^{\wedge-1/2} \right) \mathbf{Q}_t \left(I_N \odot \mathbf{Q}_t^{\wedge-1/2} \right), \tag{5}$$

$$\mathbf{Q}_t = (1-a-b)\mathbf{Q} + a\mathbf{e}_{t-1}\mathbf{e}_{t-1}' + b\mathbf{Q}_{t-1}, \tag{6}$$

where $\mathbf{Q} = \mathbb{E}(\mathbf{Q}_t) = \left[q_{ij} \right]_{i,j=1,\ldots,N}$, a and b are nonnegative scalar parameters satisfying $a + b < 1$. It is clear that Engle (2002) specifies the conditional correlations as a weighted sum of past correlations, since the matrix of the quasi correlations, $\mathbf{Q}_t$, is written as a GARCH process and then transformed to a correlation matrix. Engle (2002, 2009) used the estimator $\hat{\mathbf{Q}} = \frac{1}{T} \sum_{t=1}^{T} \hat{\mathbf{e}}_t \hat{\mathbf{e}}_t'$.

In the bivariate case the conditional correlation coefficient $\rho_{12,t}$ is expressed as follows:

$$\begin{aligned} \rho_{12,t} &= \frac{q_{12,t}}{\sqrt{q_{11,t}q_{22,t}}}, \\ q_{12,t} &= (1-a-b)q_{12} + ae_{1,t-1}e_{2,t-1} + bq_{12,t-1}, \\ q_{11,t} &= (1-a-b)q_{11} + ae_{1,t-1}^2 + bq_{11,t-1}, \\ q_{22,t} &= (1-a-b)q_{22} + ae_{2,t-1}^2 + bq_{22,t-1}. \end{aligned} \tag{7}$$

3.2. Structural breaks

In order to identify the number and timing of the potential structural breaks we employ the Awarding-Nominating procedure of Karoglou (2010). This procedure involves two stages: the "Nominating breakdates" stage and the "Awarding breakdates" stage.

The "Nominating breakdates" stage involves the use of one or more statistical tests to identify some dates as possible breakdates. In recent years, a number of statistical tests have been developed for that reason and for the purposes of this paper, we use the following ones:

(a) I&T (Inclán & Tiao, 1994)
(b) SAC1 (the first test of Sansó et al., 2003)
(c) SAC2BT, SAC2QS, SAC2VH (the second test of Sansó et al., 2003, with the Bartlett kernel, the Quadratic Spectral kernel, and the Vector Autoregressive HAC or VARHAC kernel of Den Haan & Levin, 1998 respectively)
(d) K&LBT, K&LQS, K&LVH (the version of the Kokoszka & Leipus, 2000 test refined by Andreou & Ghysels, 2002 with the Bartlett kernel, the Quadratic Spectral kernel, and the VARHAC kernel respectively).

These tests are designed to detect a structural change in the volatility dynamics, but in fact they do not discriminate between shifts in the mean and shifts in the variance. For the purpose of this paper, this is a plausible feature since all types of breaks need to be considered in order to determine if and to what extent the distributional properties change when moving from one regime to another. Furthermore, their properties for strongly dependent series have been extensively investigated (e.g. Andreou & Ghysels, 2002; Karoglou, 2006; Sansó et al., 2003) and there is evidence that they perform satisfactorily under the most common ARCH-type processes.

To identify multiple breaks in a series we incorporate the aforementioned test in the following iterative scheme (algorithm):

1. Calculate the test statistic under consideration using the available data.
2. If the statistic is above the critical value split the particular sample into two parts at the date at which the value of a test statistic is maximized.

3. Repeat steps 1 and 2 for the first segment until no more (earlier) change-points are found.
4. Mark this point as an estimated change-point of the whole series.
5. Remove the observations that precede this point (i.e. those that constitute the first segment).
6. Consider the remaining observations as the new sample and repeat steps 1 to 5 until no more change-points are found.

The above algorithm is implemented with each of the (single breakdate CUSUM-type) test statistics described above (i.e. I&T, SAC1, SAC2BT, SAC2QS, SAC2VH, K&LBT, K&LQS, K&LVH).

What differentiates this scheme from a simple binary division procedure is that it forces the existing breaks to be detected in a time-orderly fashion, which makes it more robust when transitional periods exist — in which case a simple binary division procedure is likely to produce more breaks in the interim period. In the absence of transitional periods both procedures will produce the same breaks.

The nominated breakdates for each series are simply all those which have been detected in each case. Note that at this stage we are not much concerned with detecting more breaks than those that actually exist because whichever is not an actual breakdate will be picked up in the Awarding breakdates stage.

The "Awarding breakdates" stage is a procedure which, in essence, is about uniting contiguous nominated segments (i.e. segments that are defined by the nominated breakdates) unless one of the following two conditions is satisfied:

(I) the means of the contiguous segments are statistically different (as suggested by the t-test)
(II) the variances of the contiguous segments are statistically different (as suggested by the battery of tests which is described below)

This testing procedure is repeated until no more segments can be united, that is, until no condition of the two above is satisfied for any pair of contiguous segments.

The battery of tests mentioned in (II) constitute a different approach to the CUSUM-type tests described previously in that they test for the homogeneity of variances of contiguous segments without encompassing the time-series dimension of the data. They include the standard F-test, the Siegel–Tukey test with continuity correction (Sheskin, 2004, and Siegel & Tukey, 1960), the adjusted Bartlett test (see Judge, Hill, Griffiths, Lütkepohl, & Lee, 1988, and Sokal & Rohlf, 1995), the Levene test (1960) and the Brown and Forsythe (1974) test.

Overall, we find that the stochastic behaviour of all indices yields about three to seven breaks during the sample period, roughly one every two to four years on average. The resulting break dates for each series are in the additional Appendix (which is available upon request), Table A.1. The predominant feature of the underlying segments is that mainly changes in variance are found statistically significant. Finally, there are several breakdates that are identical in all series and others that are very close to one another, which apparently signify economic events with a global impact.

Table A.2 in the additional Appendix provides a detailed account of the possible associations that can be drawn between each breakdate and a major economic event that took place at or around the breakdate period, either in the world or in each respective economy. It appears that dates for the extraordinary events of the AFC of 1997, the GFC of 2007–08 and the European sovereign-debt crisis that followed are very clearly identified in all stock return series and with very little or no variability. Other less spectacular events, such as the Russian financial crisis of 1998, the Japanese asset price bubble of 1986–1991 or the UK's withdrawal from the European Exchange Rate Mechanism (ERM), can also be associated with the breakdates that have been identified in some series. Table A.3 presents some of the descriptive statistics of the stock returns of each segment between the breakdates. The variability of the mean returns becomes particularly prominent for all countries at the end of our sample i.e. after the 2007–08 financial crisis. In exactly the same period, the stock market uncertainty as proxied by the standard deviation rises dramatically.

We selected among the breaks detected (for each series' combination for the respective bivariate and trivariate models) the two dates that correspond to the two financial crisis events, on which we will focus in our analysis. These dates are also the most common breaks of each series' combination. We intend to study the impact of the AFC of 1997 and the recent GFC of 2007–08 on the volatility and correlation dynamics of the eight stock markets. As seen in Table 1a we break the whole sample into three subsamples and rerun all the models under the same specifications. The first

Table 1a
Break dates and subsamples.

Panel A: Break dates

	1st break	2nd break
CAC–DAX	17/03/1997	15/01/2008
CAC–FTSE	17/03/1997	24/07/2007
DAX–FTSE	21/07/1997	24/07/2007
HS–NIKKEI	24/10/2001	27/07/2007
HS–STRAITS	28/08/1997	26/07/2007
NIKKEI–STRAITS	28/08/1997	26/07/2007
SP–TSE	27/03/1997	15/01/2008
ASIA	28/08/1997	26/07/2007
EUROPE	17/03/1997	24/07/2007

Panel B: Subsamples

	Subsample A	Subsample B	Subsample C
CAC–DAX	01/01/1988–17/03/1997	18/03/1997–30/06/2010	18/03/1997–15/01/2008
CAC–FTSE	01/01/1988–17/03/1997	18/03/1997–30/06/2010	18/03/1997–24/07/2007
DAX–FTSE	01/01/1988–21/07/1997	22/07/1997–30/06/2010	22/07/1997–24/07/2007
HS–NIKKEI	01/01/1988–24/10/2001	25/10/2001–30/06/2010	25/10/2001–27/07/2007
HS–STRAITS	01/01/1988–28/08/1997	29/08/1997–30/06/2010	29/08/1997–26/07/2007
NIKKEI–STRAITS	01/01/1988–28/08/1997	29/08/1997–30/06/2010	29/08/1997–26/07/2007
SP–TSE	01/01/1988–27/03/1997	28/03/1997–30/06/2010	28/03/1997–15/01/2008
ASIA	01/01/1988–28/08/1997	29/08/1997–30/06/2010	29/08/1997–26/07/2007
EUROPE	01/01/1988–17/03/1997	18/03/1997–30/06/2010	18/03/1997–24/07/2007

Table 1b
Descriptive statistics.

	CAC	DAX	FTSE	HS	NIKKEI	STRAITS	SP	TSE
Panel A: Returns descriptive statistics								
Minimum	−4.1134	−5.9525	−4.0240	−10.649	−5.2598	−4.43287	−4.1126	−4.2509
Maximum	4.6011	4.6893	4.0756	7.4903	5.7477	6.4573	4.7587	4.0695
Mean	0.0092	0.0132	0.0078	0.0160	−0.0062	0.0105	0.0106	0.0094
Median	0.0000	0.0188	0.0028	0.0000	0.0000	0.0000	0.0113	0.0153
Standard deviation	0.5948	0.6240	0.4812	0.7292	0.6403	0.5686	0.4946	0.4351
Skewness	−0.0369	−0.2220	−0.1276	−0.5687	−0.0384	−0.0362	−0.2635	−0.7959
Kurtosis	8.2136	9.3199	9.8214	19.9725	9.2271	12.6490	12.4805	15.2183
Jarque–Bera statistic	6647.14	9813.85	11393	70748.6	9482.4	22764.9	22043.6	37119.9
Panel B: Returns correlations								
CAC	1.0000							
DAX	0.7869	1.0000						
FTSE	0.7950	0.7004	1.0000					
HS	0.3110	0.3343	0.3286	1.0000				
NIKKEI	0.2775	0.2591	0.2820	0.4310	1.0000			
STRAITS	0.3203	0.3360	0.3291	0.6251	0.4100	1.0000		
SP	0.4550	0.4674	0.4598	0.1550	0.1136	0.1723	1.0000	
TSE	0.4600	0.4491	0.4785	0.2286	0.1968	0.2261	0.6986	1.0000

subsample (A) starts from our first observation of 1988 and ends on the break date near the AFC. This is the pre-AFC period. The second subsample (B) starts from the AFC and ends on our last observation of 2010. This is called the post-AFC period, which also includes the current crisis. Finally, the third subsample (C) starts from the AFC break point and ends on the GFC break. This is the period between the two crises.

4. Empirical analysis

4.1. Data

Daily stock price index data for eight countries were sourced from the Datastream database for the period 1st January 1988 to 30th June 2010, giving a total of 5869 observations. The eight countries and their respective price indices are: UK: FTSE 100 (FTSE), US: S&P 500 (SP), Germany: DAX 30 (DAX), France: CAC 40 (CAC), Japan: Nikkei 225 (NIKKEI), Singapore: Straits Times (STRAITS), Hong Kong: Hang Seng (HS) and Canada: TSE 300 (TSE). We selected the most representative indices for the European, Asian and American stock markets. Our sample is large enough to include various crisis events like the Asian (1997), the Russian (1998) and the recent Global crisis, which is still an on-going process beginning from 2007. For each national index, the continuously compounded return was estimated as $r_t = (\log p_t - \log p_{t-1}) \times 100$ where p_t is the price on day t.

The descriptive statistics of each return series and the series correlations pairwise are reported in Table 1b. The mean of all returns is positive except for NIKKEI. The Asian returns show greater standard

deviation on average than the European and the American. FTSE from Europe and the two American series have the lowest values of unconditional volatility, between 44% and 49%. HS and NIKKEI exhibit the highest volatility, 73% and 64%, respectively, and DAX follows with 62%. CAC and STRAITS volatility is calculated in the middle, 59% and 57%, respectively. It is obvious that the normality hypothesis for our daily returns is rejected. All series exhibit skewness with negative values of the relevant measure, indicating that the data are skewed left (long left tail), and excess kurtosis, far above the benchmark of 3 of the normality case, which means a more 'peaked' data distribution (leptokurtosis). The higher correlations are computed for the European

Table 3
Bivariate AR(1)-DCC-FIAPARCH(1,*d*,1) models mean equation.

		ϕ_{ii}	ϕ_{ij} $(i \neq j)$
CAC–DAX	CAC	0.02	−0.01
		(0.97)	(−0.53)
	DAX	−0.10	0.11
		(−5.18)***	(6.32)***
CAC–FTSE	CAC	−0.01	0.03
		(−0.74)	(1.27)
	FTSE	0.02	−0.01
		(0.85)	(−0.88)
DAX–FTSE	DAX	−0.07	0.11
		(−4.45***)	(5.40)***
	FTSE	0.03	−0.02
		(1.81)**	(−1.84)**
HS–NIKKEI	HS	0.04	−0.02
		(2.50)***	(−1.93)***
	NIKKEI	−0.03	0.03
		(−2.53)***	(3.15)***
HS–STRAITS	HS	0.01	0.06
		(0.53)	(3.24)***
	STRAITS	0.08	0.02
		(4.83)***	(1.28)
NIKKEI–STRAITS	NIKKEI	−0.04	0.07
		(−2.84)***	(5.03)***
	STRAITS	0.08	−0.003
		(5.66)***	(−0.31)
SP–TSE	SP	−0.02	0.01
		(−1.34)	(0.35)
	TSE	0.06	0.07
		(3.63)***	(5.57)***

Notes: The numbers in parentheses are t-statistics.
***, **, * denote significance at the 0.05, 0.10, 0.15 level respectively.

Table 2
Engle and Sheppard test for DCC E-S test $(j)^{\sim}\chi^2(j+1)$ under H$_0$: CCC model.

	E-S test(12)	p-Values
CAC–DAX	375.73	[0.00]
CAC–FTSE	414.24	[0.00]
DAX–FTSE	305.11	[0.00]
HS–NIKKEI	99.54	[0.00]
HS–STRAITS	128.76	[0.00]
NIKKEI–STRAITS	53.43	[0.00]
SP–TSE	56.62	[0.00]
ASIA	211.63	[0.00]
EUROPE	533.58	[0.00]

Table 4
Bivariate AR(1)-DCC-FIAPARCH(1,d,1) models — Variance equation.

		β_i	c_i	γ_i	δ_i	d_i
CAC–DAX	CAC	0.61	0.26	0.41	1.52	0.41
		(13.26)***	(10.31)***	(6.02)***	(17.54)***	(8.33)***
	DAX	0.57	0.24	0.31	1.64	0.39
		(10.70)***	(7.64)***	(5.10)***	(19.70)***	(8.64)***
CAC–FTSE	CAC	0.59	0.26	0.40	1.63	0.36
		(11.62)***	(8.99)***	(5.62)***	(18.64)***	(7.78)***
	FTSE	0.63	0.28	0.38	1.55	0.41
		(16.71)***	(10.80)***	(5.88)***	(17.60)***	(10.43)***
DAX–FTSE	DAX	0.55	0.20	0.31	1.62	0.39
		(9.27)***	(5.73)***	(5.25)***	(17.97)***	(8.83)***
	FTSE	0.62	0.24	0.40	1.47	0.43
		(14.75)***	(8.89)***	(6.20)***	(16.27)***	(10.58)***
HS–NIKKEI	HS	0.54	0.23	0.31	1.58	0.38
		(7.21)***	(4.86)***	(4.61)***	(19.68)***	(7.76)***
	NIKKEI	0.54	0.19	0.47	1.70	0.38
		(9.06)***	(5.05)***	(4.47)***	(15.33)***	(7.21)***
HS–STRAITS	HS	0.53	0.26	0.31	1.58	0.35
		(7.36)***	(5.46)***	(4.75)***	(21.15)***	(7.63)***
	STRAITS	0.46	0.22	0.18	1.81	0.35
		(6.25)***	(4.19)***	(4.66)***	(19.92)***	(7.80)***
NIKKEI–STRAITS	NIKKEI	0.50	0.19	0.48	1.75	0.36
		(7.68)***	(4.18)***	(4.46)***	(15.25)***	(7.23)***
	STRAITS	0.27	0.09	0.20	1.89	0.30
		(1.94)***	(0.72)***	(4.67)***	(19.52)***	(7.74)***
SP–TSE	SP	0.59	0.27	0.56	1.52	0.37
		(10.40)***	(8.54)***	(5.60)***	(16.78)***	(6.99)***
	TSE	0.57	0.24	0.23	1.66	0.41
		(10.29)***	(6.16)***	(4.52)***	(21.62)***	(10.40)***

Notes: See notes in Table 3.

returns (CAC–DAX–FTSE) and the American pair (SP–TSE). Moreover, the American variables' correlation to the Asian variables is lower than their correlation to the European. See in the Appendix the graphs of each return series.

4.2. Multivariate models

Multivariate GARCH models with time varying correlations are essential for enhancing our understanding of the relationships between the (co-)volatilities of economic and financial time series. Thus in this Section, within the framework of the multivariate DCC model, we will analyse the dynamic adjustments of the variances and the correlations for the various indices. Overall we estimate seven bivariate specifications: three for the European countries: CAC 40–DAX 30 (CAC–DAX), CAC 40–FTSE 100 (CAC–FTSE) and DAX 30–FTSE 100 (DAX–FTSE); three for the Asian countries: Hang Seng–Nikkei 225 (HS–NIKKEI), Hang Seng–Straits Times (HS–

STRAITS) and Nikkei 225–Straits Times (NIKKEI–STRAITS); one for the S&P 500 and TSE 300 indices (SP–TSE). Moreover, we estimate two trivariate models: one for the three European countries (CAC–

Table 5
Bivariate AR(1)-DCC-FIAPARCH (1,d,1) models equation for quasi correlations.

	ρ_{ij}	a	b
CAC–DAX	0.42	0.0159	0.9840
	(1.60)*	(3.52)***	(213.3)***
CAC–FTSE	0.25	0.0241	0.9758
	(1.25)	(2.81)***	(112.1)***
DAX–FTSE	0.26	0.0228	0.9771
	(1.37)	(2.79)***	(117.4)***
HS–NIKKEI	0.37	0.0119	0.9861
	(5.09)***	(2.14)***	(134.9)***
HS–STRAITS	0.52	0.0523	0.9138
	(20.95)***	(5.09)***	(43.00)***
NIKKEI–STRAITS	0.30	0.0117	0.9860
	(4.41)***	(1.56)*	(92.24)***
SP–TSE	0.64	0.0261	0.9589
	(28.31)***	(3.59)***	(61.74)***

Notes: See notes in Table 3.

Table 6
Bivariate AR(1)-DCC-FIAPARCH(1,d,1) Models degrees of freedom — Ljung–Box test statistics.

		υ	Q_{12}	Q_{12}^2
CAC–DAX	CAC	8.03	13.93	34.99
		(12.61)***	[0.31]	[0.00]
	DAX		21.37	15.06
			[0.05]	[0.24]
CAC–FTSE	CAC	9.63	18.64	11.23
		(11.41)***	[0.10]	[0.51]
	FTSE		12.11	19.94
			[0.44]	[0.07]
DAX–FTSE	DAX	9.20	19.74	8.40
		(11.50)***	[0.07]	[0.75]
	FTSE		11.03	24.94
			[0.53]	[0.02]
HS–NIKKEI	HS	7.02	32.52	57.59
		(15.22)***	[0.00]	[0.00]
	NIKKEI		11.67	7.15
			[0.47]	[0.85]
HS–STRAITS	HS	6.23	21.39	76.45
		(15.75)***	[0.04]	[0.00]
	STRAITS		16.69	1.58
			[0.16]	[1.00]
NIKKEI–STRAITS	NIKKEI	6.92	8.33	10.46
		(14.87)***	[0.76]	[0.58]
	STRAITS		23.24	1.56
			[0.03]	[1.00]
SP–TSE	SP	7.33	34.23	8.75
		(14.31)***	[0.00]	[0.72]
	TSE		19.33	5.21
			[0.08]	[0.95]

Notes: The numbers in parentheses are t-statistics.
The numbers in brackets are p-values.
***, **, * denote significance at the 0.05, 0.10, 0.15 level respectively.

Table 7
Wald tests - $\chi^2(1)$ — Bivariate models.

Panel A: Tests for restrictions on fractional differencing parameters			
H_0	$d_i's$	$d_i's = 0$	$d_i's = 1$
CAC–DAX	0.41 {0.05}–0.39 {0.04}	81.17 [0.00]	5.65 [0.02]
CAC–FTSE	0.36 {0.05}–0.41 {0.04}	95.13 [0.00]	8.75 [0.00]
DAX–FTSE	0.39 {0.04}–0.43 {0.04}	120.75 [0.00]	5.38 [0.02]
HS–NIKKEI	0.38 {0.05}–0.38 {0.05}	98.47 [0.00]	9.25 [0.00]
HS–STRAITS	0.35 {0.05}–0.35 {0.04}	93.60 [0.00]	17.17 [0.00]
NIKKEI–STRAITS	0.36 {0.05}–0.30 {0.04}	100.74 [0.00]	27.10 [0.00]
SP–TSE	0.37 {0.05}–0.41 {0.04}	100.95 [0.00]	8.58 [0.00]

Panel B: Tests for restrictions on power term parameters			
H_0	$\delta_i's$	$\delta_i's = 1$	$\delta_i's = 2$
CAC–DAX	1.52 {0.09}–1.64 {0.08}	199.12 [0.00]	57.29 [0.00]
CAC–FTSE	1.63 {0.09}–1.55 {0.09}	203.89 [0.00]	59.48 [0.00]
DAX–FTSE	1.62 {0.09}–1.47 {0.09}	199.66 [0.00]	54.28 [0.00]
HS–NIKKEI	1.58 {0.08}–1.70 {0.11}	260.08 [0.00]	82.24 [0.00]
HS–STRAITS	1.58 {0.07}–1.81 {0.09}	348.13 [0.00]	118.02 [0.00]
NIKKEI–STRAITS	1.75 {0.11}–1.89 {0.10}	285.20 [0.00]	109.83 [0.00]
SP–TSE	1.52 {0.09}–1.66 {0.08}	241.94 [0.00]	70.86 [0.00]

Notes: For each of the seven pairs of indices, Table 7 reports the values of the Wald statistics of the unrestricted bivariate DCC-FIAPARCH(1,d,1) and the restricted ($d_i = 0,1$; $\delta = 1,2$) models respectively.
The numbers in curly brackets are standard errors.
The numbers in square brackets are p-values.

DAX–FTSE) and one for the three Asian countries (NIKKEI–HS–STRAITS). We have also performed the test of Engle and Sheppard (2001) for DCC against constant conditional correlations in all models. Table 2 shows that the CCC hypothesis is always rejected at 100% significance level.

We estimate the various specifications using the approximate Quasi Maximum Likelihood Estimation (QMLE) method as implemented in the OxMetrics module G@rch 5.0 by Laurent (2007). The existence of outliers, particularly in daily data, causes the distribution of returns to exhibit excess kurtosis (Table 1b, panel A with descriptive statistics). To accommodate the presence of such leptokurtosis, we estimate the models using student-t distributed innovations.

4.2.1. Bivariate processes

For the mean equation we choose a VAR(1) process whereas in the variance equation a (1,d,1) order is chosen for the FIAPARCH formulation with DCC.

Table 3 gives the mean equation coefficients estimates. In the majority of the models (nine out of fourteen) the AR(1) coefficients (ϕ_{ii}) are significant at the 10% level or better. The mean equation of diagonal elements of Φ (ϕ_{ij}), which capture the cross effects between the series, are also significant in most of the cases (eight out of the fourteen cases). In the European stock markets we see that DAX is positively affected by the other two European indices while the German index has a negative impact on FTSE. In the Asian markets there is a mixed bidirectional feedback between HS and NIKKEI, where the latter affects the former negatively and the effect in the opposite direction is positive. STRAITS affects both HS and NIKKEI positively, but it is independent of changes from the other two Asian indices. Finally, there is a unidirectional positive feedback from SP to TSE.

Table 4 summarizes the variance equation results. In all cases the fractional differencing parameter (d_i), the power term parameter (δ_i) and the asymmetry parameter (γ_i) are highly significant. The estimates for two GARCH parameters (β_i, c_i) are also significant except for one case. The fractional parameters are very similar in the three European models with values between 0.36 and 0.43, while in the Asian models we get similar but slightly lower values of long-range volatility

Table 8
Trivariate AR(1)-DCC-FIAPARCH(1,d,1) models.

	CAC–DAX–FTSE			NIKKEI–HS–STRAITS		
	CAC	DAX	FTSE	NIKKEI	HS	STRAITS
ϕ_{ii}	−0.02	−0.10	0.03	−0.04	0.01	0.07
	(−0.78)	(−5.14)***	(1.52)*	(−2.98)***	(0.68)	(4.28)***
	D	C	C	HS	N	N
ϕ_{ij}	−0.01	0.08	−0.01	0.02	−0.02	−0.002
	(−0.46)	(4.03)***	(−0.46)	(2.05)***	(−1.99)***	(−0.19)
	F	F	D	S	S	HS
	0.05	0.05	−0.01	0.05	0.05	0.01
	(2.24)***	(2.31)***	(−0.93)	(3.37)***	(2.82)***	(0.95)
β_i	0.59	0.56	0.62	0.55	0.56	0.45
	(14.91)***	(10.94)	(18.64)***	(9.10)***	(8.67)***	(6.15)***
c_i	0.29	0.26	0.29	0.19	0.27	0.23
	(11.37)***	(7.75)***	(11.83)***	(5.05)***	(6.54)***	(4.14)***
γ_i	0.35	0.25	0.38	0.41	0.25	0.16
	(6.04)***	(4.73)***	(5.73)***	(4.91)***	(4.26)***	(4.05)***
δ_i	1.59	1.70	1.52	1.77	1.61	1.83
	(19.86)***	(20.68)***	(16.72)***	(16.48)***	(21.04)***	(20.25)***
d_i	0.35	0.35	0.38	0.40	0.36	0.32
	(10.21)***	(9.72)***	(12.12)***	(7.61)***	(7.89)***	(7.67)
	C–D	C–F	D–F	HS–N	N–S	S–HS
ρ_{ij}	0.45	0.27	0.33	0.38	0.33	0.50
	(2.65)***	(1.74)**	(0.38)***	(13.68)***	(11.43)	(18.50)***
a	0.0129			0.0326		
	(5.61)***			(2.89)***		
b	0.9870			0.9449		
	(425.2)***			(36.48)***		
v	8.57			7.42		
	(15.42)***			(17.09)***		
Q_{12}	15.89	20.01	12.96	9.55	26.81	19.40
	[0.20]	[0.07]	[0.37]	[0.66]	[0.01]	[0.08]
Q_{12}^2	46.76	23.57	24.78	9.82	95.19	1.55
	[0.00]	[0.02]	[0.02]	[0.63]	[0.00]	[1.00]

Notes: See notes in Table 6.

dependence (0.30–0.38). The SP–TSE process generated significant estimates (0.37 and 0.41), similar to the other six models. The power terms are also similar, with the values from the Asian pairs being higher than in the other four bivariate formulations. The three Asian processes gave powers between 1.58 and 1.89, while in the rest of the models we obtained power terms between 1.47 and 1.66. It is worth mentioning that STRAITS exhibits the highest power terms (1.89 and 1.81) and the lowest degree of (long-memory) persistence (0.30 and 0.35) in the two bivariate formulations that are included (NIKKEI–STRAITS and HS–STRAITS, respectively). Finally, the asymmetric response of volatility to positive and negative shocks is strong in all cases. The value of the corresponding parameter γ_i is between 0.18 (STRAITS) and 0.56 (SP).

The unconditional correlation coefficient ρ_{ij} is highly significant in most cases (five out of the seven cases, see the first column of Table 5). CAC–FTSE and DAX–FTSE generated insignificant coefficients. It is interesting that CAC–FTSE also gave insignificant cross effects in

Table 9
Wald tests − $\chi^2(1)$ − Trivariate models.

H_0	CAC–DAX–FTSE	NIKKEI–HS–STRAITS
$d_i's$	0.35 {0.03}–0.35 {0.04}–0.38 {0.03}	0.40 {0.05}–0.36 {0.05}–0.32 {0.04}
$d_i's = 0$	157.89 [0.00]	137.46 [0.00]
$d_i's = 1$	0.92 [0.34]	0.73 [0.39]
$\delta_i's$	1.59 {0.08}–1.70 {0.08}–1.52 {0.09}	1.77 {0.11}–1.61 {0.08}–1.83 {0.09}
$\delta_i's = 1$	358.65 [0.00]	593.48 [0.00]
$\delta_i's = 2$	194.97 [0.00]	345.16 [0.00]

Notes: The numbers in curly brackets are standard errors.
The numbers in square brackets are p-values.

the mean of returns (see Table 3). Among the other models, SP–TSE gave the highest unconditional correlation parameter, 0.64, whereas the lowest significant value is obtained from NIKKEI–STRAITS, that is 0.30. The DCC parameters a and b are also highly significant, indicating a considerable time varying comovement. The persistence of the conditional correlations, measured by the sum of a and b, is always high and close to unity, that is between 0.9661 and 0.9999. b is always above 0.90 and a is below 0.05, revealing slight response to innovations and major persistence.

The degrees of freedom ($\mathbf{R}_t$) parameters are highly significant and fluctuate around 7 for the Asian and American models and around 9 for the European processes. In the majority of the cases the hypothesis of uncorrelated standardized and squared standardized residuals is well supported (see the last two columns of Table 6).

Next, the Wald testing procedure applied on the estimated models provides support for the consideration of long memory and power features in our modelling. We examine the Wald statistics for the linear constraints $d_i's = 0$ (stable APARCH) and $d_i's = 1$ (IAPARCH). As seen in panel A of Table 7, the Wald tests clearly reject both the stable and the integrated null hypotheses against the FIAPARCH one. We also test whether the estimated power terms are significantly different from unity or two using Wald tests. All the estimated power coefficients are significantly different from either unity or two (see Table 7, panel B). We observe in all cases higher Wald statistics for the $d_i's = 0$ and the $\delta_i's = 1$ hypotheses in comparison with their alternatives: $d_i's = 1$ and $\delta_i's = 2$, which means that the former hypotheses are more 'rejectable' than the latter ones.

4.2.2. Trivariate processes

Table 8 reports the parameters of interest for the two trivariate AR(1)-DCC-FIAPARCH(1,d,1) models for the three European and the three Asian indices. The cross effects in the mean equation are similar to the bivariate results. DAX is positively affected by both CAC and FTSE as in the bivariate processes, while FTSE is independent of changes from the other two markets in the trivariate model. In the trivariate model of the Asian countries we obtain the same results for the cross effects as in the bivariate ones. The ARCH and GARCH parameters (β_i, c_i) are highly significant in all cases. The fractional parameters (d_i) are all

significant and similar to the ones obtained from the bivariate models. FTSE gives the highest value for d_i among the three European series as in the bivariate case and the same stands for NIKKEI (0.40) in the Asian countries. The power terms δ_i are also significant and in accordance with the corresponding results from the bivariate models. The Asian indices give higher power terms on average in comparison with the European indices. The asymmetry parameter γ_i is strong in both models and similar to the bivariate cases. STRAITS again gives the lowest value of d_i (0.32), the highest value of δ_i (1.83) and the lowest value of γ_i (0.16). Both trivariate models generate strong unconditional correlation coefficients ρ_{ij}, which are all highly significant unlike the bivariate cases of the European countries. In Europe the highest unconditional correlation is between CAC and DAX (0.45). The highest correlation between the French and the German financial markets is justified since they are both Continental European markets. FTSE is the Anglo-Saxon market with characteristics that differ traditionally from the Continental European markets because of more advanced financial liberalisation and deregulation. So, the correlation of FTSE to CAC or DAX is found to be lower. In Asia the highest unconditional correlation is between HS and STRAITS (0.50), the same as in the bivariate models. The conditional correlations' is again high (close to unity) and significant in both models. Finally, the degrees of freedom ($\sum_{t=1}^{T}$) parameters are highly significant and lower in Asia than in Europe, which also confirms the bivariate results.

Next, again we examine the Wald statistics for the linear constraints $d_i's = 0$ (stable APARCH) and $d_i's = 1$ (IAPARCH). As seen in Table 9 the Wald tests reject the stable null hypothesis but not the integrated one, unlike the bivariate results, where both hypotheses are rejected against the FIAPARCH one. Regarding the Wald tests of the power terms, all the estimated power coefficients are significantly different from either unity or two as in the bivariate models.

4.3. Subsamples

4.3.1. Bivariate processes

All bivariate models run for the whole sample period are re-estimated for each subsample period under the same specification, that is the AR(1)-DCC-FIAPARCH(1,d,1) with student-t distributed

Table 10
Bivariate AR(1)-DCC–FIAPARCH(1,d,1) models.

Variance equation: Fractional parameter d_i								
	Whole sample		Subsample A		Subsample B		Subsample C	
	CAC	DAX	CAC	DAX	CAC	DAX	CAC	DAX
CAC–DAX	0.41	0.39	0.29	0.43	0.42	0.39	0.29	0.29
	(8.33)***	(8.64)***	(3.76)***	(3.64)***	(5.17)***	(5.78)***	(3.61)***	(5.06)***
	CAC	FTSE	CAC	FTSE	CAC	FTSE	CAC	FTSE
CAC–FTSE	0.36	0.41	0.40	0.45	0.31	0.36	0.37	0.37
	(7.78)***	(10.43)***	(4.46)***	(2.68)***	(6.19)***	(9.01)***	(5.66)***	(8.30)***
	DAX	FTSE	DAX	FTSE	DAX	FTSE	DAX	FTSE
DAX–FTSE	0.39	0.43	0.40	0.31	0.35	0.39	0.40	0.39
	(8.83)***	(10.58)***	(5.11)***	(3.80)***	(6.97)***	(8.76)***	(6.51)***	(7.93)***
	HS	NIKKEI	HS	NIKKEI	HS	NIKKEI	HS	NIKKEI
HS–NIKKEI	0.38	0.38	0.39	0.34	0.41	0.43	0.48	0.39
	(7.76)***	(7.21)***	(5.29)***	(4.90)***	(5.32)***	(5.61)***	(2.43)***	(2.64)***
	HS	STRAITS	HS	STRAITS	HS	STRAITS	HS	STRAITS
HS–STRAITS	0.35	0.35	0.27	0.05	0.19	0.29	0.19	0.31
	(7.63)***	(7.80)***	(5.54)***	(1.02)***	(2.80)***	(4.10)***	(2.77)***	(4.64)***
	NIKKEI	STRAITS	NIKKEI	STRAITS	NIKKEI	STRAITS	NIKKEI	STRAITS
NIKKEI–STRAITS	0.36	0.30	0.34	0.22	0.37	0.32	0.28	0.26
	(7.23)***	(7.74)***	(3.78)***	(7.27)***	(6.44)***	(5.44)***	(4.55)***	0.96
	SP	TSE	SP	TSE	SP	TSE	SP	TSE
SP–TSE	0.37	0.41	0.32	0.12	–	–	–	–
	(6.99)***	(10.40)***	(5.01)***	(5.06)***				

Notes: See notes in Table 3.

Table 11
Tests for restrictions on fractional differencing parameters — Wald tests — $\chi^2(1)$ — Bivariate models.

H_0	Whole sample		Subsample A		Subsample B		Subsample C	
	$d_i's = 0$	$d_i's = 1$	$d_i's = 0$	$d_i's = 1$	$d_i's = 0$	$d_i's = 1$	$d_i's = 0$	$d_i's = 1$
C–D	81.17 [0.00]	5.65 [0.02]	22.53 [0.00]	3.46 [0.06]	31.85 [0.00]	1.62 [0.20]	20.97 [0.00]	11.21 [0.00]
C–F	95.13 [0.00]	8.75 [0.00]	14.72 [0.00]	0.50 [0.48]	64.91 [0.00]	15.60 [0.00]	57.71 [0.00]	7.66 [0.01]
D–F	120.75 [0.00]	5.38 [0.02]	35.58 [0.00]	5.88 [0.02]	78.46 [0.00]	9.71 [0.00]	69.90 [0.00]	4.68 [0.03]
HS–N	98.47 [0.00]	9.25 [0.00]	49.17 [0.00]	7.23 [0.01]	48.55 [0.00]	1.76 [0.18]	11.09 [0.00]	0.23 [0.63]
HS–S	93.60 [0.00]	17.17 [0.00]	21.81 [0.00]	105.70 [0.00]	15.11 [0.00]	17.39 [0.00]	22.86 [0.00]	24.01 [0.00]
N–S	100.74 [0.00]	27.10 [0.00]	35.87 [0.00]	22.39 [0.00]	59.74 [0.00]	12.41 [0.00]	30.79 [0.00]	22.38 [0.00]
SP–T	100.95 [0.00]	8.58 [0.00]	41.01 [0.00]	64.98 [0.00]	–	–	–	–

Notes: The numbers in brackets are p-values.

errors. Only the model for SP–TSE did not converge for subsamples B and C. The leverage parameter γ_i is significant in most models in the three subsamples and the estimated values are similar to those for the whole sample (see the additional Appendix B, Tables B.3-B.5, which is available upon request).

The fractional parameter results in Table 10 show that all estimates are significant except for one. In most cases the subsample models' values of d_i fluctuate around the respective value of the original model (for the whole sample). We cannot conclude on a certain direction of this fluctuation. The degree of the series' long-memory 'persistence' across the different subperiods remains at the same level for the majority of the models. Table 11 reports the Wald statistics for the linear constraints $d_i's = 0$ and $d_i's = 1$ across the subperiods. Both hypotheses are rejected against the FIAPARCH in most cases.

The power term parameter δ_i is highly significant across all subsamples' estimates (see Table 12). As in the case of the fractional parameter, the power terms for the sub-periods' models also fluctuate around the level of the value in the corresponding model for the entire period. Interestingly, for most cases the power term estimates of the period between the two crises (subsample C) are higher than the estimates in the other two subsamples (A and B) and the whole sample's values. The Wald tests (Table 13) show that δ_i is significantly different from either unity or two for all the cases across the three subsamples.

The dynamic correlation estimates follow the predictable pattern according to the financial crisis literature. They are always lower before the crisis. After the crisis break they are much higher and remain on a higher level. These findings are depicted on the graphs of the dynamic conditional correlations for each bivariate model presented in the Appendix. It is obvious that the DCCs estimated after the second break for the GFC period are much higher than those after the AFC break, revealing that the recent crisis has caused stronger contagion effects in the market and leads the investors to exhibit more evident herding behaviour. During the GFC the international financial integration is complete in comparison with the AFC in 1997, where the financial liberalisation and deregulation was still in process. As seen in Table 14, the correlation coefficient ρ_{ij}, which is significant in most cases, in the pre-AFC period (subsample A) always receives lower values than in the post-AFC period and the period between the two crises (subsamples B and C, respectively). For the majority of the models, we also observe that the ρ_{ij} value of the whole period model approaches mostly the level of the pre-crisis model.

Finally, in the additional Appendix B (which is available upon request) with all the parameters' estimations we observe that the AR(1) coefficients (ϕ_{ii}) are significant at the 15% level or better for the majority of the models in the subsamples. The cross effects are significant in many cases (see also panel A in Table 17). DAX, as with the whole sample, is affected positively by the other two European indices before the AFC (subsample A) and between the two crises (subsample C).

Table 12
Bivariate AR(1)-DCC-FIAPARCH(1,d,1) Models.

Variance equation: Power term parameter δ_i

	Whole sample		Subsample A		Subsample B		Subsample C	
	CAC	DAX	CAC	DAX	CAC	DAX	CAC	DAX
CAC–DAX	1.52	1.64	1.65	1.53	1.51	1.61	2.17	2.14
	(17.54)***	(19.70)***	(8.16)***	(5.81)***	(13.60)***	(14.54)***	(8.13)***	(8.94)***
	CAC	FTSE	CAC	FTSE	CAC	FTSE	CAC	FTSE
CAC–FTSE	1.63	1.55	1.42	1.36	1.63	1.47	1.73	1.48
	(18.64)***	(17.60)***	(6.22)***	(4.28)***	(14.90)***	(15.02)***	(12.65)***	(10.88)***
	DAX	FTSE	DAX	FTSE	DAX	FTSE	DAX	FTSE
DAX–FTSE	1.62	1.47	1.37	1.34	1.62	1.43	1.64	1.55
	(17.97)***	(16.27)***	(6.74)***	(4.21)***	(14.16)***	(14.54)***	(11.08)***	(11.01)***
	HS	NIKKEI	HS	NIKKEI	HS	NIKKEI	HS	NIKKEI
HS–NIKKEI	1.58	1.70	1.51	1.91	1.83	1.79	1.81	2.04
	(19.68)***	(15.33)***	(14.84)***	(10.14)***	(11.61)***	(9.74)***	(3.24)***	(5.49)***
	HS	STRAITS	HS	STRAITS	HS	STRAITS	HS	STRAITS
HS–STRAITS	1.58	1.81	1.39	2.18	2.07	1.98	2.27	2.09
	(21.15)***	(19.92)***	(16.01)***	(6.42)***	(11.30)***	(14.31)***	(11.11)***	(12.68)***
	NIKKEI	STRAITS	NIKKEI	STRAITS	NIKKEI	STRAITS	NIKKEI	STRAITS
NIKKEI–STRAITS	1.75	1.89	2.10	1.87	1.85	1.98	2.23	2.05
	(15.25)***	(19.52)***	(8.32)***	(10.93)***	(9.05)***	(16.46)***	(7.09)***	(13.51)***
	SP	TSE	SP	TSE	SP	TSE	SP	TSE
SP–TSE	1.52	1.66	1.89	2.22	–	–	–	–
	(16.78)***	(21.62)***	(7.17)***	(6.34)***				

Notes: See notes in Table 3.

Table 13
Tests for restrictions on power term parameters — Wald tests — $\chi^2(1)$ — Bivariate models.

H_0	Whole sample		Subsample A		Subsample B		Subsample C	
	$\delta_i\text{'s}=1$	$\delta_i\text{'s}=2$	$\delta_i\text{'s}=1$	$\delta_i\text{'s}=2$	$\delta_i\text{'s}=1$	$\delta_i\text{'s}=2$	$\delta_i\text{'s}=1$	$\delta_i\text{'s}=2$
C–D	199.12 [0.00]	57.29 [0.00]	43.01 [0.00]	12.66 [0.00]	103.78 [0.00]	29.06 [0.00]	49.67 [0.00]	24.19 [0.00]
C–F	203.89 [0.00]	59.48 [0.00]	14.11 [0.00]	2.70 [0.10]	129.00 [0.00]	35.45 [0.00]	88.42 [0.00]	26.55 [0.00]
D–F	199.66 [0.00]	54.28 [0.00]	19.14 [0.00]	3.32 [0.07]	126.57 [0.00]	33.18 [0.00]	85.87 [0.00]	25.46 [0.00]
HS–N	260.08 [0.00]	82.24 [0.00]	126.62 [0.00]	43.41 [0.00]	94.59 [0.00]	36.13 [0.00]	16.13 [0.00]	6.80 [0.01]
HS–S	348.13 [0.00]	118.02 [0.00]	47.15 [0.00]	17.59 [0.00]	125.28 [0.00]	56.59 [0.00]	134.03 [0.00]	66.07 [0.00]
N–S	285.20 [0.00]	109.83 [0.00]	92.78 [0.00]	40.88 [0.00]	125.11 [0.00]	52.24 [0.00]	77.70 [0.00]	37.53 [0.00]
SP–T	241.94 [0.00]	70.86 [0.00]	37.37 [0.00]	17.20 [0.00]	–	–	–	–

Notes: The numbers in brackets are p-values.

Interestingly, these two effects disappear in subsample B, that is in the period after the AFC until the end of the sample. Similarly, the negative effect of the German index on FTSE disappears in the three subsamples.

For the HS–NIKKEI pair, there is still a mixed bidirectional feedback in the periods after the AFC and in between the two crises. However, the negative effect of NIKKEI on HS disappears in the pre AFC period. In the other two Asian pairs with STRAITS the ϕ_{ij} coefficients indicate a positive effect from STRAITS to HS and NIKKEI for all three subsamples, as with the whole sample. The higher values of the cross effect coefficients in the period with the two crises taking place indicate a more sound market integration in Asia during the turbulent times. For the American pair in the pre-AFC period, as in the whole period, SP affects TSE positively.

4.3.2. Trivariate processes

Finally, we re-estimate the two trivariate models, one for the Asian indices and one for the European, for the three subsamples. The Asian model did not converge for the third sub-period and the European for the second one. Our findings are very similar to the ones for the bivariate processes. The fractional parameters and the power terms (Table 15, panels A and B) fluctuate around the values of the whole sample and are always significant. The Wald tests show that δ_i is significantly different from either unity or two, and they also reject the $d_i\text{'s}=0$ hypothesis, but do not reject the $d_i\text{'s}=1$ (see panels A and B in Table 16). The correlation coefficients (Table 15, panel C) are again higher in the post-AFC periods (subsamples B and C) than in the pre-AFC period (subsample A). See also the graphs of the conditional correlations for the two trivariate models in the appendix. The asymmetric response of volatility to positive and negative shocks is strong in most subsamples' models, with γ_i

Table 14
Bivariate AR(1)-DCC-FIAPARCH(1,*d*,1) Models.

Unconditional correlations ρ_{ij}				
	Whole sample	Subsample A	Subsample B	Subsample C
CAC–DAX	0.42	0.53	0.66	0.68
	(1.60)*	(21.82)***	(1.98)***	(1.97)***
CAC–FTSE	0.25	0.24	0.88	0.84
	(1.25)	(1.57)*	(41.51)***	(34.38)***
DAX–FTSE	0.26	0.26	0.82	0.77
	(1.37)	(1.39)	(23.70)***	(25.75)***
HS–NIKKEI	0.37	0.30	0.55	0.50
	(5.09)***	(12.40)***	(20.81)***	(13.73)***
HS–STRAITS	0.52	0.38	0.63	0.57
	(20.95)***	(10.66)***	(37.52)***	(22.50)***
NIKKEI–STRAITS	0.30	0.20	0.46	0.22
	(4.41)***	(5.50)***	(18.75)***	(2.23)***
SP–TSE	0.64	0.54	–	–
	(28.31)***	(9.56)***		

Notes: See notes in Table 3

fluctuating around the respective estimated values of the whole sample (see Tables B.6–B.8 in the additional Appendix B).

Regarding the cross effects in the additional Appendix B (see also Table 17), DAX, similarly to the whole sample, is positively affected by both CAC and FTSE in the pre-AFC period but only by CAC in the period between the two crises, where the FTSE index affects the French index positively as in the model for the whole sample. In the Asian case, HS positively affects both NIKKEI and STRAITS before the AFC, while STRAITS has a positive impact on the other two indices in the post-AFC period, including also the GFC, as in the whole sample. During this period NIKKEI affects HS negatively, as in the whole sample.

4.4. Discussion

Our analysis gives strong evidence that conditional volatility is best modelled with the FIAPARCH specification, which combines long memory, leverage effects and power transformations of the conditional variances. These three features augment the traditional GARCH model in a suitable way to adequately fit the volatility process. The Wald tests applied support the particular augmented model and are in line with the results of Conrad et al. (2011). The corresponding parameters are found robust to the structural breaks in the returns' and volatilities' series, since their estimated values in the subsamples are similar to those of the whole sample. The volatility 'persistence', as measured by the long memory parameter d_i, is significant in almost all cases and different from either zero or unity. In the whole sample it hovers around the same level for the eight stock markets, which indicates that a common factor of 'persistence' may affect the markets and due to the financial integration their co-persistence is apparent. The asymmetry parameter γ_i is always significant and positive, meaning a leverage for negative returns. That is, negative shocks have stronger influence on the volatility of returns than the positive shocks of the same level. The power term δ_i allows us to increase the flexibility of our modelling. The power transformation of returns, which is significantly different from one and two, gives the appropriate formulation to model the volatility process. One or more cross effects between the dependent variables in the majority of the multivariate specifications are also significant for the mean of returns and show a time varying behaviour across the subsamples. Finally, the implementation of the DCC model of Engle (2002) provides a thorough insight into the time varying pattern of conditional correlations, which accounts for structural breaks that correspond to major financial crisis events.

5. Contagion effect

In order to complete our empirical modelling of the main equity markets during the two crisis periods we perform two contagion tests. We intend to clarify whether the higher correlations observed in the post crisis periods are due to the contagion between the financial

Table 15
Trivariate AR(1)-DCC-FIAPARCH(1,d,1) models.

Panel A: Variance equation: Fractional parameter d_i						
	ASIA			EUROPE		
	NIKKEI	HS	STRAITS	CAC	DAX	FTSE
Whole sample	0.40 (7.61)***	0.36 (7.89)***	0.32 (7.67)***	0.35 (10.21)***	0.35 (9.72)***	0.38 (12.12)***
Subsample A	0.43 (3.06)***	0.26 (3.00)***	0.19 (7.36)***	0.32 (4.61)***	0.41 (4.15)***	0.42 (3.77)***
Subsample B	0.36 (6.80)***	0.22 (3.30)***	0.32 (5.90)***	–	–	–
Subsample C	–	–	–	0.36 (2.70)***	0.39 (3.28)***	0.34 (3.80)***

Panel B: Variance equation: Power term parameter δ_i						
	ASIA			EUROPE		
	NIKKEI	HS	STRAITS	CAC	DAX	FTSE
Whole sample	1.77 (16.48)***	1.61 (21.04)***	1.83 (20.25)***	1.59 (19.56)***	1.70 (20.68)***	1.52 (16.72)***
Subsample A	2.19 (6.65)***	1.61 (10.09)***	2.01 (11.95)***	1.72 (9.58)***	1.57 (6.29)***	1.48 (5.47)***
Subsample B	1.95 (9.40)***	2.07 (12.72)***	1.94 (17.43)***	–	–	–
Subsample C	–	–	–	1.70 (6.98)***	1.63 (6.00)***	1.45 (5.69)***

Panel C: Unconditional correlations ρ_{ij}						
	ASIA			EUROPE		
	NIKKEI-HS	NIKKEI–STRAITS	HS–STRAITS	CAC–DAC	CAC–FTSE	DAX–FTSE
Whole sample	0.38 (13.68)***	0.33 (11.43)***	0.50 (18.50)***	0.45 (2.65)***	0.27 (1.74)***	0.33 (2.38)***
Subsample A	0.22 (6.77)***	0.20 (5.97)***	0.37 (11.29)***	0.54 (19.38)***	0.58 (23.08)***	0.44 (14.94)***
Subsample B	0.51 (25.07)***	0.47 (21.22)***	0.62 (36.10)***	–	–	–
Subsample C	–	–	–	0.62 (0.84)	0.57 (1.18)	0.54 (1.62)*

Notes: See notes in Table 3

markets or their interdependence. Following Forbes and Rigobon (2002), contagion is characterised by the increased spillovers between different markets after a crisis shock in one market and interdependence is their high inter-linkages during all states of the economy. The higher volatilities after a shock result in higher correlation coefficients

Table 16
Wald tests − $\chi^2(1)$ − Trivariate models.

Panel A: Tests for restrictions on fractional differencing parameters				
	Asia		Europe	
H_0	$d_i's = 0$	$d_i's = 1$	$d_i's = 0$	$d_i's = 1$
Whole sample	137.46 [0.00]	0.73 [0.39]	157.89 [0.00]	0.92 [0.34]
Subsample A	27.92 [0.00]	0.49 [0.48]	37.98 [0.00]	0.62 [0.43]
Subsample B	53.55 [0.00]	0.65 [0.42]	–	–
Subsample C	–	–	10.76 [0.00]	0.07 [0.79]

Panel B: Tests for restrictions on power term parameters				
	Asia		Europe	
H_0	$\delta_i's = 1$	$\delta_i's = 2$	$\delta_i's = 1$	$\delta_i's = 2$
whole sample	593.48 [0.00]	345.16 [0.00]	358.65 [0.00]	194.97 [0.00]
subsample A	137.19 [0.00]	86.16 [0.00]	74.46 [0.00]	40.16 [0.00]
subsample B	230.53 [0.00]	146.89 [0.00]	–	–
subsample C	–	–	27.05 [0.00]	14.63 [0.00]

Notes: The numbers in brackets are p-values.

calculations due to heteroskedasticity and omitted variables. This can mislead the analysis in favour of contagion, while the interdependence is the actual spillover phenomenon. Forbes and Rigobon (2002) proposed an adjustment to the correlation coefficient calculation in order to test it during crisis events. We will use the DCC coefficients generated by (the estimated) Engle's model in order to overcome the limitations of the classic correlations coefficients. Cho and Parhizgari (2008) point out the superiority of the DCCs in comparison to the Forbes and Rigobon (2002) modified coefficients, since Engle's model estimates not only volatility-adjusted correlations but also correlations that consider the time-varying behaviour of the volatility pattern.

Our model's DCCs computed from the multivariate framework (with cross effects in the mean equation, and long memory, asymmetries and power transformations in the variance equation) are suitable to test the contagion effect during both crises (AFC and GFC). We perform two contagion tests used broadly in the empirical literature: the t-test in the difference of the means of DCCs across the subsamples to detect the significant increase after crisis episodes (see for example Cho & Parhizgari, 2008) and the DCCs regression analysis with crisis intercept dummies to observe the upward shift of the correlations' mean (see, for example, Chiang et al., 2007). The DCCs from the whole sample's bivariate models are used for both tests. The two crisis breaks (see Table 1a) are applied to determine the pre- and post-crisis periods of the t-test and to form the dummies for the regressions.

The t-test is calculated for the difference of the dynamic correlations means of each period before and after both crises. Tables 18 and 19 report the main statistical properties of the correlations for the whole

Table 17
Cross Effects (ϕ_{ij}, $i \neq j$, coefficients).

Whole sample	Pre-AFC period	Post-AFC period	Subsample C
Panel A: Bivariate models			
CAC, FTSE $\overset{+}{\rightarrow}$ DAX	CAC, FTSE $\overset{+}{\rightarrow}$ DAX	–	CAC, FTSE $\overset{+}{\rightarrow}$ DAX
DAX $\rightarrow$ FTSE	–	–	–
STRAITS $\overset{+}{\rightarrow}$ NIKKEI, HS	STRAITS $\overset{+}{\rightarrow}$ NIKKEI, HS	STRAITS $\overset{+}{\rightarrow}$ NIKKEI, HS	STRAITS $\overset{+}{\rightarrow}$ NIKKEI, HS
HS $\overset{+}{\rightleftharpoons}$ NIKKEI	HS $\overset{+}{\rightarrow}$ NIKKEI	HS $\rightleftharpoons^{+}$ NIKKEI	HS $\overset{+}{\rightleftharpoons}$ NIKKEI
SP $\overset{+}{\rightarrow}$ TSE	SP $\overset{+}{\rightarrow}$ TSE	NC	NC
Panel B: Trivariate Models			
CAC, FTSE $\rightarrow^{+}$ DAX	CAC, FTSE $\overset{+}{\rightarrow}$ DAX	NC	CAC $\overset{+}{\rightarrow}$ DAX
FTSE $\rightarrow^{+}$ CAC	–	NC	FTSE $\overset{+}{\rightarrow}$ CAC
STRAITS $\overset{+}{\rightarrow}$ NIKKEI, HS	–	STRAITS $\overset{+}{\rightarrow}$ NIKKEI, HS	NC
HS $\overset{+}{\rightleftharpoons}$ NIKKEI	HS $\overset{+}{\rightarrow}$ NIKKEI, STRAITS	NIKKEI $\rightarrow$ HS	NC

Notes: CAC, FTSE $\overset{+}{\rightarrow}$ DAX: CAC and FTSE affect DAX positively. HS $\overset{+}{\rightleftharpoons}$ NIKKEI: there is a mixed bidirectional feedback between HS and NIKKEI, where the latter affects the former negatively. NC: No convergence.

sample and each subsample around the crises, as well as the t-test's p-value for the means' difference. For both crises, we always reject the null hypothesis that the means are equal (two-sided test). We conclude that their difference is statistically significant and their increase after the crisis event denotes sound contagion effects due to the financial shocks of the AFC and the GFC. For the AFC shock, in particular, we also confirm the contagion effect by excluding the GFC period from the post-AFC sub-sample. It is interesting that the lowest correlation shift after both crises

Table 18
DCC mean difference t-tests for the Asian Financial Crisis.

		C–D	C–F	D–F	HS–N	HS–S	N–S	SP–T
Whole	Mean	0.7277	0.7090	0.6217	0.3972	0.5297	0.3543	0.6505
Sample	Median	0.7527	0.7425	0.6494	0.4172	0.5439	0.3655	0.6658
	Std dev	0.1877	0.1803	0.2060	0.1478	0.1467	0.1426	0.0975
	N	5867	5867	5867	5867	5867	5867	5867
Pre-AFC	Mean	0.5538	0.5532	0.4327	0.3220	0.4666	0.2648	0.6198
	Median	0.5476	0.5858	0.4367	0.3504	0.4793	0.2615	0.6430
	Std dev	0.1222	0.1500	0.1372	0.1317	0.1593	0.1449	0.1126
	N	2400	2400	2490	3602	2518	2518	2408
Post AFC	Mean	0.8481	0.8168	0.7611	0.5167	0.5772	0.4216	0.6720
	Median	0.8818	0.8382	0.7772	0.5228	0.5863	0.4386	0.6863
	Std dev	0.1178	0.1048	0.1185	0.0758	0.1157	0.0964	0.0786
	N	3467	3467	3377	2265	3349	3349	3459
Post AFC	Mean	0.8250	0.7852	0.7224	0.4817	0.5579	0.4008	0.6663
Excl. GFC	Median	0.8645	0.8060	0.7413	0.4904	0.5596	0.4030	0.6783
	Std dev	0.1187	0.0970	0.1065	0.0665	0.1121	0.0939	0.0763
	N	2826	2701	2611	1502	2585	2585	2818
AFC mean	Increase	0.2943	0.2637	0.3284	0.1947	0.1107	0.1568	0.0522
Difference	(%) increase	53.15	47.67	75.88	60.47	23.72	59.22	8.43
	t-Test p value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AFC mean	Increase	0.2712	0.2320	0.2897	0.1597	0.0914	0.1360	0.0466
Difference	(%) increase	48.97	41.94	66.94	49.59	19.59	51.35	7.51
Excl. GFC	t-Test, p value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 19
DCC mean difference t-tests for the recent Global Financial Crisis.

		C–D	C–F	D–F	HS–N	HS–S	N–S	SP–T
Whole sample	Mean	0.7277	0.7090	0.6217	0.3972	0.5297	0.3543	0.6505
	Median	0.7527	0.7425	0.6494	0.4172	0.5439	0.3655	0.6658
	Std dev	0.1877	0.1803	0.2060	0.1478	0.1467	0.1426	0.0975
	N	5867	5867	5867	5867	5867	5867	5867
Pre-GFC	Mean	0.7004	0.6760	0.5810	0.3690	0.5128	0.3337	0.6449
	Median	0.7120	0.7088	0.5902	0.3892	0.5247	0.3432	0.6601
	Std dev	0.1809	0.1702	0.1896	0.1372	0.1448	0.1394	0.0976
	N	5226	5101	5101	5104	5103	5103	5226
Post GFC	Mean	0.9501	0.9285	0.8930	0.5857	0.6424	0.4921	0.6969
	Median	0.9530	0.9324	0.8947	0.5866	0.6630	0.4872	0.7182
	Std dev	0.0137	0.0223	0.0272	0.0342	0.1032	0.0668	0.0834
	N	641	766	766	763	764	764	641
GFC mean difference	Increase	0.2497	0.2525	0.3120	0.2167	0.1296	0.1584	0.0521
	(%) increase	35.64	37.36	53.70	58.73	25.27	47.46	8.07
	t-Test, p value	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 20
DCC AR(1) mean equation with crisis dummies.

$\rho_{ij,t} = \psi_0 + \psi_1 DUM_1 + \psi_2 DUM_2 + \chi_1 \rho_{ij,t-1} + \varepsilon_{ij,t}$

	ψ_0	ψ_1	ψ_2	ψ_3
CAC–DAX	0.0030	0.0015	0.0006	0.9946
	(2.43)***	(3.03)***	(2.54)***	(526.3)***
CAC–FTSE	0.0046	0.0017	0.0012	0.9921
	(2.84)***	(2.56)***	(2.81)***	(386.0)***
DAX–FTSE	0.0046	0.0032	0.0017	0.9895
	(3.99)***	(4.32)***	(3.79)***	(435.9)***
HS–NIKKEI	0.0111	0.0035	0.0025	0.9673
	(7.95)***	(3.77)***	(2.22)***	(282.0)***
HS–STRAITS	0.0200	0.0041	0.0036	0.9569
	(8.67)***	(3.72)***	(2.81)***	(246.6)***
NIKKEI–STRAITS	0.0085	0.0029	0.0020	0.9703
	(7.10)***	(3.06)***	(1.77)***	(297.6)***
SP–TSE	0.0100	0.0008		0.9840
	(4.86)***	(1.83)***		(324.7)***
SP–TSE	0.0100		0.0010	0.9846
	(4.81)***		(1.49)*	(323.9)***

Notes: See notes in Table 3.

is observed between the US and the Canadian stock indices. We recalculate the t-statistics for shorter periods around the crisis breaks (500 observations before and after each crisis) and again the DCCs mean difference is statistically significant (results not reported due to space considerations). Our empirical results confirm the contagion phenomenon for all the main financial markets under study for both crises using the t-test irrespective of the sample size.

In the regression analysis we run the DCCs ($\rho_{ij,t}$) on a constant (ψ_0), the two crisis intercept dummies DUM_1 for the AFC and DUM_2 for the GFC (with coefficients ψ_1 and ψ_2, respectively) and the AR(1) lag with the coefficient χ_1 to remove any serial correlation:

$$\rho_{ij,t} = \psi_0 + \psi_1 DUM_1 + \psi_2 DUM_2 + \chi_1 \rho_{ij,t-1} + u_{ij,t}.$$

We limit our correlation model to the mean equation without conditional variance estimation, since no ARCH effect is neglected.
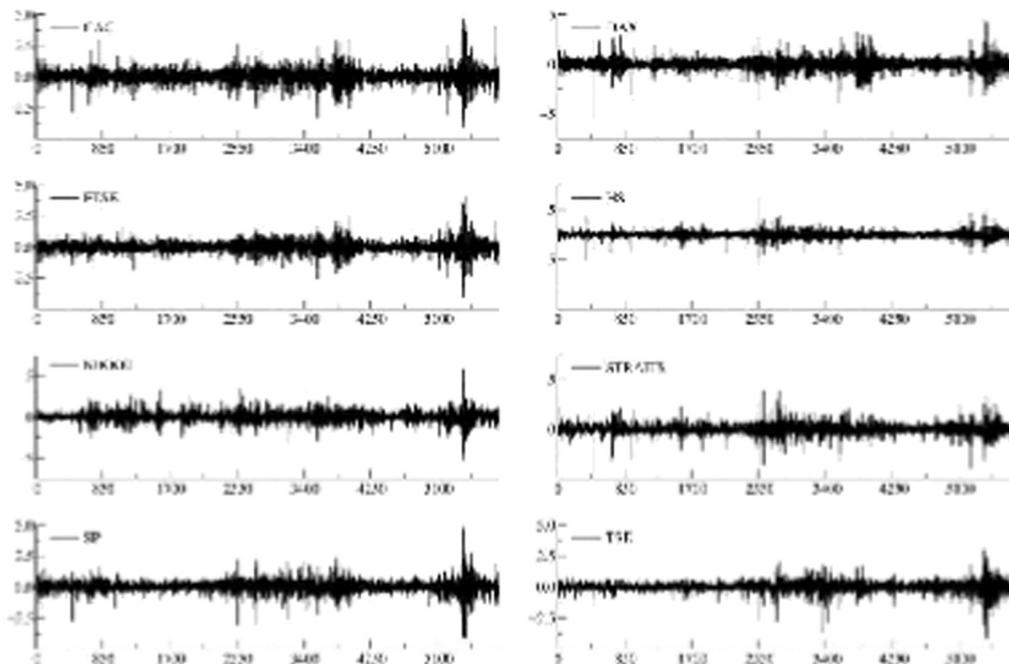
Table 20 presents the regression results. The AR(1) coefficient is always above 0.95, denoting very high correlation persistence. The intercept dummies are always positive and significant confirming the significant correlations' increase, which means contagion effects after both crises. For the SP–TSE pair the GFC dummy is insignificant when both dummies are included, so we run two regressions for each crisis dummy separately. We observe the lowest dummy coefficients with the smallest t-statistic for the US and Canada, which is in accordance with the t-test procedure for the DCCs mean difference. Our dynamic correlations analysis proves that both contagion tests are in favour of contagion rather than simple interdependence after the crisis shocks.

6. Conclusion

The purpose of the current analysis was to investigate the applicability of the multivariate FIAPARCH model with DCC to eight stock market indices returns, also taking into account the structural breaks corresponding to financial crisis events. The VAR-DCC-FIAPARCH model is proved to capture thoroughly the volatility and correlation processes compared to simpler specifications, like the multivariate GARCH with CCC.

We have provided strong evidence that conditional volatilities are better modelled incorporating long memory, power effects and leverage features. We further prove that time varying conditional correlations across markets, estimated by the DCC model, are highly persistent and follow a sound upward pattern during financial crises. The cross-border contagion effects depicted on the increasing correlations and the herding behaviour among investors as the correlations remain high confirm the existing empirical evidence. We also compare two different crises in terms of correlations to observe higher correlations in the recent Global financial crisis than in the Asian one. The financial liberalisation, deregulation and integration of the markets has led to more apparent market interdependence nowadays. Such a conclusion has major policy implications and a substantial impact on the current risk management practices.

Appendix A. Graphs

Fig. A.1. Returns graphs.
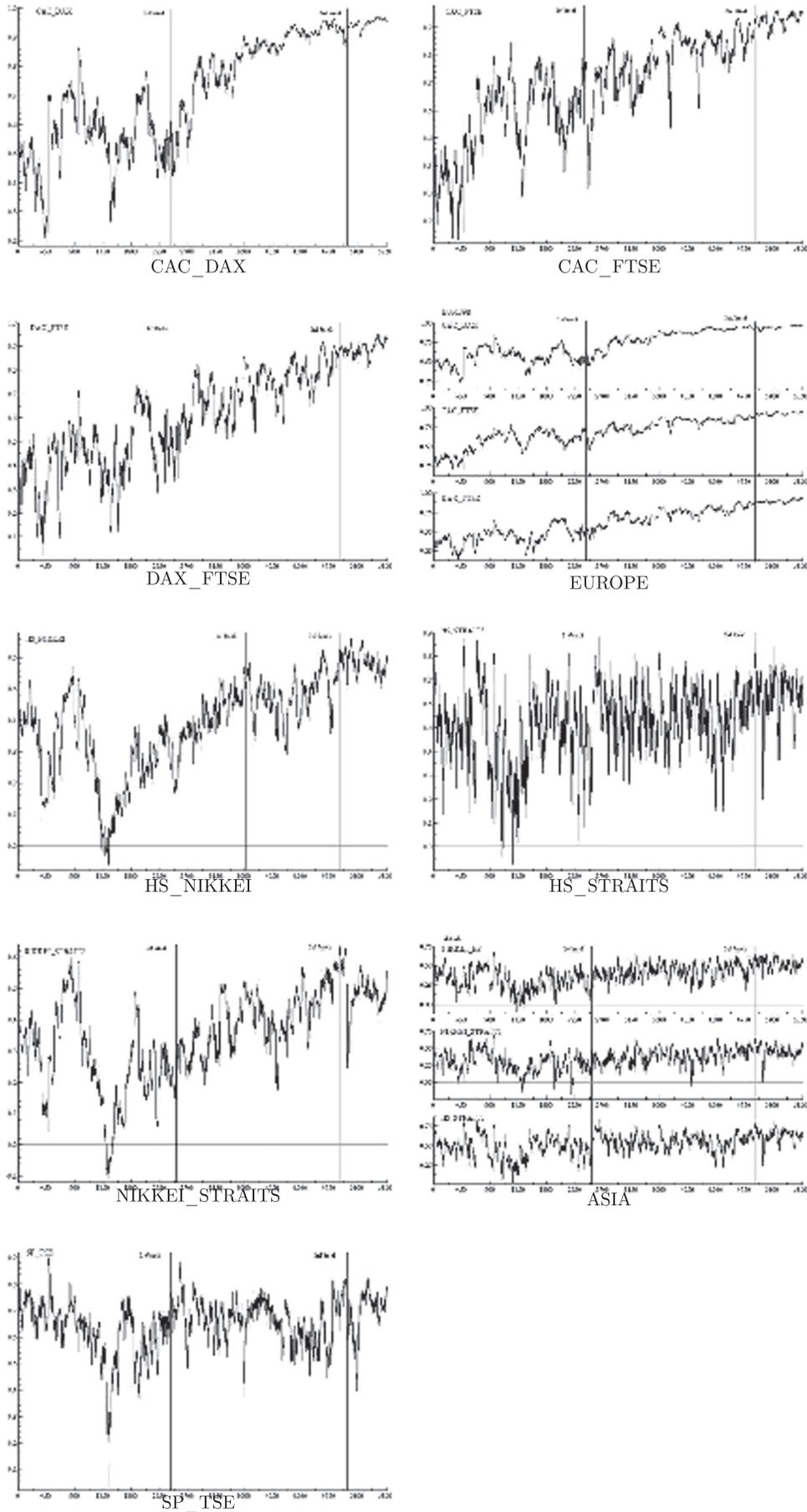
**Fig. A.2.** Dynamic conditional correlations graphs whole sample.
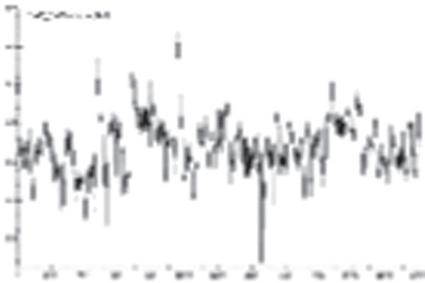
cac_dax A   cac_dax B   cac_dax C
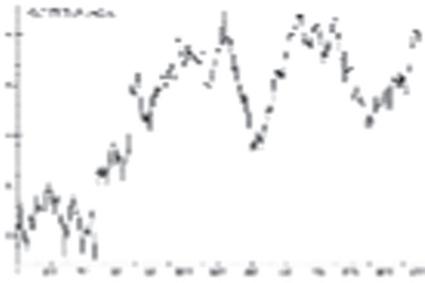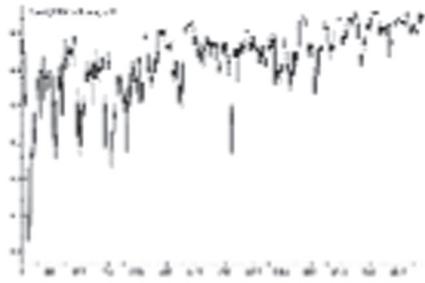
cac_ftse A   cac_ftse B   cac_ftse C
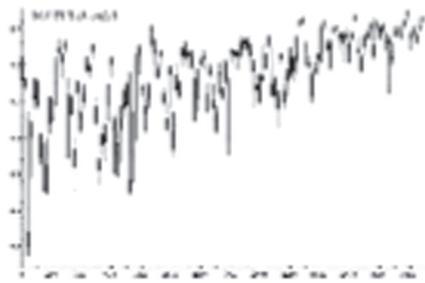
dax_ftse A   dax_ftse B   dax_ftse C

Europe A   Europe B

hs_nikkei A

hs_nikkei B

hs_nikkei C

hs_straits A

hs_straits B

hs_straits C

nikkei_straits A

nikkei_straits B

nikkei_straits C

asia A

asia B

sp_tse A
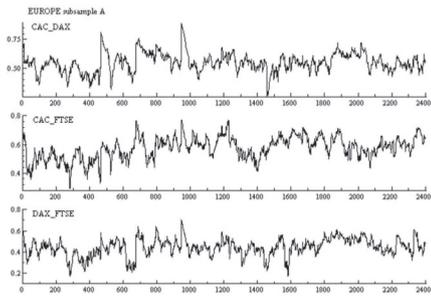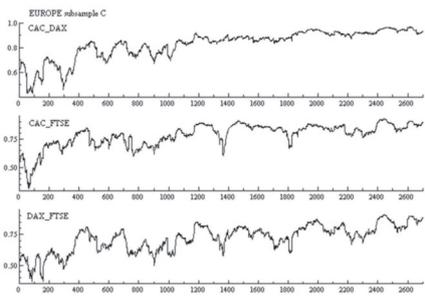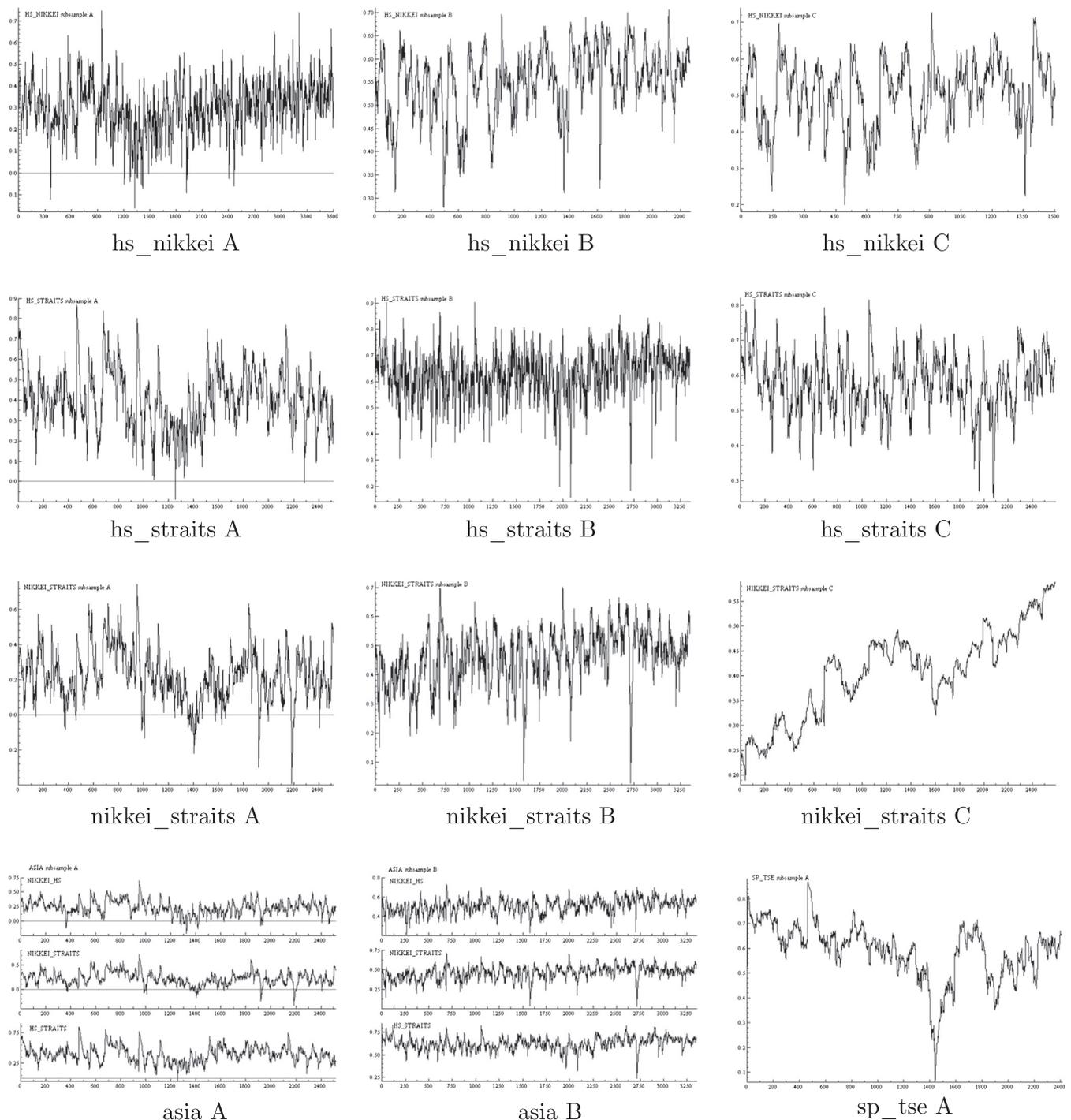
Fig. A.3 Dynamic conditional correlations graphs subsamples.

References

Aloui, C. (2011). Latin American stock markets' volatility spillovers during the financial crises: A multivariate FIAPARCH-DCC framework. *Macroeconomics and Finance in Emerging Market Economies, 4*, 289–326.

Andreou, E., & Ghysels, E. (2002). Detecting multiple breaks in financial market volatility dynamics. *Journal of Applied Econometrics, 17*, 579–600.

Ang, A., & Bekaert, G. (1999). *International asset allocation with time-varying correlations.* NBER working paper no. 7056.

Bai, J., & Perron, P. (2003). Computation and analysis of multiple structural change models. *Journal of Applied Econometrics, 18*, 1–22.

Baillie, R. T., Bollerslev, T., & Mikkelsen, H. O. (1996). Fractionally integrated generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics, 74*, 3–30.

Bartram, S. M., & Wang, Y. -H. (2005). Another look at the relationship between cross-market correlation and volatility. *Finance Research Letters, 2*, 75–88.

Bauwens, L., Laurent, S., & Rombouts, J. V. K. (2006). Multivariate GARCH models: A survey. *Journal of Applied Econometrics, 21*, 79–109.

Billio, M., & Pelizzon, L. (2003). Contagion and interdependence in stock markets: Have they been misdiagnosed? *Journal of Economics and Business, 55*, 405–426.

Bollerslev, T., & Mikkelsen, H. O. (1996). Modeling and pricing long memory in stock market volatility. *Journal of Econometrics, 73*, 151–184.

Boyer, B. H., Kumagai, T., & Yuan, K. (2006). How do crises spread? Evidence from accessible and inaccessible stock indices. *Journal of Finance, 61*, 957–1003.

Brown, M. B., & Forsythe, A. B. (1974). Robust tests for equality of variances. *Journal of the American Statistical Association, 69*, 364–367.

Cappiello, L., Engle, R. F., & Sheppard, K. (2006). Asymmetric dynamics in the correlations of global equity and bond returns. *Journal of Financial Econometrics, 4*, 537–572.

Chakrabarti, R., & Roll, R. (2002). East Asia and Europe during the 1997 Asian collapse: A clinical study of a financial crisis. *Journal of Financial Markets, 5*, 1–30.

Chiang, T. C., Jeon, B. N., & Li, H. (2007). Dynamic correlation analysis of financial contagion: Evidence from Asian markets. *Journal of International Money and Finance, 26*, 1206–1228.

Chiang, T. C., & Zheng, D. (2010). An empirical analysis of herd behavior in global stock markets. *Journal of Banking and Finance, 34*, 1911–1921.

Cho, J. H., & Parhizgari, A. M. (2008). East Asian financial contagion under DCC-GARCH. *The International Journal of Banking and Finance, 6*, 17–30.

Conrad, C. (2010). Non-negativity conditions for the hyperbolic GARCH model. *Journal of Econometrics, 157*, 441–457.

Conrad, C., & Haag, B. R. (2006). Inequality constraints in the fractionally integrated GARCH model. *Journal of Financial Econometrics, 4*, 413–449.

Conrad, C., Karanasos, M., & Zeng, N. (2011). Multivariate fractionally integrated APARCH modeling of stock market volatility: A multi-country study. *Journal of Empirical Finance, 18*, 147–159.

Corsetti, G., Pericoli, M., & Sbracia, M. (2001). *Correlation analysis of financial contagion: What one should know before running a test.* Yale University, Economic Growth Center discussion paper No. 822.

Den Haan, W.J., & Levin, A.T. (1998). Vector autoregressive covariance matrix estimation. University of California, San Diego, Unpublished Results.

Dimitriou, D., & Kenourgios, D. (2013). Financial crises and dynamic linkages among international currencies. *Journal of International Financial Markets Institutions and Money, 26*, 319–332.

Dimitriou, D., Kenourgios, D., & Simos, T. (2013). Global financial crisis and emerging stock market contagion: A multivariate FIAPARCH–DCC approach. *International Review of Financial Analysis, 30*, 46–56.

Ding, Z., Granger, C. W. J., & Engle, R. F. (1993). A long memory property of stock market returns and a new model. *Journal of Empirical Finance, 1*, 83–106.

Engle, R. F. (2002). Dynamic conditional correlation: Dynamic conditional correlation. A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics, 20*, 339–350.

Engle, R. F. (2009). *Anticipating correlations. A new paradigm for risk management.* Princeton and Oxford: Princeton University Press.

Engle, R. F., & Sheppard, K. (2001). *Theoretical and empirical properties of dynamic conditional correlation multivariate GARCH.* NBER working paper no. 8554.

Forbes, J. K., & Rigobon, R. (2002). No contagion, only interdependence: Measuring stock market comovements. *The Journal of Finance, 57*, 2223–2261.

Ho, K. -Y., & Zhang, Z. (2012). Dynamic linkages among financial markets in the greater China region: A multivariate asymmetric approach. *The World Economy, 35*, 500–523.

Inclán, C., & Tiao, G. C. (1994). Use of cumulative sums of squares for retrospective detection of changes of variance. *Journal of the American Statistical Association, 89*, 913–923.

Judge, G. G., Hill, R. C., Griffiths, W. E., Lütkepohl, H., & Lee, T. -C. (1988). *Introduction to the theory and practice of econometrics.* New York: John Wiley and Sons.

Karoglou, M. (2006). *On the detection of structural changes in volatility dynamics with applications.* Leicester: University of Leicester (PhD Thesis).

Karoglou, M. (2010). Breaking down the non-normality of stock returns. *The European Journal of Finance, 16*, 79–95.

Kazi, I. A., Guesmi, K., & Kaabia, O. (2011). *Contagion effect of financial crisis on OECD stock markets.* Kiel Institute for the World Economy, economics discussion papers, no. 2011–15.

Kenourgios, D., & Padhi, P. (2012). Emerging markets and financial crises: Regional, global or isolated shocks? *Journal of Multinational Financial Management, 22*, 24–38.

Kenourgios, D., & Samitas, A. (2011). Equity market integration in emerging Balkan markets. *Research in International Business and Finance, 25*, 296–307.

Kenourgios, D., Samitas, A., & Paltalidis, N. (2011). Financial crises and stock market contagion in multivariate time-varying asymmetric framework. *Journal of International Financial Markets Institutions and Money, 21*, 92–106.

Khan, S., & Park, K. W. (2009). Contagion in the stock markets: The Asian financial crisis revisited. *Journal of Asian Economics, 20*, 561–569.

Kokoszka, P., & Leipus, R. (2000). Change-point estimation in ARCH models. *Bernoulli, 6*, 1–28.

Kotkatvuori-Örnberg, J., Nikkinen, J., & Äijö, J. (2013). Stock market correlations during the financial crisis of 2008–2009: Evidence from 50 equity markets. *International Review of Financial Analysis, 28*, 70–78.

Laurent, S. (2007). *Estimating and forecasting ARCH models using G@RCH 5.* Timberlake Consultants.

Levene, H. (1960). Robust tests for equality in variances. In I. Olkin, S. G. Ghurye, W. Hoeffding, W. G. Madow, & H. B. Mann (Eds.), *Contributions to Probability and Statistics: Essays in Honor of Harold Hotelling* (pp. 278–292). California: Stanford University Press.

Lin, W. L., Engle, R. F., & Ito, T. (1994). Do bulls and bears move across borders? International transmission of stock returns and volatility. *The Review of Financial Studies, 7*, 507–538.

Longin, F., & Solnik, B. (2001). Extreme correlation of international equity markets. *Journal of Finance, 56*, 649–676.

Moldovan, I. (2011). Stock markets correlation: Before and during the crisis analysis. *Theoretical and Applied Economics, 18*, 111–122.

Sandoval, L. J., & Franca, I. de. P. (2012). Correlation of financial markets in times of crisis. *Physica A: Statistical Mechanics and its Applications, 391*, 187–208.

Sansó, A., Aragó,V., & Carrion-i-Silvestre, J.L. (2003). Testing for changes in the unconditional variance of financial time series. Universitat de les illes Balears, Unpublished Results.

Sheskin, D. J. (2004). *Handbook of parametric and nonparametric statistical procedures.* Boca Raton, Florida: Chapman & Hall CRC Press.

Siegel, S., & Tukey, J. W. (1960). A nonparametric sum of ranks procedure for relative spread in unpaired samples. *Journal of the American Statistical Association, 55*, 429–445.

Silvennoinen, A., & Teräsvirta, T. (2009). Multivariate GARCH models. In T. G. Andersen, R. A. Davis, J. -P. Kreiß, & T. Mikosch (Eds.), *Handbook of Financial Time Series* (pp. 201–228). Berlin: Springer-Verlag.

Sokal, R. R., & Rohlf, F. J. (1995). *Biometry: The principles and practice of statistics in biological research.* New York: W.H. Freeman and Co.

Syllignakis, M. N., & Kouretas, G. P. (2011). Dynamic correlation analysis of financial contagion: Evidence from the Central and Eastern European markets. *International Review of Economics & Finance, 20*, 717–732.

Tse, Y. K. (1998). The conditional heteroscedasticity of the yen–dollar exchange rate. *Journal of Applied Econometrics, 13*, 49–55.

Tse, Y. K., & Tsui, A. K. C. (2002). A Multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations. *Journal of Business & Economic Statistics, 20*, 351–362.

Yang, T., & Lim, J. J. (2004). *Review of Pacific Basin Financial Markets and Policies, 7*, 119–151.